Crossing Relations for Double-Gluon Emission Antenna Functions

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1 Aims & Objectives

In this project we explore the physics behind double gluon emission from quark - antiquark pairs. The likelihood of any given configuration of this system is calculated through its *matrix element*, depicted visually on the left hand side of Figure 1. This matrix element is quite complex, however in certain *soft* and *collinear* physical configurations (Section 2.2) it can be simplified using the A_4^0 antenna function (Section 2.1). A_4^0 contains singularities and poles at various points in its domain (referred to as its *phase space*), corresponding to limiting cases of important physical states. Exploring these singularities and poles within the structure of A_4^0 is important as it allows us to characterise crucial underlying particle interactions. Moreover, determining the antenna function's structure in these singular limits may provide a means of overestimating the original function, useful for application in Monte Carlo simulations. This overestimate must be verified, which can be accomplished by evaluating these functions at uniformly sampled points within the phase space (Section 2.3).

2 Investigations

2.1 Crossing Relations for A_4^0

In certain physical configurations (Section 2.2), matrix elements of complex physical processes can be decomposed into simpler components using structures called *antenna functions*, as demonstrated in Figure 1. On the left hand side, we have the matrix element squared of all possible Feynman diagrams involving double gluon emission from quark - antiquark pairs, which has been decomposed into the simpler matrix element of quark - antiquark annihilation, multiplied by the antenna function. This antenna function for double gluon emission from $q\bar{q}$ pairs is denoted A_4^0 in [2] (henceforth GGG). The antenna function for $q\bar{q}$ pairs in the final state is available in GGG, however we want to explore configurations with $q\bar{q}$ pairs in the initial state. To derive this, we use relations from crossing symmetry demonstrated in Figure 2.

The A_4^0 from GGG is implemented in *Mathematica*, and algebraic substitutions have been implemented to obtain our desired antenna function. We will verify that these substitutions have resulted in the correct antenna function using methods described in Section 2.3.

2.2 Double Soft & Triple Collinear Limits

Soft gluon emission refers to configurations where a gluon is emitted with negligible momentum that goes to zero. Alternatively, the *collinear* limit refers to emission of a gluon whose momentum becomes collinear with that of its parent. In these limits, the matrix-element structure simpifies such that we can apply the factorisation of Figure 1.

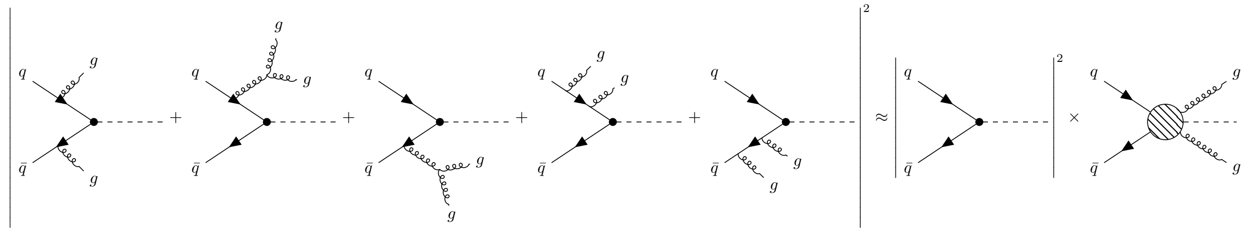

Figure 1: Factorisation of the $q + \bar{q} \to g + g + Z$ matrix element using the A_4^0 antenna function.

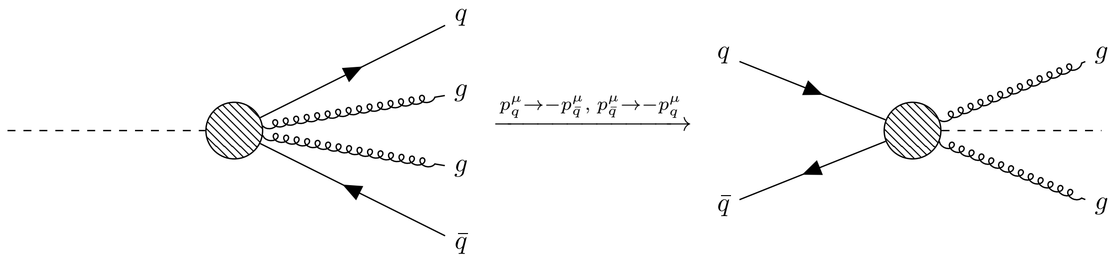

Figure 2: *Left:* Feynman diagram where the quark, antiquark and gluons are all in the final state. *Right:* Feynman diagram where the quark and antiquark are in the initial state, with both gluons in the final state.

However, these antenna functions themselves contain further interesting structure. Considering the left hand side of Figure 1, squaring the matrix element results in the cross-multiplication and product between different Feynman diagrams. In many physical configurations it is a single diagram that dominates all others, such that these cross-multiplied terms can be ignored. However, in the case of *double soft emission* where both gluons are emitted with negligible momenta, these gluons become so similar that different diagrams begin to converge. This has the effect of increasing the cross-multiplied terms, resulting in *interference effects*. Similar interference effects arise in the *triple collinear* limit, where both gluon momenta become collinear with that of the quarks.

We can parameterize soft gluons with the transformation $p \to \lambda p$ for each soft momenta, then taking the limit $\lambda \to 0$. Similarly, we parameterize a gluon and quark becoming collinear by substituting $p_q \cdot p_g \to \lambda p_q \cdot p_g$ and again with $\lambda \to 0$. These λ factors then appear in the denominators of A_4^0, such that *Mathematica* can be used to obtain series expansions of A_4^0 about $\lambda = 0$, denoted the *double soft* and *triple collinear* limits of A_4^0 depending on which substitution has been used. This expansion enables us to explore the mathematical structure of these interference effects, while also providing likely candidates to overestimate the original function.

2.3 Phase Space Sampling

To verify that these derived functions are indeed correct, we compare numerical results of our functions to directly computed matrix elements (left hand side of Figure 1) available in *PYTHIA* software [3]. For these comparisons we rely on the RAMBO algorithm [1], also implemented in *PYTHIA*, to generate uniformly random configurations of $q\bar{q}$ and gluons while still obeying energy and momentum conservation. We then compare the numerical results of the matrix element against our functions evalutated at these random phase-space points to discern agreeance.

3 Future Plans

PYTHIA software is written in $C++$, thus we have exported the A_4^0 function and derived limits from *Mathematica* into $C++$. We are currently configuring a test of 100,000 randomly generated phase-space points, at each point recording the ratio of matrix element to A_4^0. This ratio should converge to values corresponding to the simplified matrix element on the right hand side of Figure 1. We then require some criteria with which we can determine if a given phase-space point lies a domain corresponding to soft or collinear configurations. Using this criterion, we apply a similar testing of the soft and collinear limits in these domains to verify their accuracy. If these functions can be verified as overestimates, this enables Monte Carlo integration of the original, complex matrix element to further explore these emission processes.

References

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[3] Torbjörn Sjöstrand, Stefan Ask, Jesper R. Christiansen, Richard Corke, Nishita Desai, Philip Ilten, Stephen Mrenna, Stefan Prestel, Christine O. Rasmussen, and Peter Z. Skands. An introduction to PYTHIA 8.2. *Computer Physics Communications*, 191:159–177, Jun 2015.