

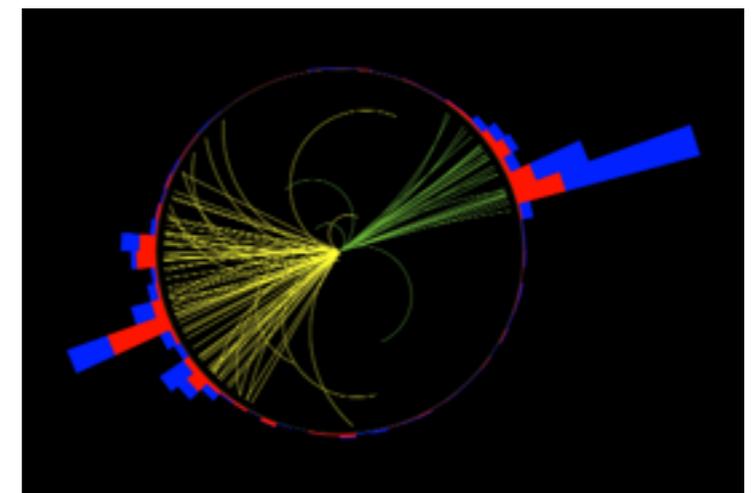
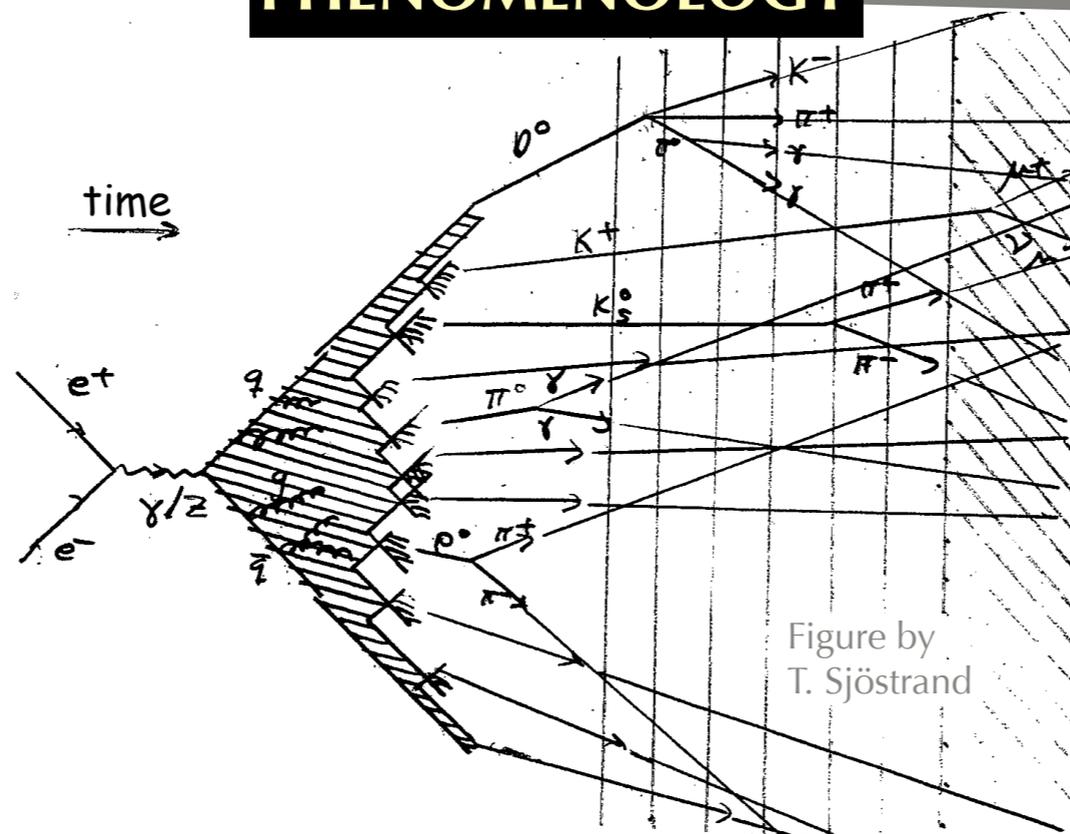
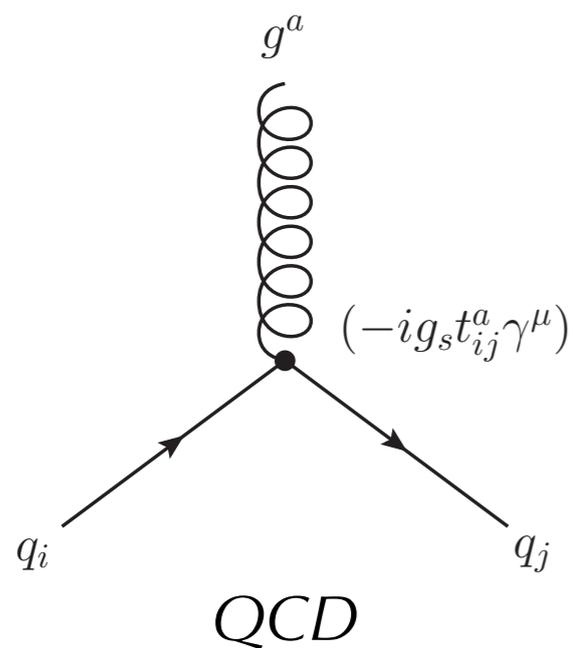
# Particle Physics Phenomenology

Peter Skands, Monash University

**THEORY**

**PHENOMENOLOGY**

**EXPERIMENT**



"Jets"

**INTERPRETATION**

October 1-2, 2015, Sydney  
Spring School on Particle Physics & Cosmology



# The Phenomenology Pipeline

→ Come up with theory **idea** (e.g., SM, SUSY, QGP, ~~CP~~, ...)

... should be testable in experiments

Formulate phenomenological **model** (based on theoretical ideas)

... working-hypothesis physics model capturing essence of idea

Propose new sensitive **observables** (based on models)

... which can be measured in experiments

Make (detailed and precise) **calculations**

... which can be compared (statistically) against experiments

Provide theoretical **interpretations**

... of the experimental results



# Masses (& units)

The main particle-physics units of energy is MeV, GeV, TeV

1 electron-Volt = kinetic energy obtained by an electron accelerated by potential difference of 1 Volt

$$1 \text{ eV} = Q_e \cdot 1 \text{ V} = 1.602176565(35) \times 10^{-19} \text{ C} \cdot 1 \text{ J/C} = 1.6 \times 10^{-19} \text{ J}$$

(So for accelerators, the beam energy in eV is a measure of the equivalent electrostatic potential difference, for unit charge)

Planned **linear** accelerators (ILC, CLIC) could reach  $E_{\text{CM}} \sim 1000 \text{ GeV}$ .

The highest-energy (**circular**) accelerator LHC  $\sim 6500 \text{ GeV/beam}$ .

Using  $E=mc^2$  we typically express mass in units of  $\text{eV}/c^2$

$$m_e = 9.11 \times 10^{-31} \text{ kg} = 0.511 \text{ MeV}/c^2$$

$$m_\mu = 106 \text{ MeV}/c^2$$

$$m_\tau = 1780 \text{ MeV}/c^2$$

$$m_{\text{proton}} = 938 \text{ MeV}/c^2 \sim 1 \text{ GeV}/c^2$$

*(sometimes we don't even say the  $1/c^2$ ; it is implied by the quantity being mass)*

# Natural Units

FOR A RELATIVISTIC  
QUANTUM THEORY

In fact, we use MeV and GeV for *everything!*

**Define** a set of units in which  $\hbar = c = 1$

Action [Energy\*Time] : dimensionless ( $\hbar = 1$ )

All actions are measured in units of  $\hbar$

Velocity [Length/Time] : dimensionless ( $c = 1$ )

All velocities are measured in units of  $c$  (i.e.,  $\beta = v/c$ )

Energy : dimension 1

Mass : dimension 1 ( $E=m$ )

E.g.,  $m_p = 0.94$  GeV; masses  $\sim$  measured in units of  $m_p$

Time : dimension -1 ( $\Delta E \Delta t \geq 1$ ;  $E = 2\pi \nu$ )

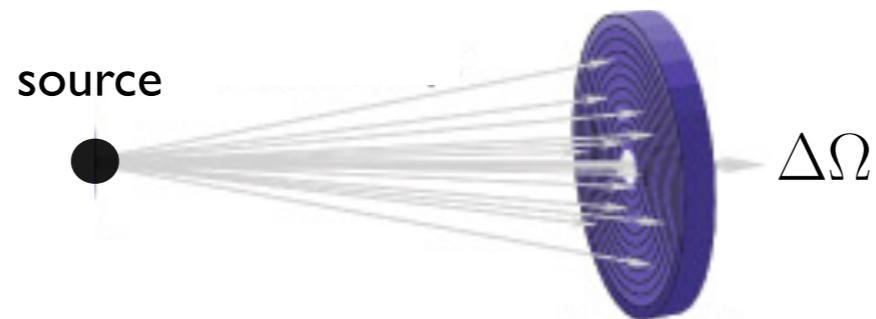
Length : dimension -1 (velocity is dimensionless)

Momentum : dimension 1 ( $\Delta p \Delta x \geq 1$ )

Example:  
lengths  $\rightarrow$  energies

	$\lambda$	$\rightarrow$	E
HEP	< 1 fm		> 1 GeV
gamma	1 pm		1 MeV
X-rays	0.1 nm		10 keV
UV	100 nm		10 eV

# Scattering Experiments



LHC detector  
Cosmic-Ray detector  
Neutrino detector  
X-ray telescope  
...

→ Integrate differential **cross sections**  
over specific **phase-space** regions

Predicted number of counts  
= integral over solid angle

$$N_{\text{count}}(\Delta\Omega) \propto \int_{\Delta\Omega} d\Omega \frac{d\sigma}{d\Omega}$$

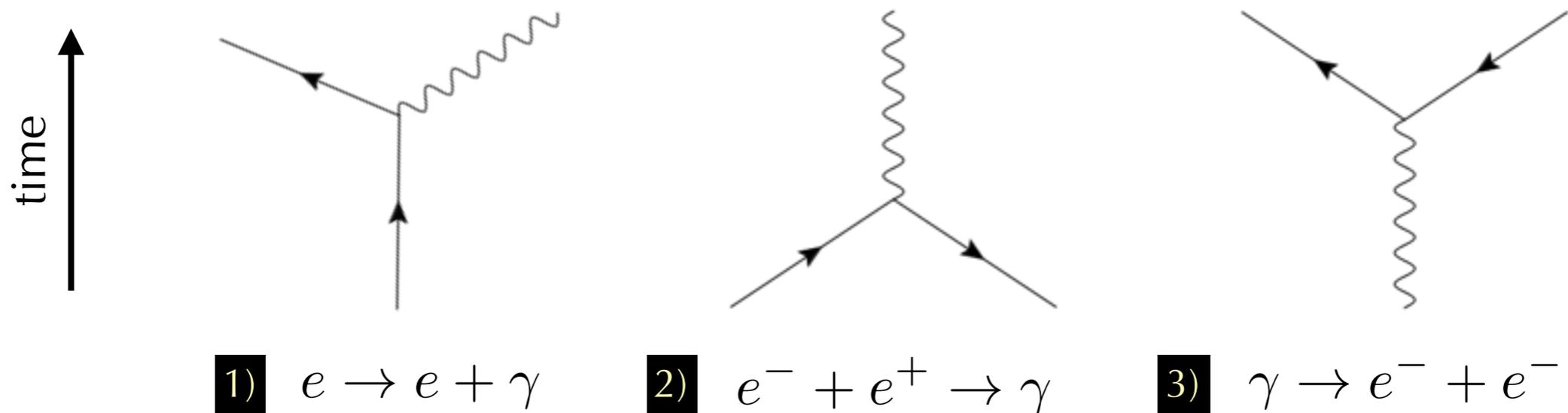
$$d\Omega = d \cos \theta d\phi$$

**In particle physics:**  
Integrate over all quantum histories  
(+ interferences)

# Preview of Interacting Quantum Field Theory

Consider Electromagnetism = electron-photon interactions

All based on the same vertex



What about 4-momentum conservation?

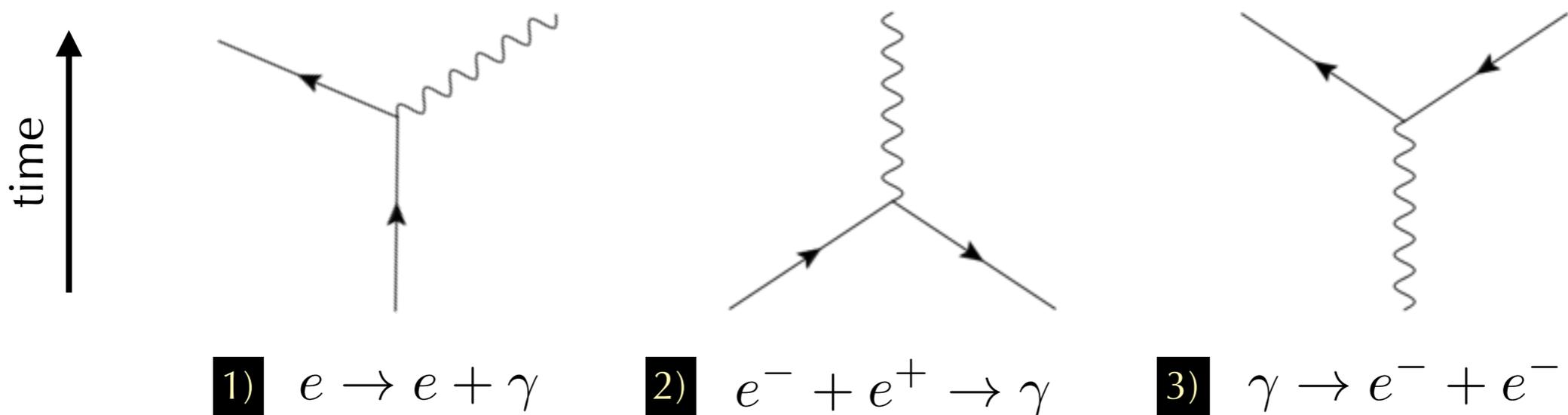
- 1) Electron at rest decaying to a recoiling electron + a photon?
- 2) Two massive particles reacting to produce a massless photon?
- 3) Massless photon decaying to two massive electrons?

This all sounds very strange (even for relativity)

# Virtual Particles

Let us consider first the pure electromagnetic interactions

All based on the same vertex



What about 4-momentum conservation?

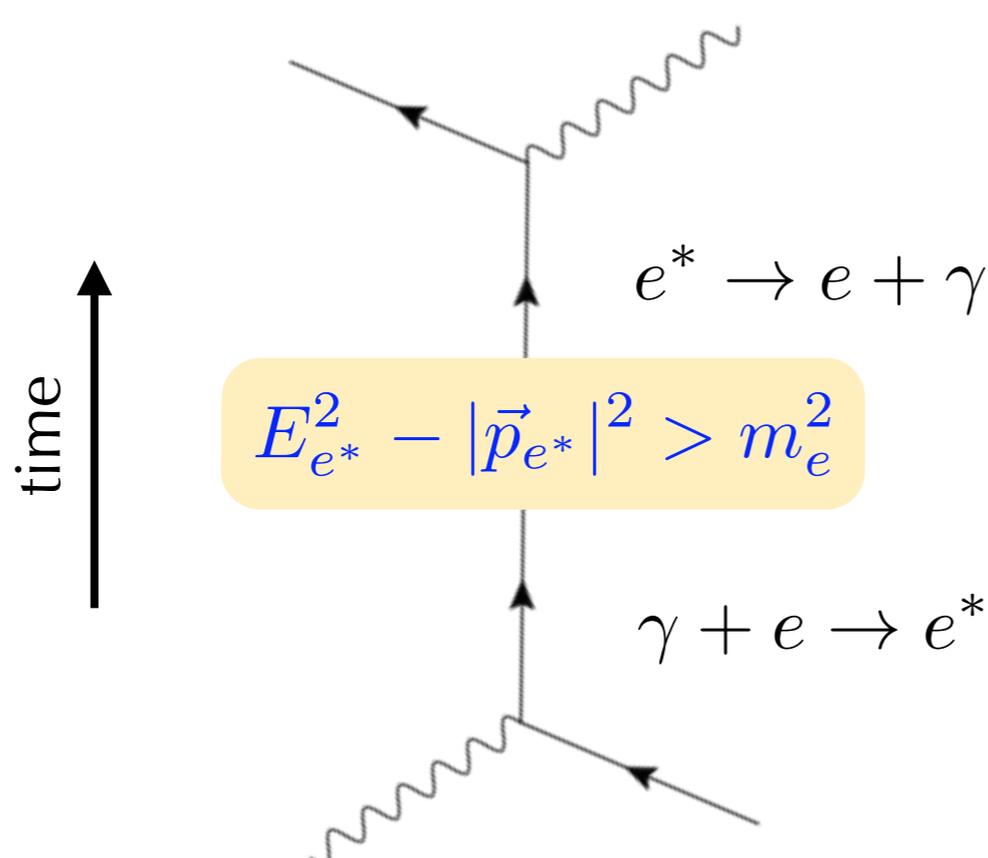
At least one of the involved particles must have  $E^2 - p^2 \neq m^2$   
(Can exist for a brief time due to Heisenberg)

We call such particles **virtual**; and say they are **off mass shell**

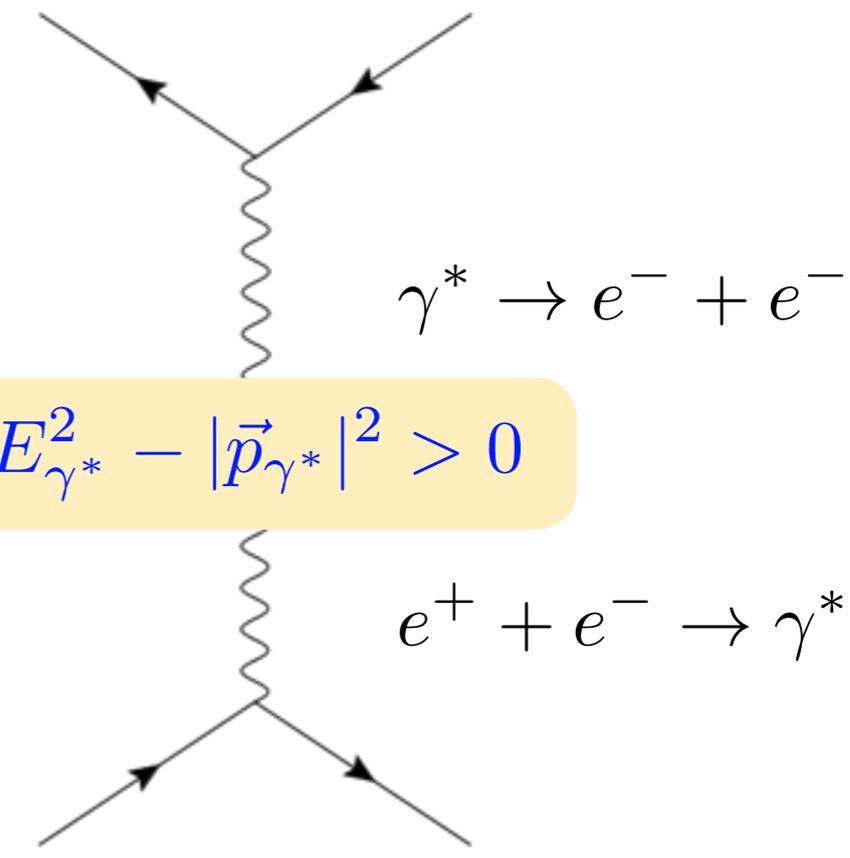
# Virtual Particles: Examples

Stitch vertices together to form **Feynman diagrams**

**EXTERNAL PARTICLES: "ON SHELL";  
INTERNAL ONES CAN BE "OFF SHELL"**

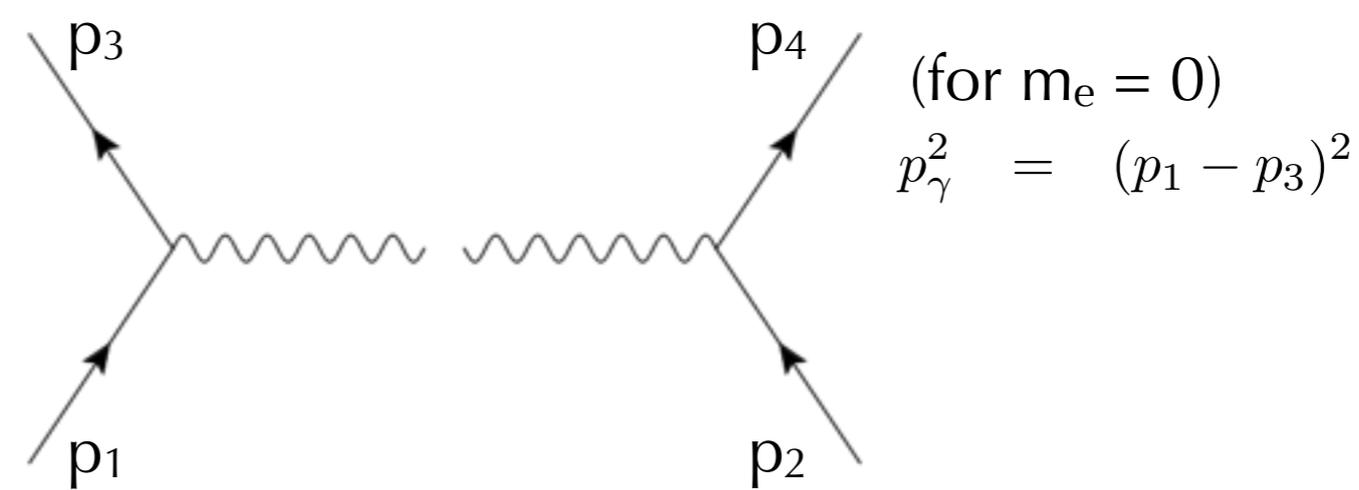


$$E_{e^*}^2 - |\vec{p}_{e^*}|^2 > m_e^2$$



$$E_{\gamma^*}^2 - |\vec{p}_{\gamma^*}|^2 > 0$$

- A)  $E_{\gamma^*}^2 - |\vec{p}_{\gamma^*}|^2 > 0 ?$
- B)  $E_{\gamma^*}^2 - |\vec{p}_{\gamma^*}|^2 < 0 ?$



# Feynman Diagrams

## Quantum Field Theory (QFT)

We use **Feynman diagrams** to draw the possible histories

These are symbolic (correspond to state changes in the underlying QFT)

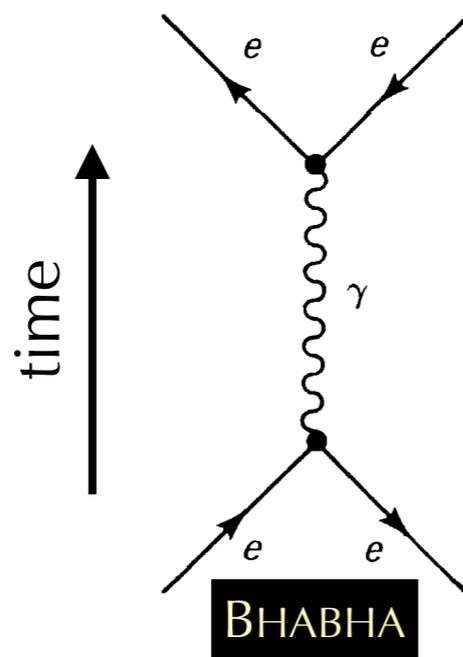


Diagram representing QM amplitude for:

*“an electron and a positron annihilated to produce a (virtual) photon, which then split back up into an  $e^+ e^-$  pair again.”*

$$e^-(p_1) + e^+(p_2) \rightarrow e^-(p_3) + e^+(p_4)$$

**EXAMPLE (UNITS IN GeV):**

$$p_1 = (5, 0, 0, 5)$$

$$p_2 = (5, 0, 0, -5)$$

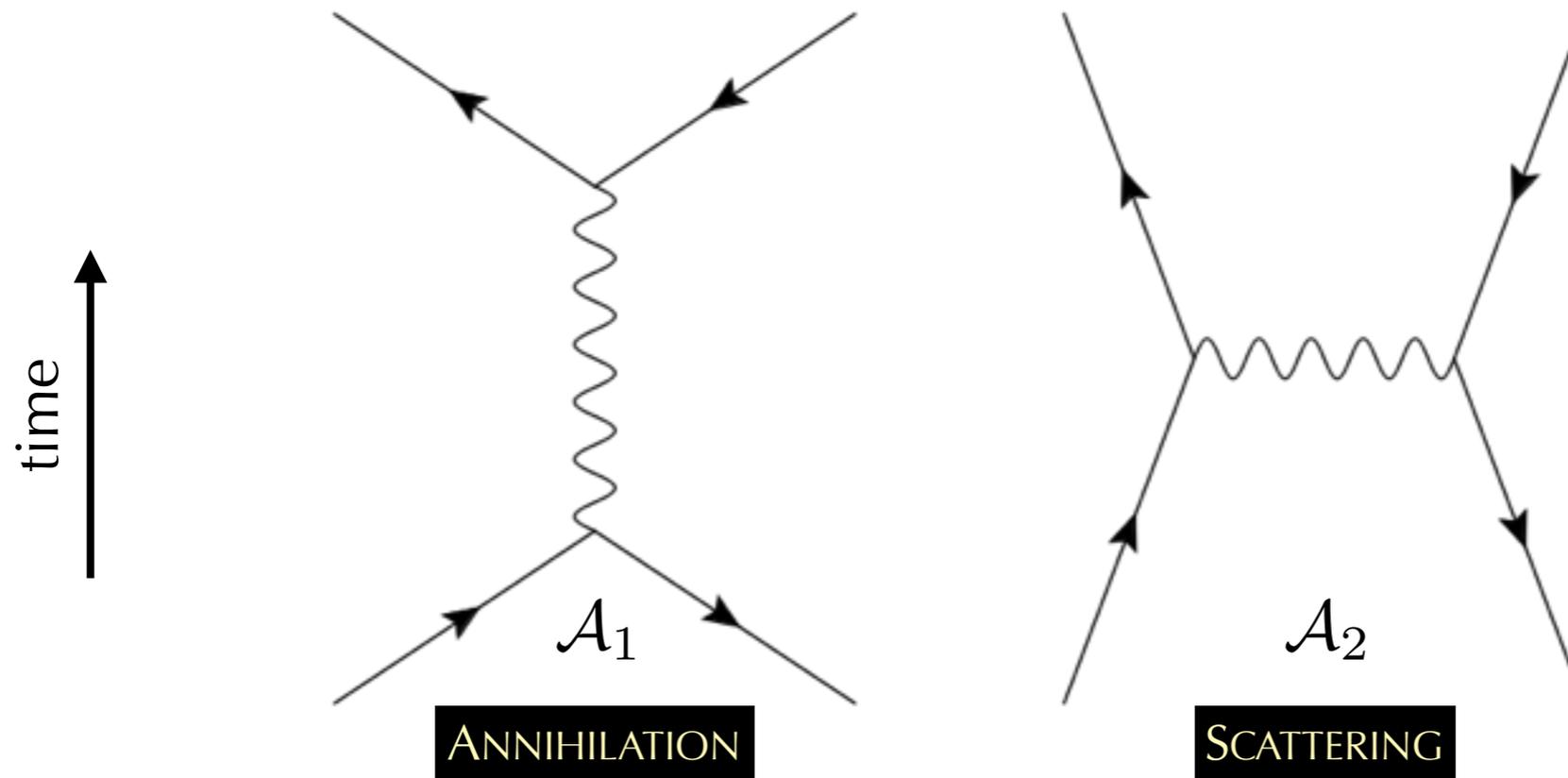
$$p_3 = (5, 0, 3, 4)$$

$$p_4 = (5, 0, -3, -4)$$

E.g., in Bhabha scattering, the force is attractive (an electron and a positron attract) but we still draw them as heading away from each other)

# (Interferences)

Actually, two Quantum “histories” contribute to Bhabha



Must sum both amplitudes; then square to get probability  
(two “paths”; analogously to double-slit experiment)

$$|\mathcal{A}|^2 = |\mathcal{A}_1 + \mathcal{A}_2|^2 = |\mathcal{A}_1|^2 + |\mathcal{A}_2|^2 + 2\text{Re}[\mathcal{A}_1 \mathcal{A}_2^*]$$

Q.M. interference

# Physical Observables = Event Rates

## Scattering : cross sections

Want to express scattering probability independently of the intensity (flux) of incident particles (beams)

$$N_{\text{events}} = \text{Probability-per-particle} * \text{Number-of-particles}$$

$$\rightarrow N_{\text{events}} = \text{Probability [area/particle]} * N_{\text{particles/area}}$$

$$\rightarrow N_{\text{events/time}} = \text{Probability [area/particle]} * N_{\text{particles/area/time}}$$

$$\text{Event Rate} = \text{Cross Section} * \text{Luminosity}$$

Measure in Experiment  
Compare with Prediction

Calculate from  
fundamental theory

Determined by  
accelerator parameters.  
In principle measurable  
(eg Van der Meer scans)

" $\sigma$ "

# Physical Observables = Event Rates

## Disintegration : decay rates

Murphy's law for particles: anything that **can** decay, **will** decay

What we actually measure is typically a cross section times a branching fraction

E.g., the event rate for  $h^0 \rightarrow \gamma\gamma$  observed at LHC is compared to a theoretical calculation of

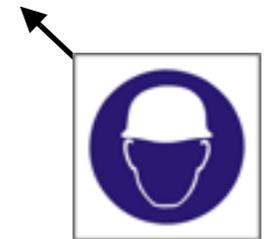
$$N(h^0 \rightarrow \gamma\gamma)_{\text{LHC}} = \sigma(pp \rightarrow h^0) * \text{BR}(h^0 \rightarrow \gamma\gamma) * L_{pp} * \langle \text{efficiency} \rangle$$

## How does a particle decay?

It sits in its rest frame and gets time evolved, by  $e^{iHt}$

Unstable  $\rightarrow$  H contains operators that want to kill it ...

They compete about which one goes first (can only decay once)



# Decay Rates

Particles are **elementary**, indistinguishable.

Any “history” information is encoded in their quantum numbers

An “old” particle doesn’t have a higher probability of decaying in the next second than a “young” one

What matters is the instantaneous decay rate per unit time

$$dN = -\Gamma N dt \quad \rightarrow \quad N(t) = N(0)e^{-\Gamma t} \quad \rightarrow \quad \tau = \frac{1}{\Gamma}$$

Generalise to multiple different decay modes

$$\Gamma = \sum_i \Gamma_i$$

$\Gamma_i$  : “Partial Width”

Branching Ratio : BR( $i$ ) =  $\frac{\Gamma_i}{\sum_j \Gamma_j}$   
or “branching fraction”

**$\pi^+$  DECAY MODES** K.A. Olive et al. (Particle Data Group), Chin. Phys. C38, 090001 (2014) (URL: <http://pdg.lbl.gov>)

	Mode	Fraction ( $\Gamma_i/\Gamma$ )	Confidence level
$\Gamma_1$	$\mu^+ \nu_\mu$	[a] (99.98770 ± 0.00004) %	
$\Gamma_2$	$\mu^+ \nu_\mu \gamma$	[b] ( 2.00 ± 0.25 ) × 10 <sup>-4</sup>	
$\Gamma_3$	$e^+ \nu_e$	[a] ( 1.230 ± 0.004 ) × 10 <sup>-4</sup>	
$\Gamma_4$	$e^+ \nu_e \gamma$	[b] ( 7.39 ± 0.05 ) × 10 <sup>-7</sup>	
$\Gamma_5$	$e^+ \nu_e \pi^0$	( 1.036 ± 0.006 ) × 10 <sup>-8</sup>	
$\Gamma_6$	$e^+ \nu_e e^+ e^-$	( 3.2 ± 0.5 ) × 10 <sup>-9</sup>	
$\Gamma_7$	$e^+ \nu_e \nu \bar{\nu}$	< 5 × 10 <sup>-6</sup>	90%

example from the “PDG book” [pdg.lbl.gov](http://pdg.lbl.gov)

# + Kinematics (E & p cons)

## Thresholds

An object cannot be produced unless the colliding particles have enough CM energy to create its rest mass

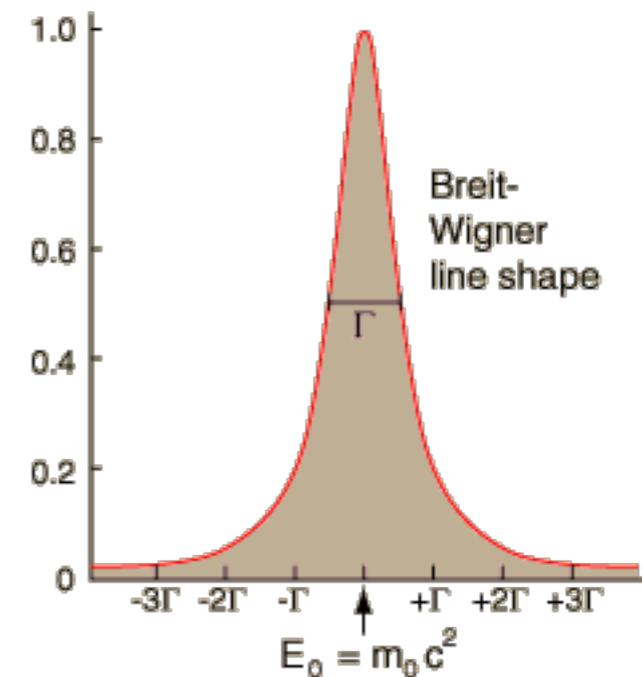
An object cannot decay to any (combination of) particles heavier than itself

## Unless ...

*Heisenberg*: the energy is uncertain...

If a particle is unstable (has a non-zero decay rate), then we at most have the duration of its life to measure its energy.

Analogous to line-broadening of lines in spectra of excited atoms



→ Shapes like this: “Breit-Wigner” “resonances”

# The Cross Section

## Quantity of interest:

Effective cross-sectional area presented by a “target particle” to a stream of “incident particles”

Relativity → must get the same if we swap the roles of incident and target particles, or in any other frame

So more precisely it's really the cross-sectional area two streams of particles present to each other



## Complications

This isn't classical physics: each particle has a probability to go through the target unaffected, + all possible scatterings

A plane wave comes in

An interaction Hamiltonian (of which the incoming plane wave is not an eigenstate) evolves it for a while

→ the evolved state is a superposition of **all possible outgoing states**

+ not only **elastic** scattering. Creation + annihilation : **inelastic**.

# Scattering off a Hard Spherical Cow

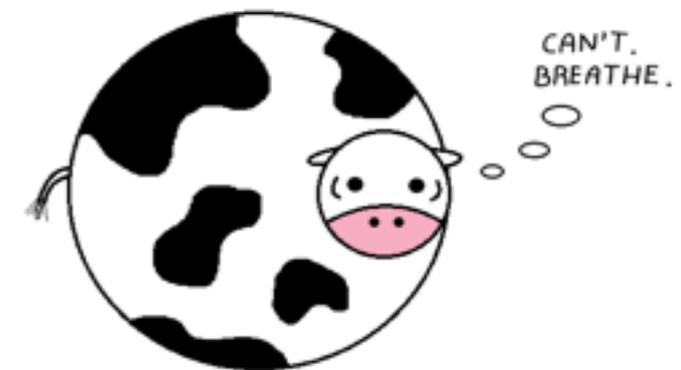
Assume a spherical cow of uniform density.



...while ignoring the effects of gravity.



...in a vacuum.



**bastard theoretical physicists**  
How do you sleep at night?

What's the total cross section?  
(Scattering off a hard sphere)

Generalise to quantum scattering of relativistic particles: *Quantum Field Theory*

# Fermi's Golden Rule

Two basic ingredients to **calculate** decay rates and cross sections

1) The **amplitude** for the process:  $\mathcal{M}$

Contains all the *dynamical* information; couplings, propagators, ...  
Calculated by evaluating the relevant Feynman Diagrams, using the  
“**Feynman Rules**” for the interaction(s) in question

2) The **phase space** available for the process

Contains only *kinematical* information;  
Depends only on external masses, momenta, energies;  
“Counts” the number/density of available final states

**The Golden Rule is\*:**  $\text{transition rate} = \frac{2\pi}{\hbar} |\mathcal{M}|^2 \times (\text{phase space})$

\*For a derivation, see QM (nonrelativistic) or QFT (relativistic)

# Pheno at the LHC

## Many from One (well ... from Two, really)

Quantum processes convert the kinetic energy of the beam particles into rest energy (mass) + momentum of outgoing particles

$$E = mc^2 \sqrt{1 + p^2 / (m^2 c^2)}$$

*E = energy*

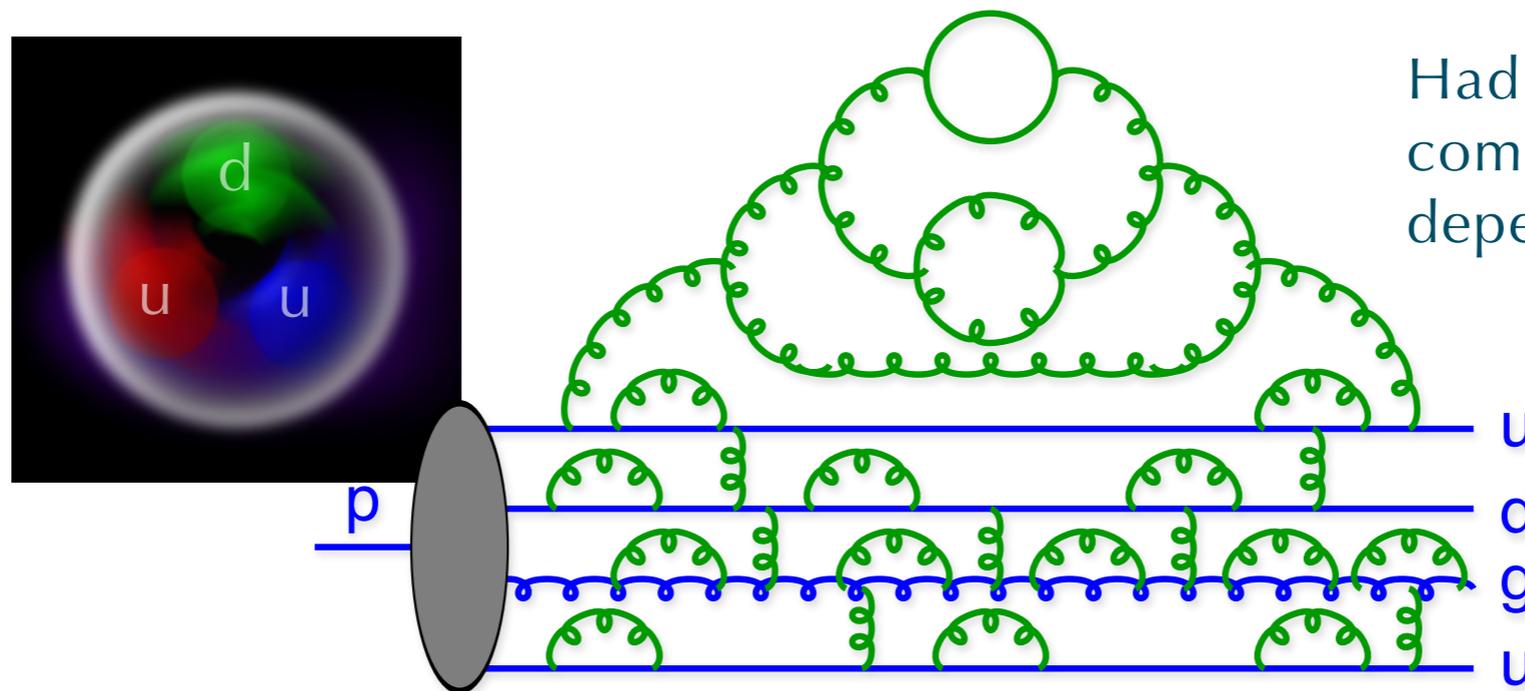
*m = mass*

*p = momentum*

*c = speed of light*

## What are we really colliding?

Take a look at the quantum level



Hadrons are composite, with time-dependent structure

# Pheno at the LHC

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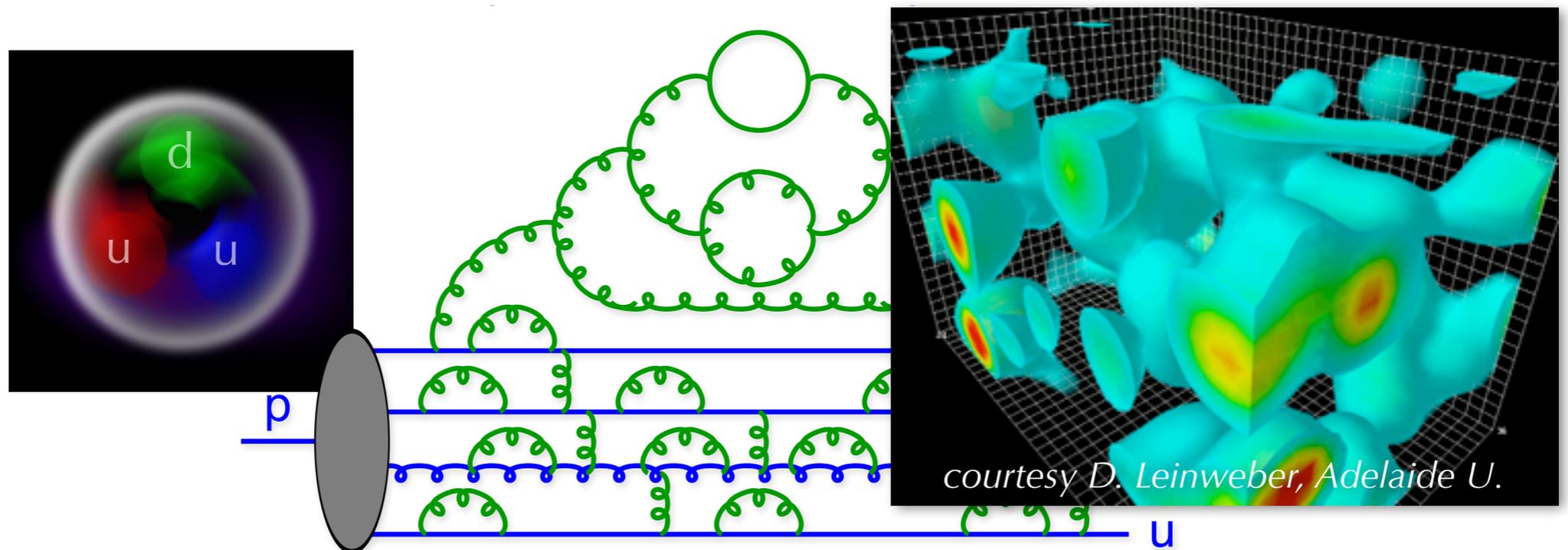
*m = mass*

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*c = speed of light*

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# Such Stuff as Beams are Made Of

Lifetime of typical fluctuation  $\sim r_p/c$  (=time it takes light to cross a proton)

$\sim 10^{-23}$  s; Corresponds to a frequency of  $\sim 500$  billion THz

To the LHC, that's slow! (reaches "shutter speeds" thousands of times faster)

Planck-Einstein:  $E=h\nu \rightarrow \nu_{\text{LHC}} = 13 \text{ TeV}/h = 3.14$  million billion THz

→ Protons look "frozen" at moment of collision

But they have a lot more than just two "u" quarks and a "d" inside

Hard to calculate, so use statistics to parametrise the structure: **parton distribution functions** (PDFs)

*Every so often I will pick a gluon, every so often a quark (antiquark)*

**Measured** at previous colliders, as function of energy fraction

Then compute the probability for all possible quark and gluon reactions and compare with experiments ...

# Rates and Triggers



We get  $\sim 40$  million collisions / sec.

We can save  $\sim 100$  / sec to disk.

## *WHICH ONES?*

Automated “trigger” systems decide which collisions may be interesting

## Not all reactions are created equally

The most likely collision type is  $gg \rightarrow gg$

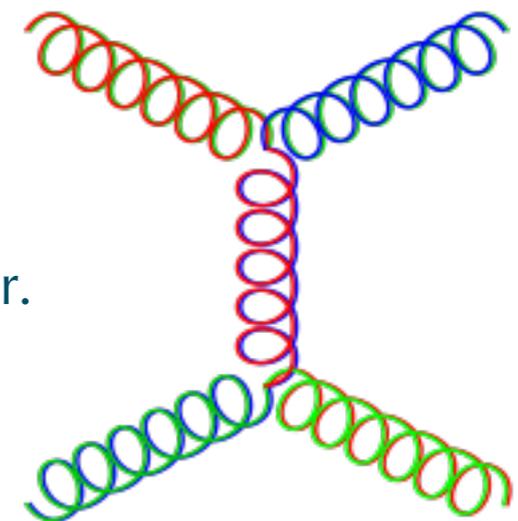
The top quark is the heaviest elementary particle

Discovered in 1995 by Fermilab’s “Tevatron” accelerator.

The LHC can make  $\sim 1$  top quark / second.

The reaction  $gg \rightarrow \text{Higgs}$  will happen  $\sim 1$  / minute

We don’t want to lose too many of them ...



Easy to collect millions of events of “high-cross-section-physics”

→ Test models of “known physics” to high precision

Triggers target the *needles in the haystack*

Trigger on signatures of decays of heavy particles, violent reactions

“Photons”

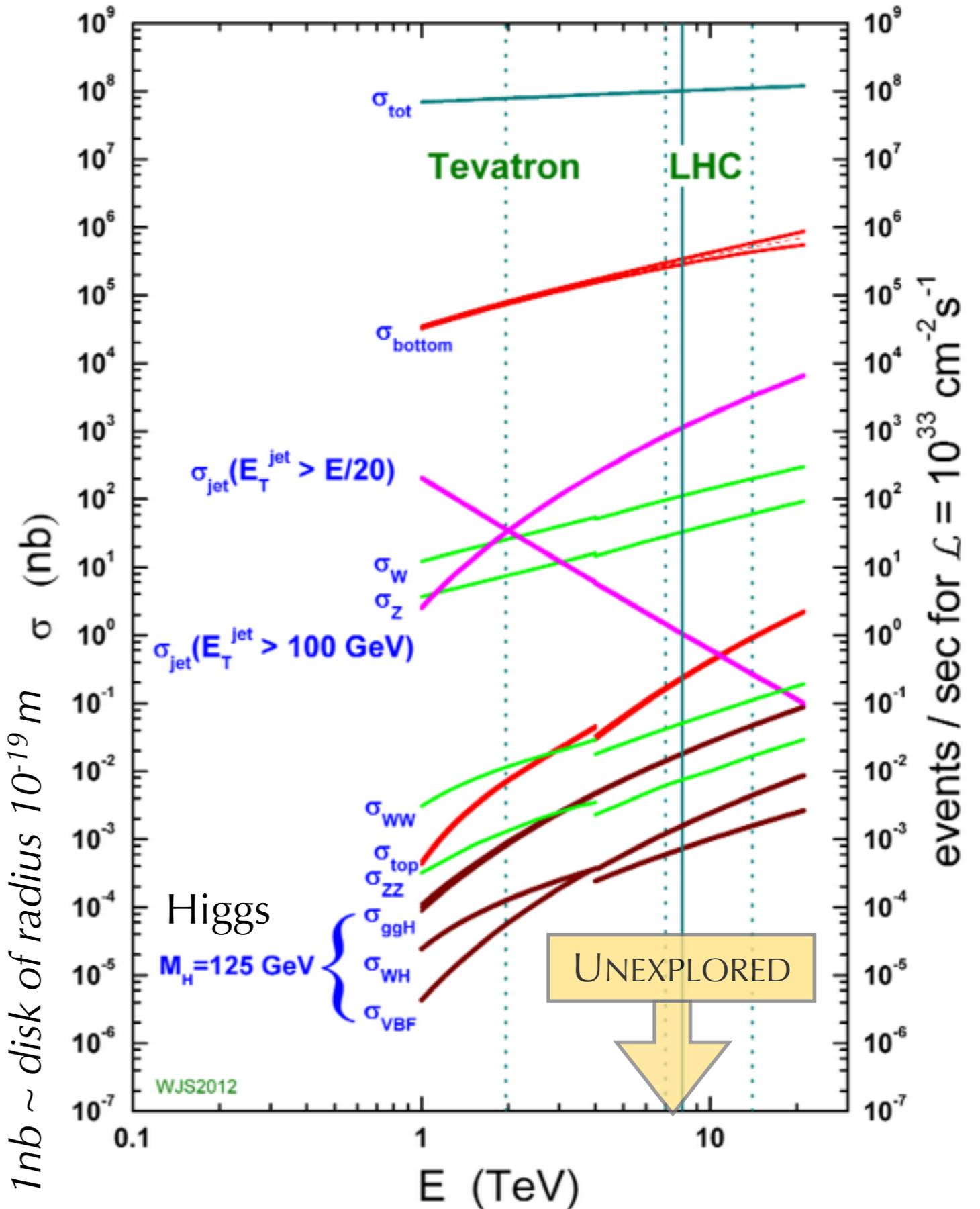
“Leptons”

“Missing Energy”

“Jets”

Effective (quantum) area;  
1nb ~ disk of radius  $10^{-19}$  m

proton - (anti)proton cross sections



# Precision

## Precision & Discovery go hand in hand

E.g., after the Higgs discovery, now comes *precision study*

**Recognise** the unknown: understand the known

Calibrate your methods, test your strategies, ...

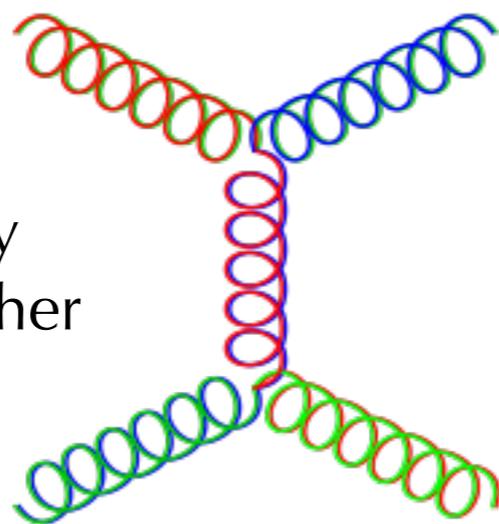
& occasionally discover that you didn't understand "the known" ...

## My own work focuses on the modelling of "jets"

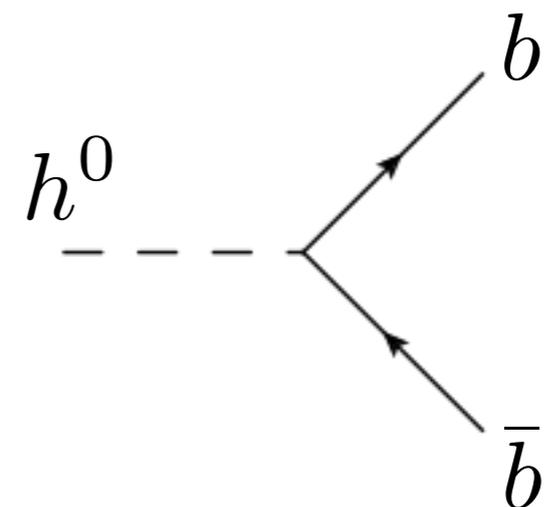
Sprays of nuclear matter, produced by energetic quarks and gluons

quantum structure

Such as when they scatter off each other



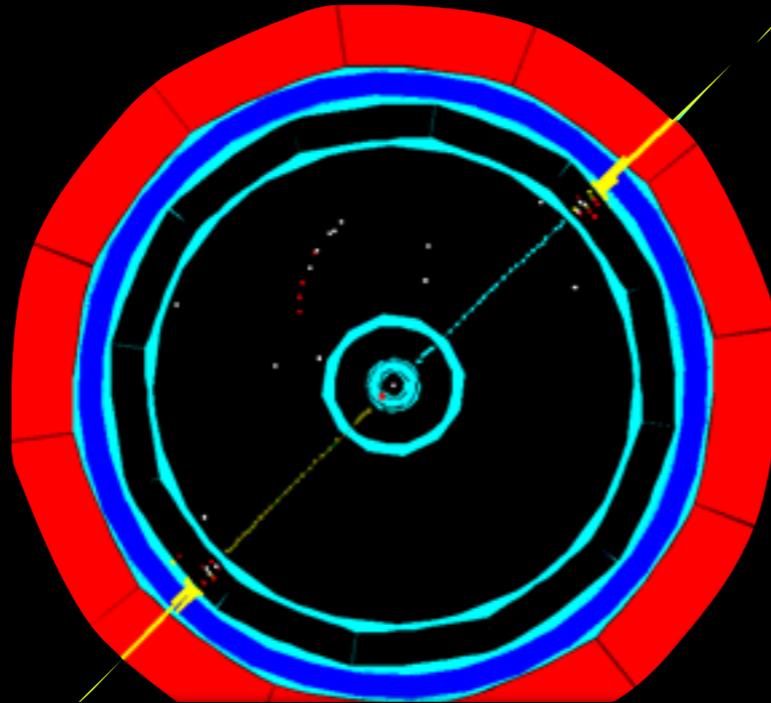
Or when a heavy particle decays to quarks / gluons



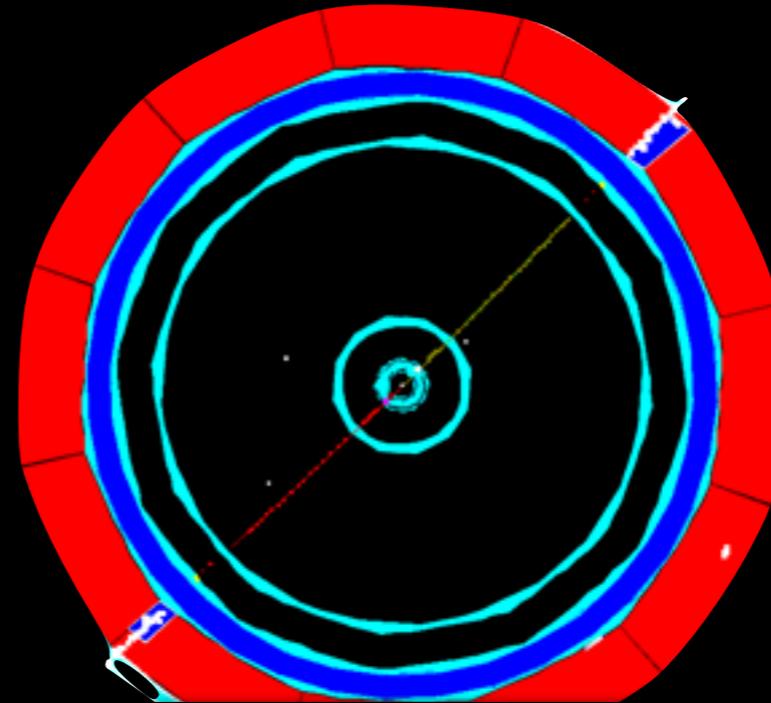
# Example: Decays of the Z boson

## Leptons

electron-  
positron  
pair creation



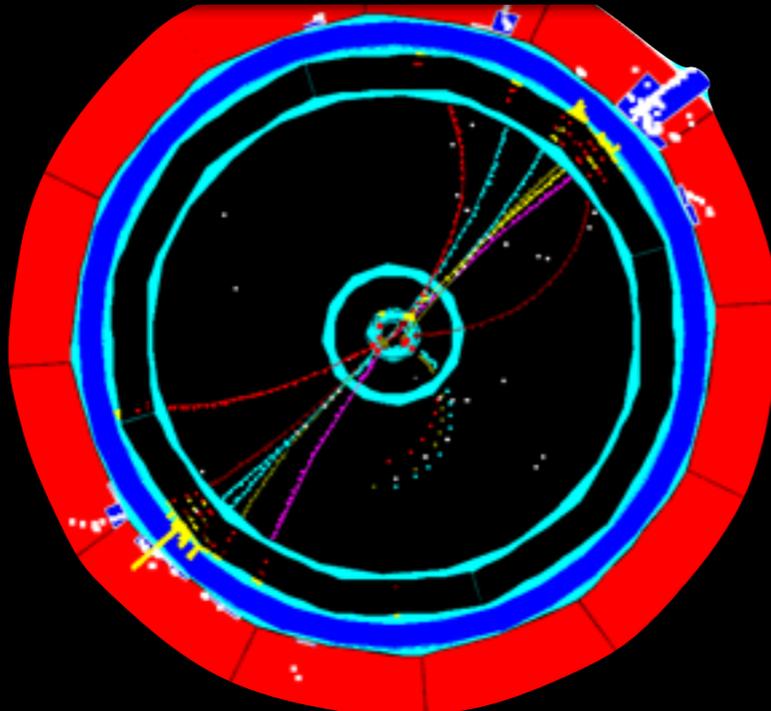
muon-  
antimuon  
pair creation



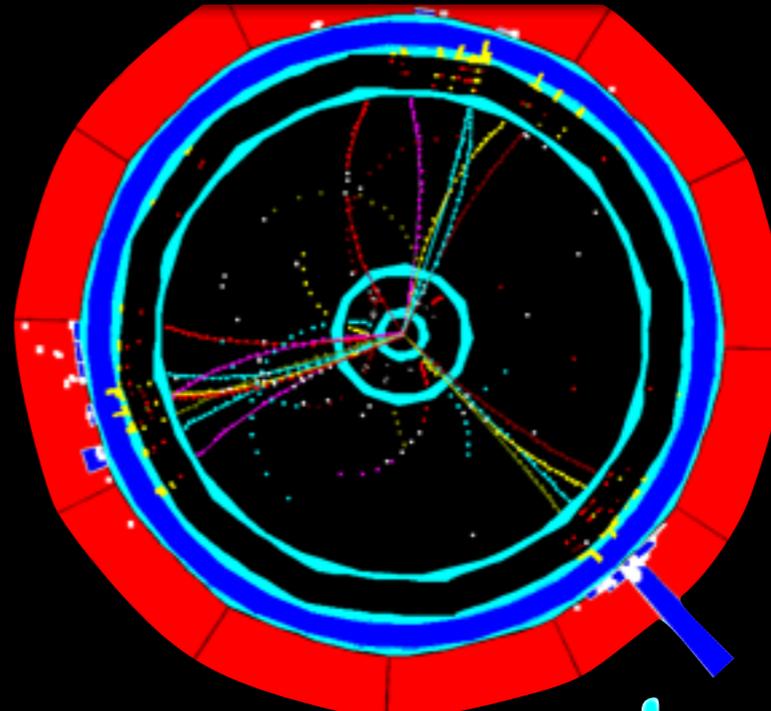
(from the ALEPH experiment at the Large Electron Positron Collider)

## Jets

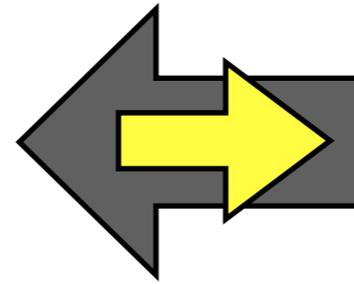
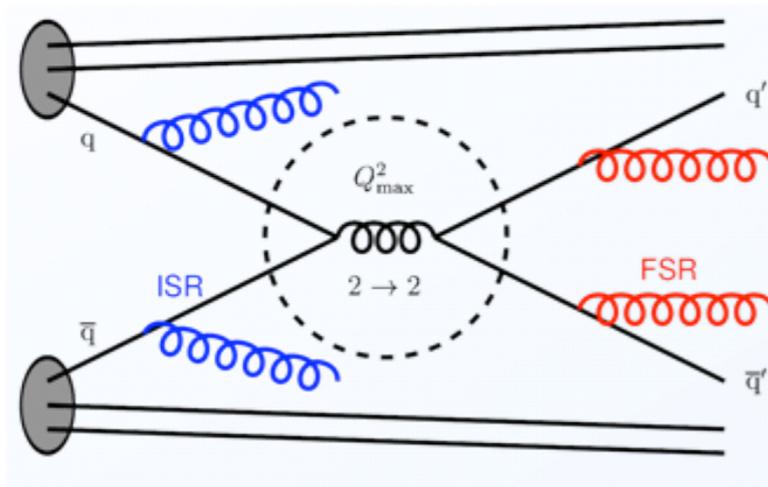
quark-  
antiquark  
pair creation  
→ 2 Jets



quark-  
antiquark  
+ gluon  
→ 3 Jets



# Collider Calculations



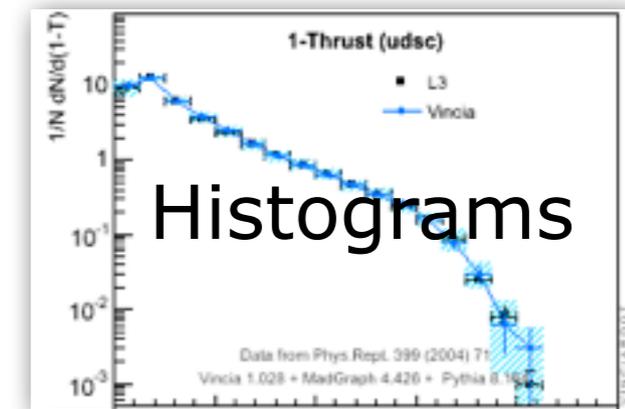
Calculate Everything  $\approx$  solve QFT\*  $\rightarrow$  requires compromise!

Start from lowest-order perturbation theory,  
 Include the 'most significant' corrections  
 $\rightarrow$  **Monte Carlo event generators**

(g)	-51	14	17	34	34	132	172
(d)	-71	29	29	42	63	171	0
(g)	-71	30	30	42	63	172	171
(g)	-71	31	31	42	63	132	172
(g)	-71	26	26	42	63	157	132
(g)	-71	27	27	42	63	158	157
(g)	-71	28	28	42	63	156	158
(g)	-71	25	25	42	63	149	156
(g)	-71	21	21	42	63	150	149
(g)	-71	21	21	42	63	108	150
(dbar)	-71	1	1	63	0	108	0
(k*0)	-83	32	41	66	66	0	0
(kbar0)	-83	32	41	66	66	0	0
(rho-)	-83	32	41	67	68	0	0
(p10)	-83	32	41	69	70	0	0
p+	83	32	41	0	0	0	0
nbar0	83	32	41	0	0	0	0
p1-	83	32	41	0	0	0	0
(p10)	-83	32	41	71	72	0	0
p1+	83	32	41	0	0	0	0

Events

**connect** with the observable world  
 of hadrons, photons, and leptons



Histograms

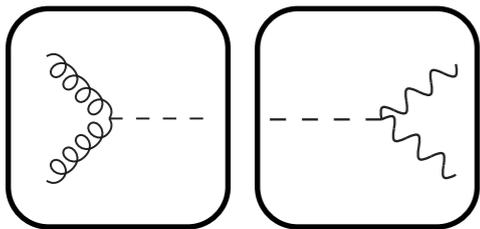
\*QFT = Quantum Field Theory

# Organising the Calculation

**Divide and Conquer** → Split the problem into many (nested) pieces

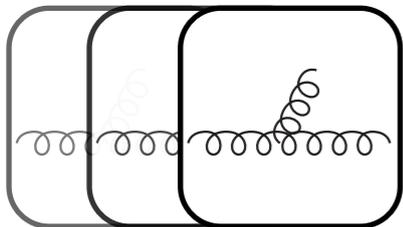
+ Quantum mechanics → Probabilities → Random Numbers

$$\mathcal{P}_{\text{event}} = \mathcal{P}_{\text{hard}} \otimes \mathcal{P}_{\text{dec}} \otimes \mathcal{P}_{\text{ISR}} \otimes \mathcal{P}_{\text{FSR}} \otimes \mathcal{P}_{\text{MPI}} \otimes \mathcal{P}_{\text{Had}} \otimes \dots$$



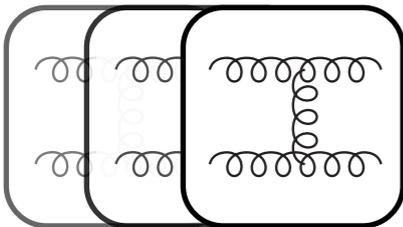
## Hard Process & Decays:

The basic hard process. E.g.,  $gg \rightarrow H^0 \rightarrow \gamma\gamma$   
→ Sets highest resolvable scale:  $Q_{\text{MAX}}$



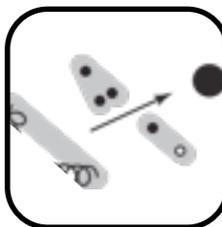
## Initial- & Final-State Radiation (ISR & FSR):

Bremsstrahlung, driven by differential evolution equations,  $dP/dQ^2$ , as function of resolution scale; run from  $Q_{\text{MAX}}$  to  $\sim 1$  GeV



## MPI (Multi-Parton Interactions)

Protons contain lots of partons → can have additional (soft) parton-parton interactions → Additional (soft) “Underlying-Event” activity



## Hadronization

Non-perturbative modeling of parton → hadron transition

# Bremsstrahlung

cf. equivalent-photon  
approximation  
Weiszäcker, Williams ~ 1934

a.k.a. Initial- and Final-state radiation

Radiation

Radiation

Accelerated  
Charges

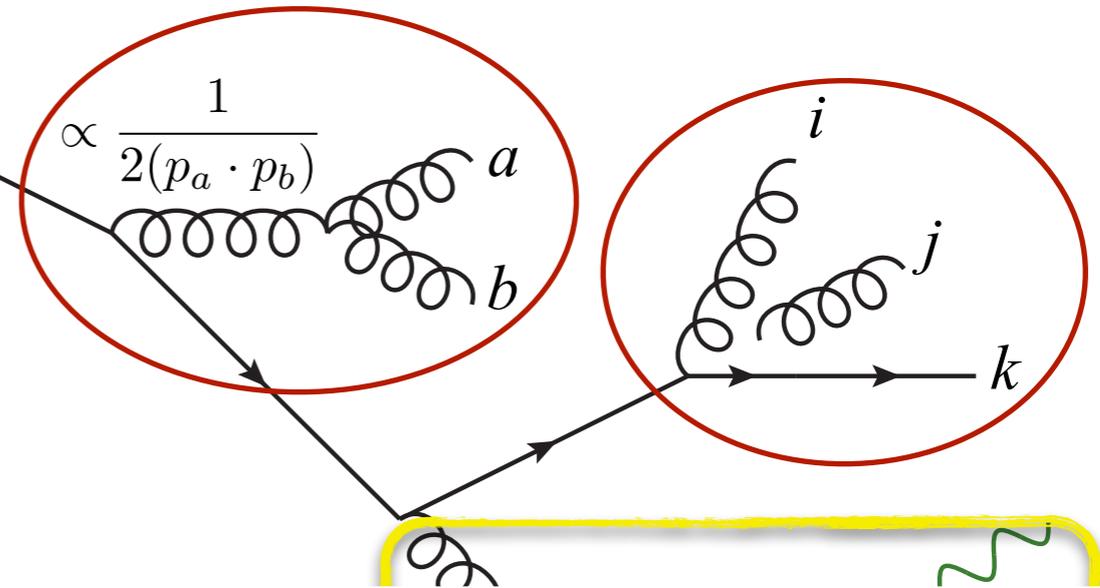
The harder they get kicked, the harder the  
fluctuations that continue to become strahlung



# The Structure of Jets

Most bremsstrahlung is driven by divergent propagators  $\rightarrow$  simple structure

Amplitudes *factorize* in singular limits ( $\rightarrow$  universal “scale-invariant” or “conformal” structure)



Partons  $ab \rightarrow$   
“collinear”:

$P(z) =$  DGLAP splitting kernels, with  $z =$  energy fraction  $= E_a/(E_a+E_b)$

$$|\mathcal{M}_{F+1}(\dots, a, b, \dots)|^2 \xrightarrow{a||b} g_s^2 C \frac{P(z)}{2(p_a \cdot p_b)} |\mathcal{M}_F(\dots, a + b, \dots)|^2$$

Gluon  $j \rightarrow$  “soft”:

Coherence  $\rightarrow$  Parton  $j$  really emitted by  $(i,k)$  “colour antenna”

$$|\mathcal{M}_{F+1}(\dots, i, j, k, \dots)|^2 \xrightarrow{j_g \rightarrow 0} g_s^2 C \frac{(p_i \cdot p_k)}{(p_i \cdot p_j)(p_j \cdot p_k)} |\mathcal{M}_F(\dots, i, k, \dots)|^2$$

+ scaling **violation**:  $g_s^2 \rightarrow 4\pi\alpha_s(Q^2)$

See: PS, *Introduction to QCD*, TASI 2012, [arXiv:1207.2389](https://arxiv.org/abs/1207.2389)

Can apply this many times  
 $\rightarrow$  nested factorizations

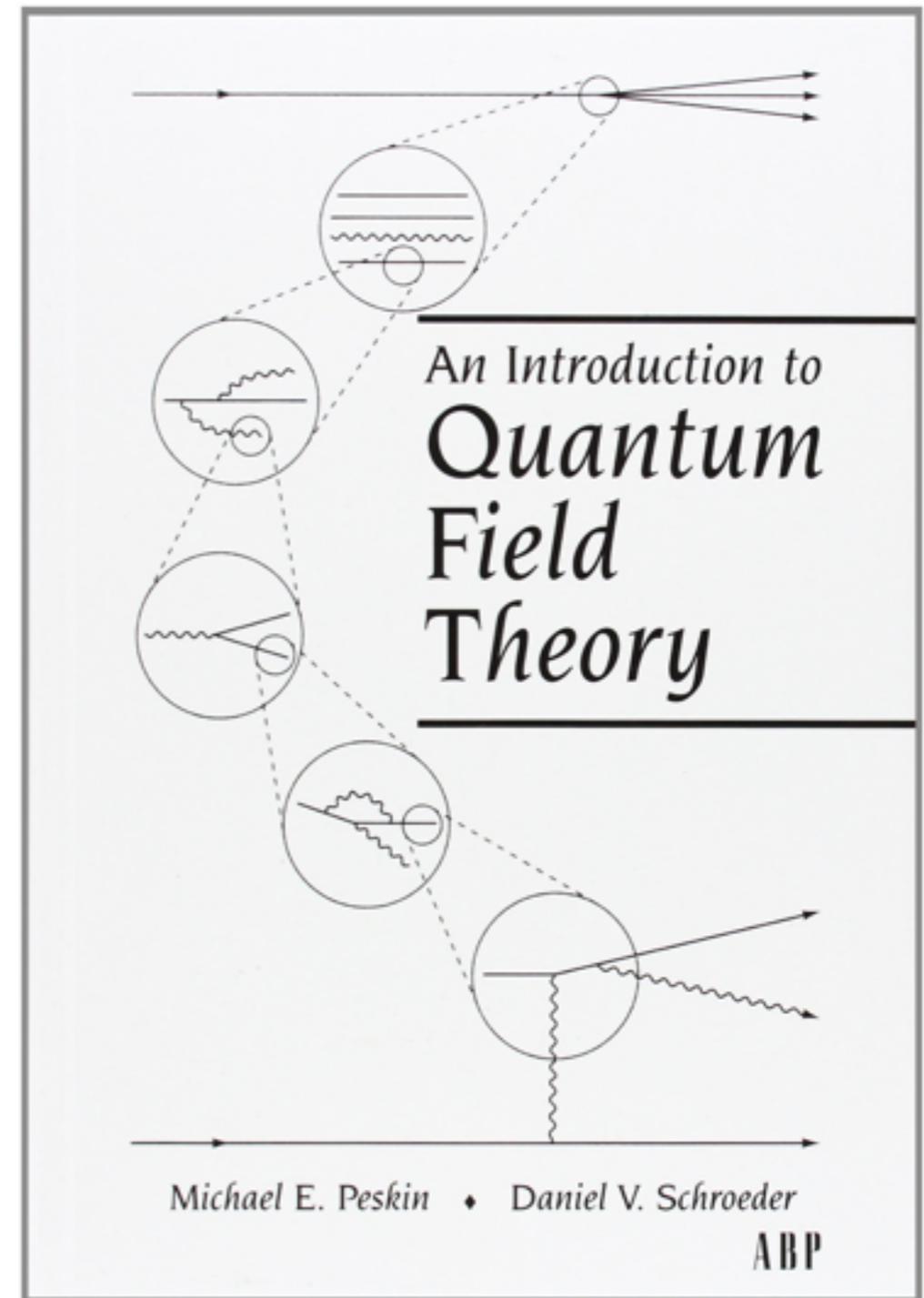
# The Structure of Quantum Fields

What we actually see when we look at a “jet”, or inside a proton

An ever-repeating self-similar pattern of quantum fluctuations

At increasingly smaller energies or distances : **scaling** (modulo  $\alpha(Q)$  scaling violation)

To our best knowledge, this is what a fundamental (‘elementary’) particle really looks like



# The Structure of Quantum Fields

What we actually see when we look at a “jet”, or inside a proton

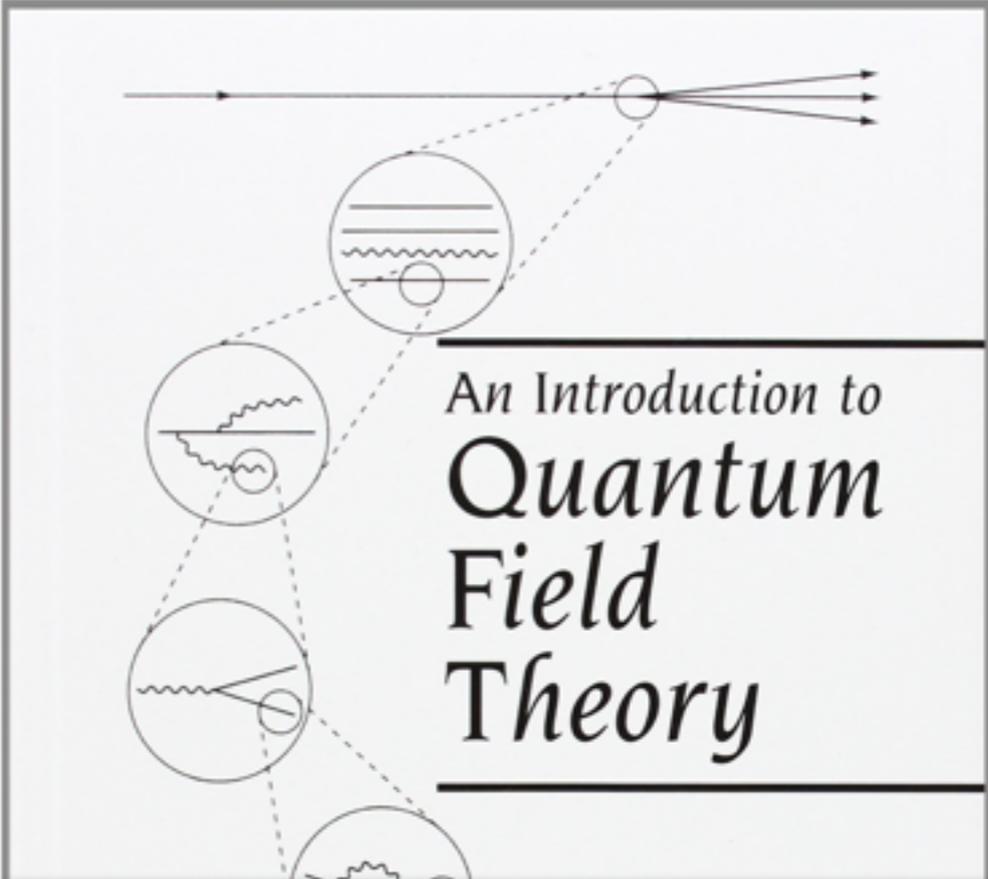
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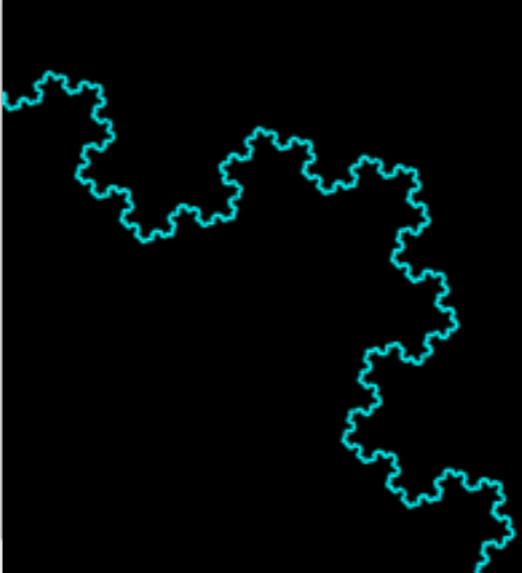
To our best knowledge, this is what a fundamental (‘elementary’) particle really looks like

Nature makes copious use of such structures

Called **Fractals**



An Introduction to  
**Quantum  
Field  
Theory**

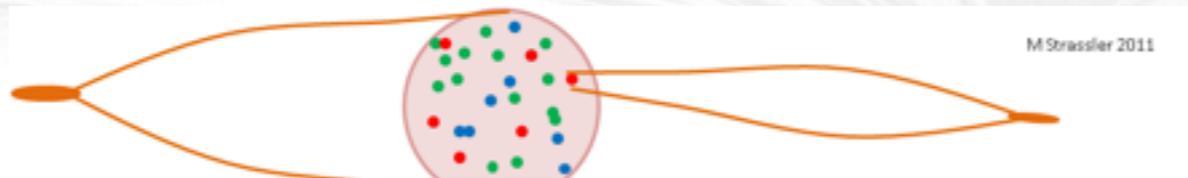
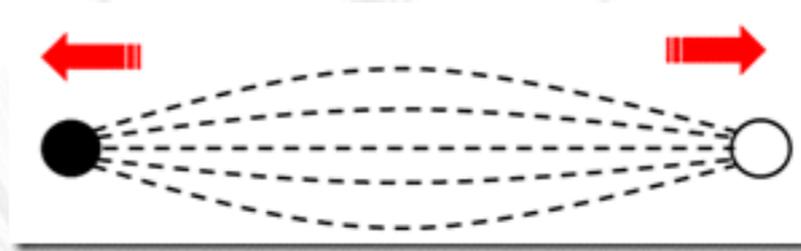


Note: this is not an elementary particle, but a different fractal, illustrating the principle

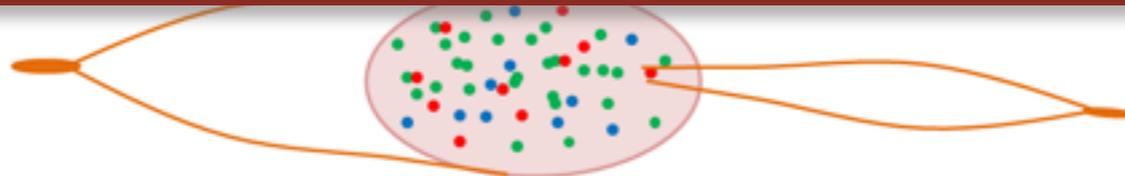
# Confinement

When highly energetic quarks fly apart, a very strong potential builds up between them

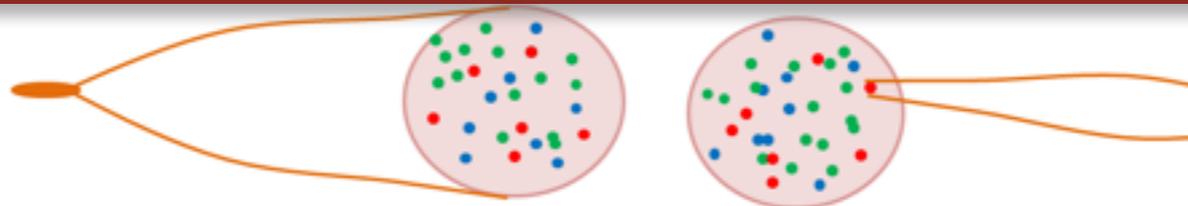
Increase in potential energy  
~ 1 GeV / femtometer  
(~ energy density of pure nuclear matter)



This is the force that normally keeps quarks locked inside hadrons



But when the kick is hard enough,  $E=mc^2$  gets a second chance to act



As the quarks separate, this happens multiple times → **Jets**



16-TON TRUCK

# Vortices Through the Vacuum

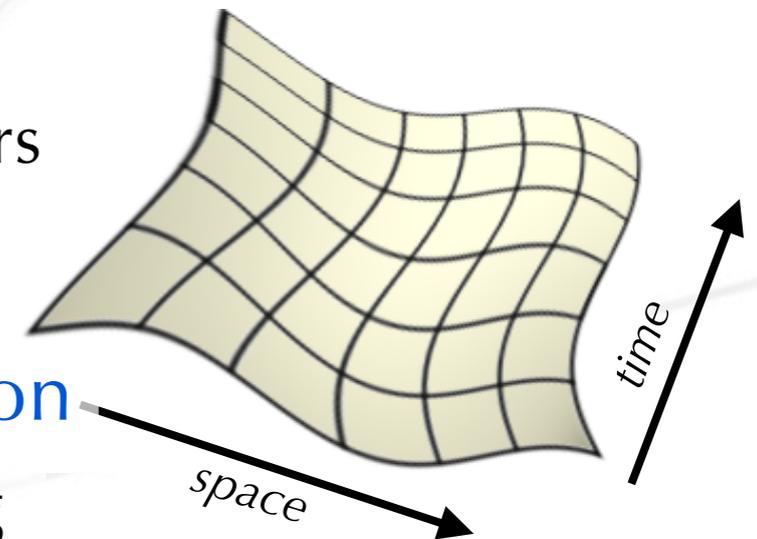
The force is approximately **constant** with distance

Suggestive of **strings** (aka vortex lines)

Similar to those in superfluids and superconductors

Inspired the “**string model**” of jet fragmentation

Breakup process modelled by **quantum tunnelling**



Used for 30 years

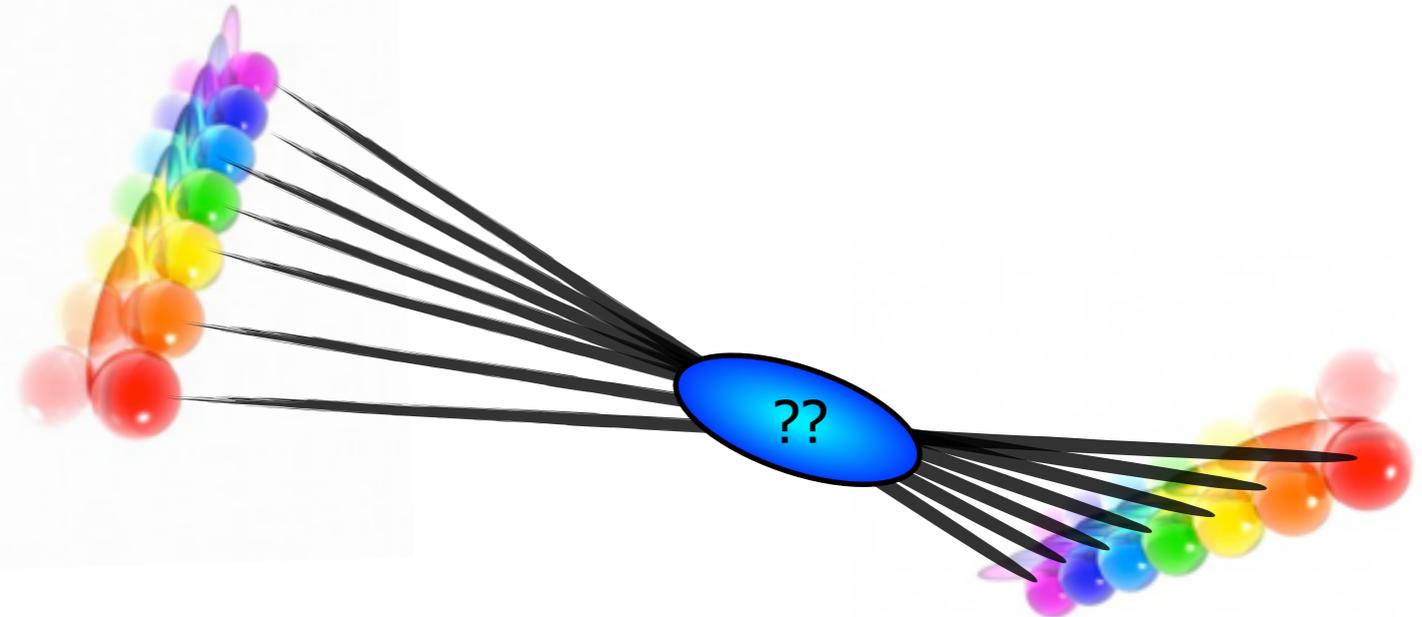
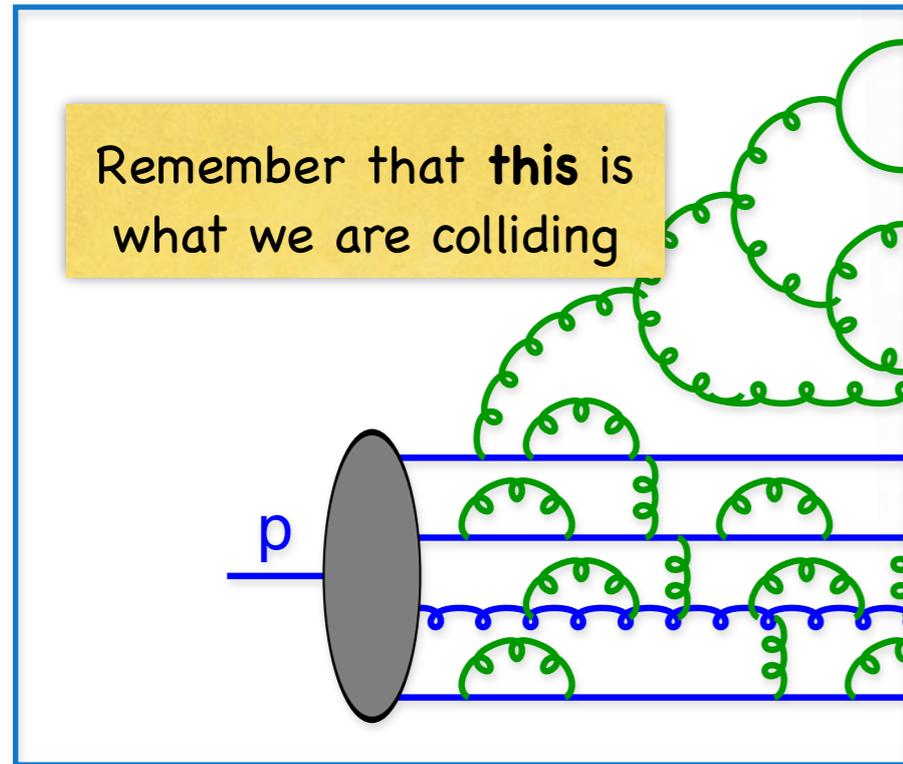
Generally good agreement with collider experiments

Until we started looking closely at the LHC Run-1 data ...

**More high-mass hadrons appear to be produced** (than predicted)

**And they appear to be moving faster** (than predicted)

# What's Going On?



*This is one of the main problems that are currently causing me to scratch my head*

Heat? Hydrodynamics?

Fat Strings?

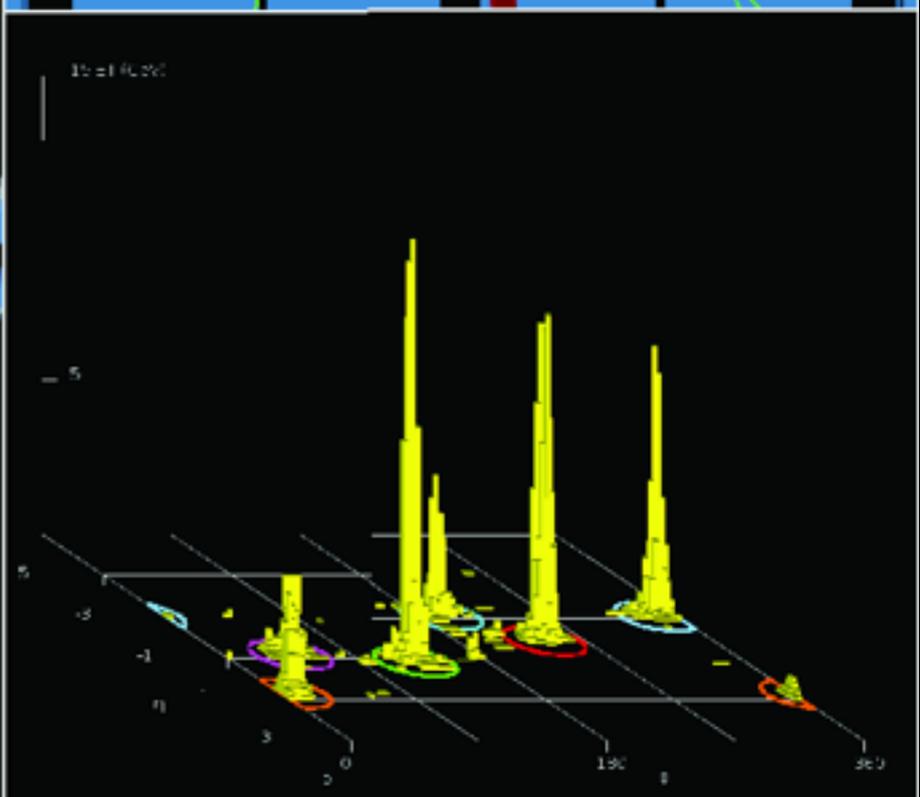
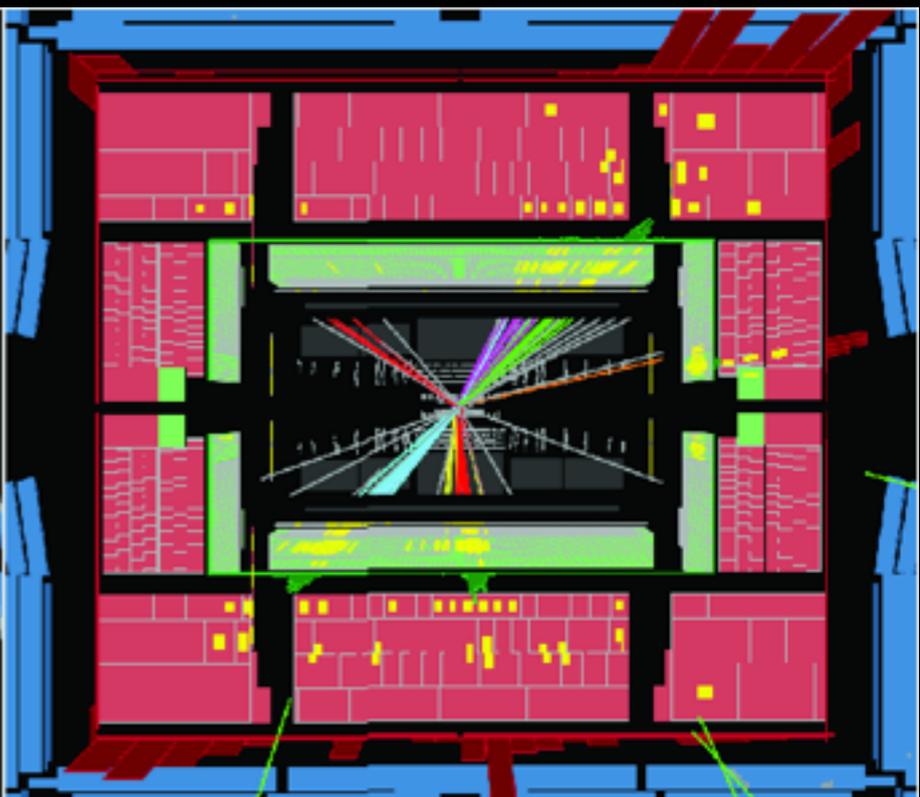
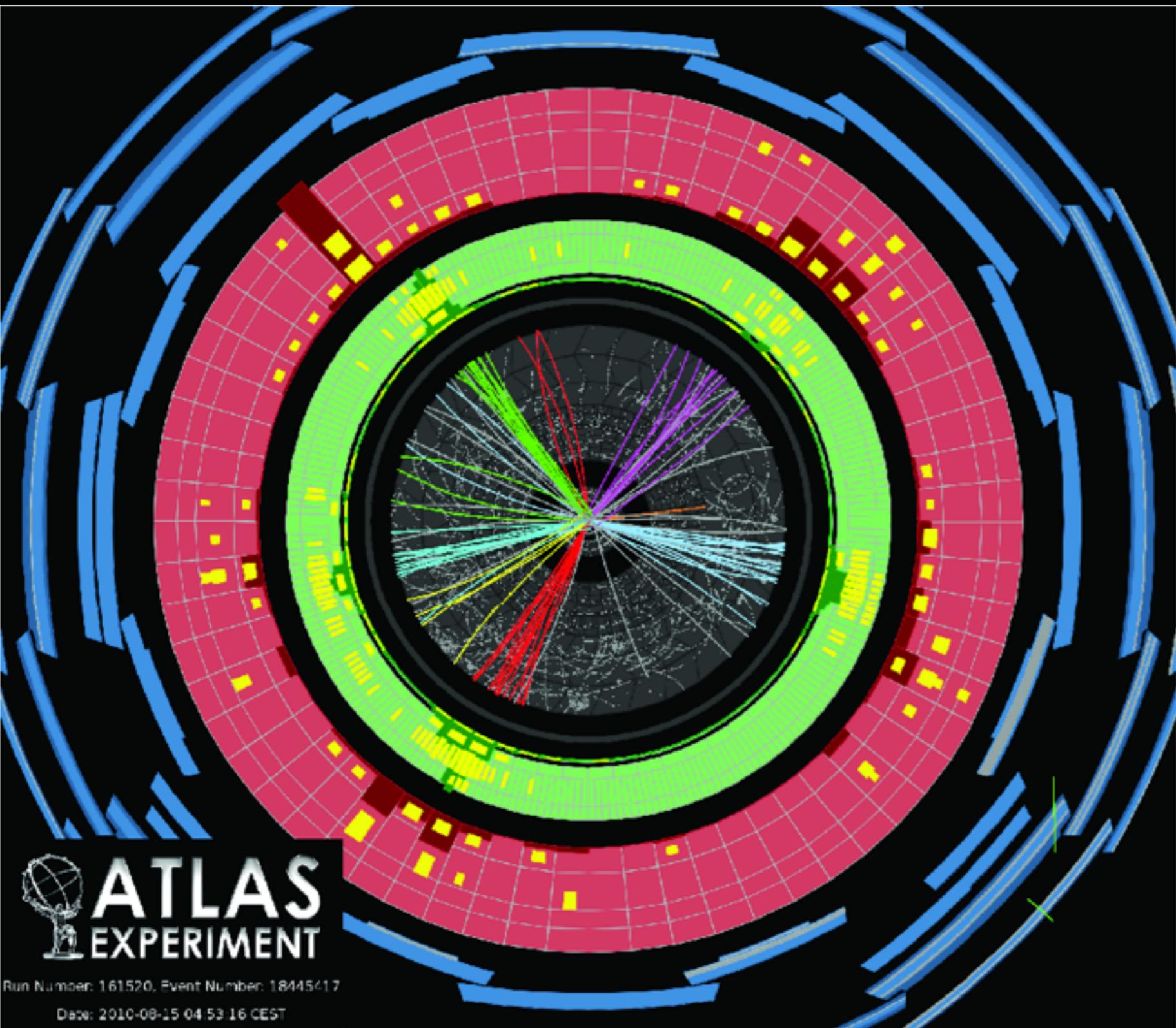
String-String Forces?

Black Strings?

String Reconnections?

Hadron-Gas Rescattering?

# Thank You



 **ATLAS**  
EXPERIMENT

Run Number: 161520, Event Number: 18445417  
Date: 2010-08-15 04:53:16 CEST

# What *is* a Fundamental Particle?

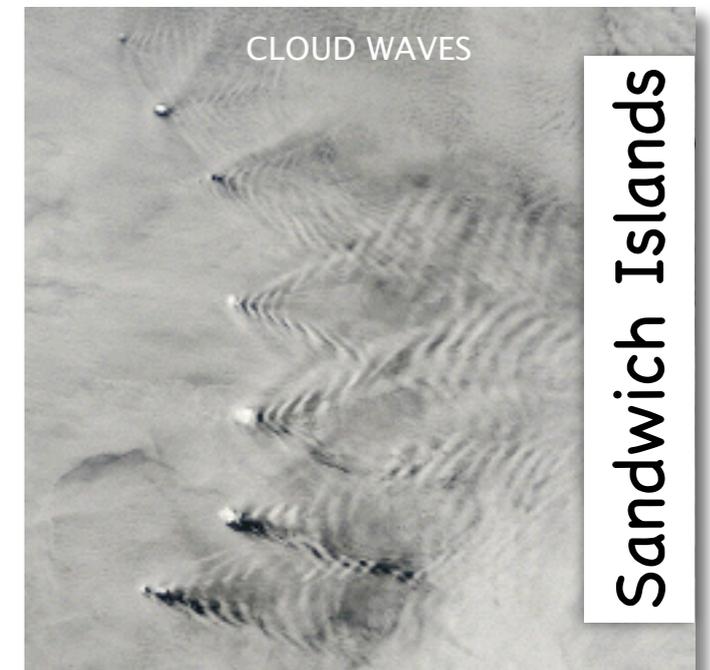
Abstractly, we think of an idealised “pointlike” particle

But could we ever really see “a point”?

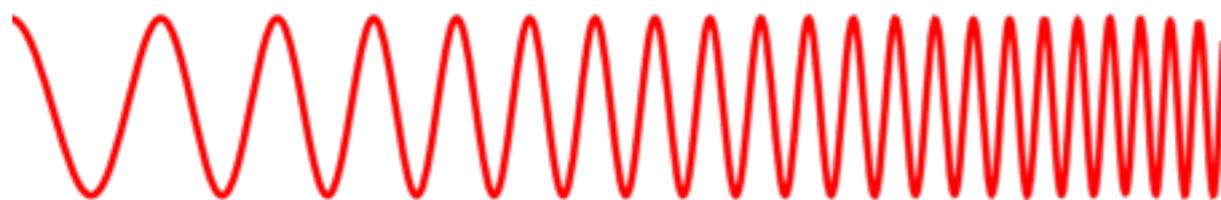
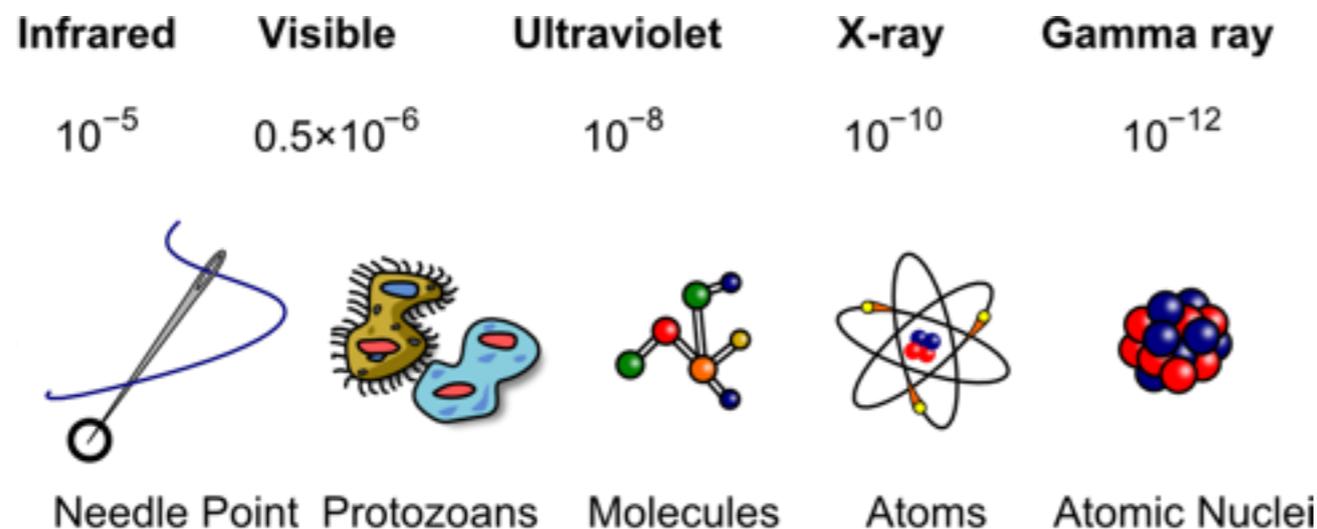
How do we see, in the quantum world?

To see something small, we scatter waves off it

→ Heisenberg’s uncertainty principle.



NASA – MODIS



To resolve “a point”, we would need infinitely short wavelengths

Heisenberg would then give it an infinitely hard kick

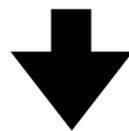
# Hard-Sphere Scattering

(Classical) particle bouncing off a (classical) hard sphere

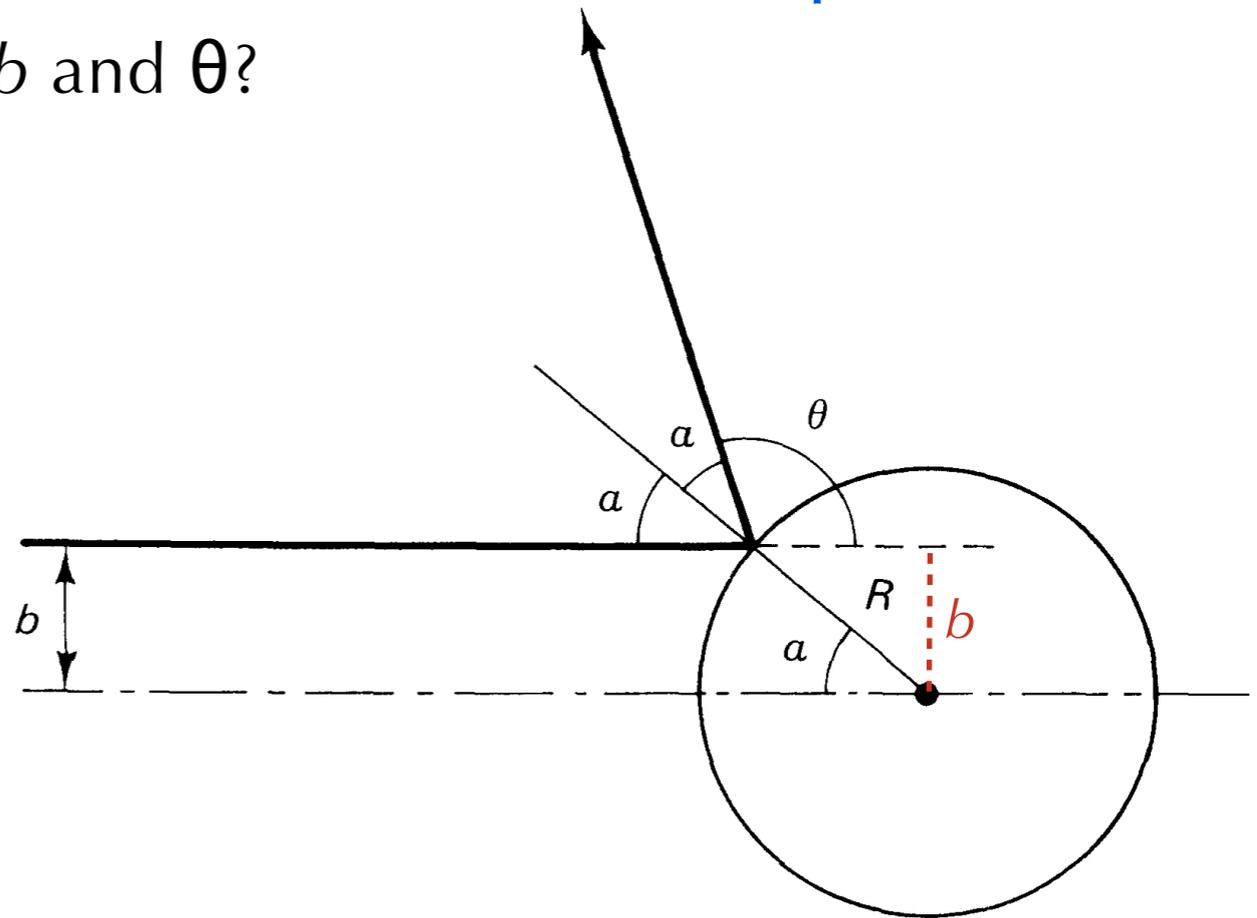
What is the relation between  $b$  and  $\theta$ ?

$$\theta = 2 \cos^{-1}(b/R)$$

If the particle comes in with an **impact parameter** between  $b$  and  $b+db$  it will emerge with a **scattering angle** between  $\theta$  and  $\theta+d\theta$ .



If the particle passes through an infinitesimal **area  $d\sigma$** , it will scatter into a corresponding **solid angle  $d\Omega$**

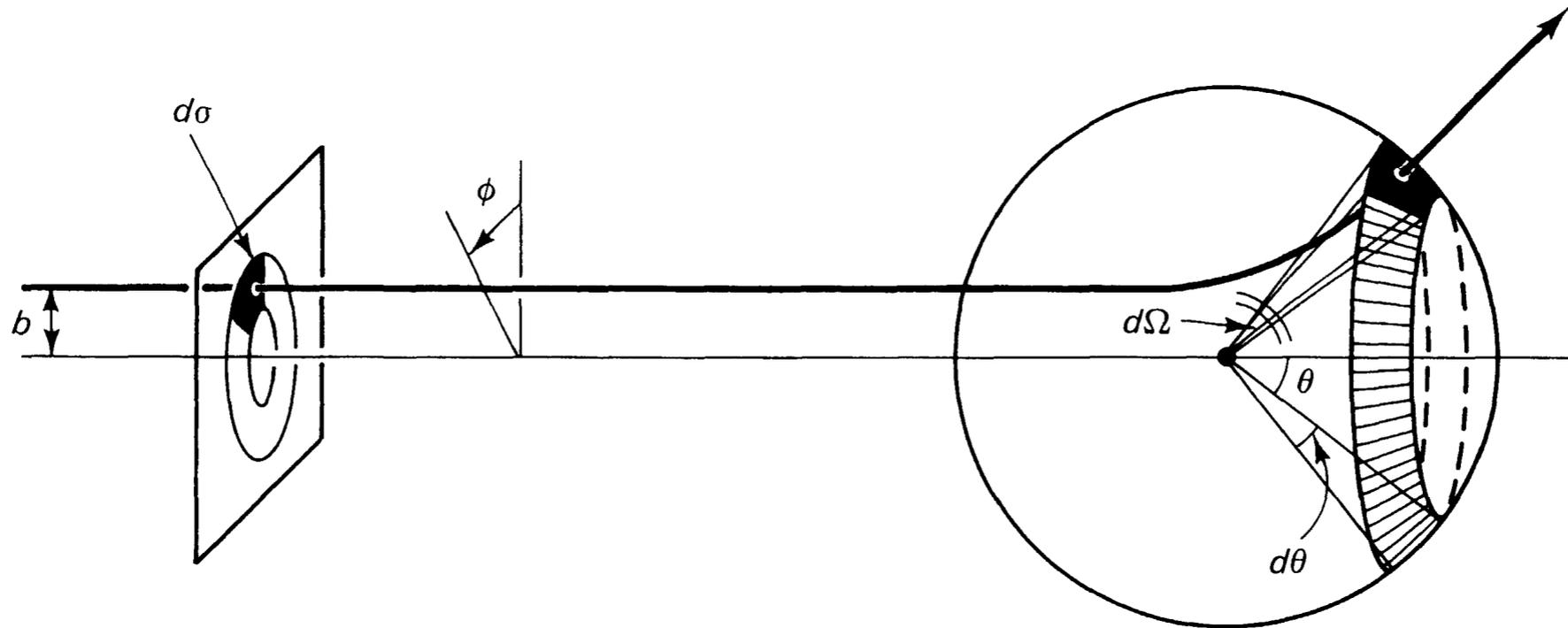


**Figure 6.2** Hard-sphere scattering.

Thus  
and hence

$$\begin{aligned} b &= R \sin \alpha, & 2\alpha + \theta &= \pi \\ \sin \alpha &= \sin(\pi/2 - \theta/2) = \cos(\theta/2) \\ b &= R \cos(\theta/2) \quad \text{or} \quad \theta = 2 \cos^{-1}(b/R) \end{aligned}$$

# $d\Omega$ Solid Angle



**Figure 6.3** Particle incident in area  $d\sigma$  scatters into solid angle  $d\Omega$ .

A differential quantity of interest for  $2 \rightarrow 2$  is thus

The differential scattering cross section per unit solid angle

$$\frac{d\sigma}{d\Omega}$$

# Back to the Hard Sphere

We found the relation

$$b = R \cos(\theta/2)$$

Hence:

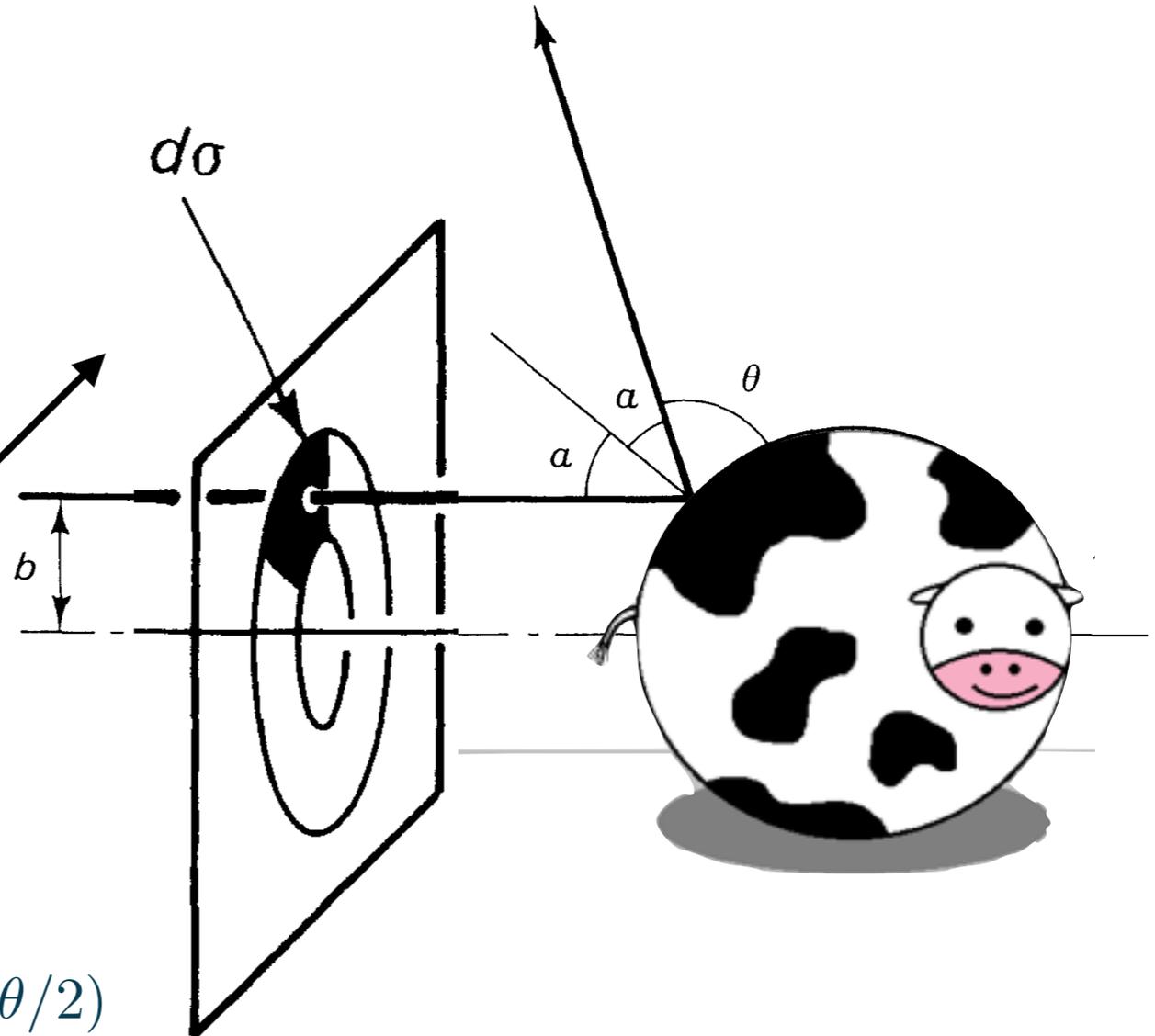
$$\frac{db}{d\theta} = -\frac{R}{2} \sin(\theta/2)$$

Note:

$$d\sigma = |b d\phi db|$$
$$d\Omega = |d\phi d \cos \theta|$$

So the differential cross section is:

$$\frac{d\sigma}{d\Omega} = \frac{|b d\phi db|}{|\sin \theta d\phi d\theta|} = \frac{R b \sin(\theta/2)}{2 \sin \theta}$$
$$= \frac{R^2 \cos(\theta/2) \sin(\theta/2)}{2 \sin \theta} = \frac{R^2}{4}$$



Integration yields:

$$\sigma = \int \frac{R^2}{4} d\Omega = \pi R^2$$