# QFT with Hadrons
## Introduction to B Physics

### 1. Leptonic Decays of Hadrons: from $\pi \to \ell\nu$ to $B \to \ell\nu$

*QFT in Hadron Decays. Decay Constants. Helicity Suppression in the SM.*

### 2. On the Structure and Unitarity of the CKM Matrix

*The CKM Matrix. The GIM Mechanism. The Unitarity Triangle.*

### 3. Semi-Leptonic Decays and the "Flavour Anomalies"

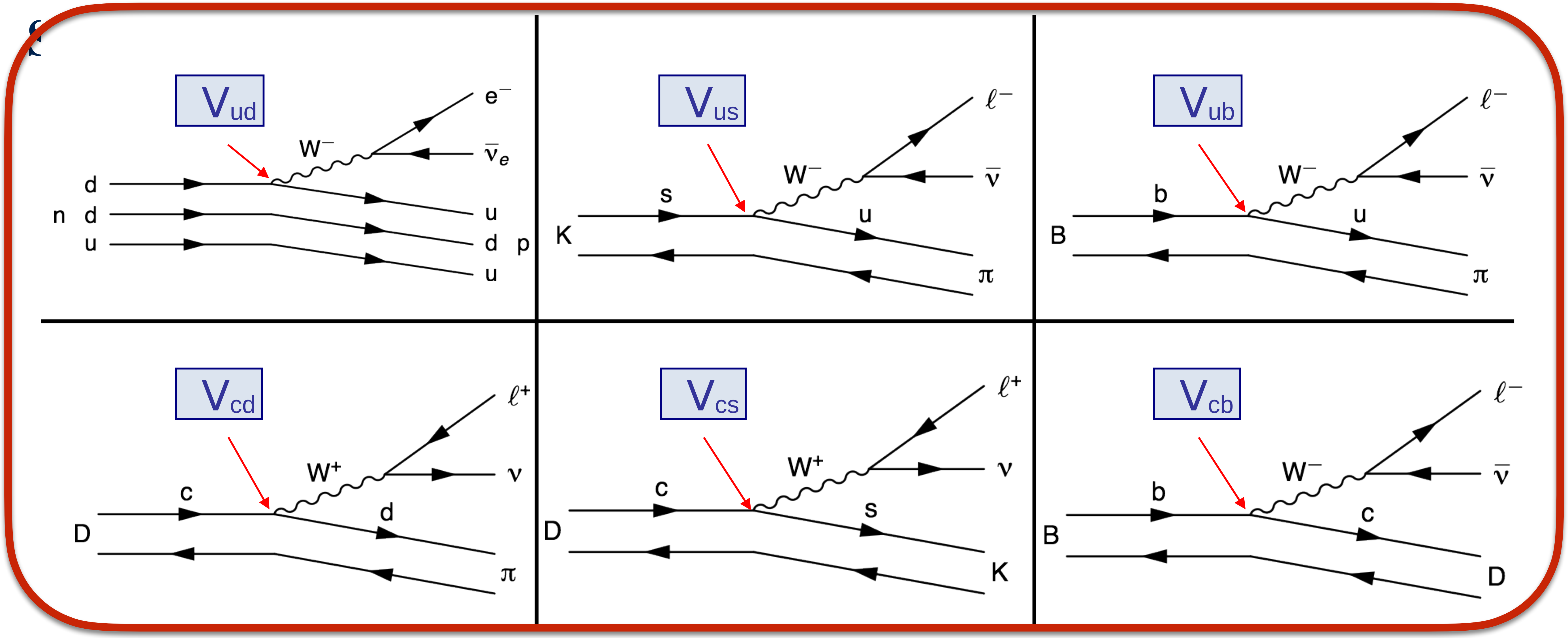
➡ *$B \to D^{(*)} \ell \nu$. The Spectator Model. Form Factors. Heavy Quark Symmetry.*

*$B \to K^{(*)} \ell^+ \ell^-$. FCNC. Aspects beyond tree level. Penguins. The OPE.*



**Peter Skands**
Monash University

# Charged-Current Semi-Leptonic Decays of Hadrons




What does it mean: ***Charged Current***?

What does it mean: ***Semi-Leptonic***?

# Cabibbo Favoured vs Cabibbo Suppressed

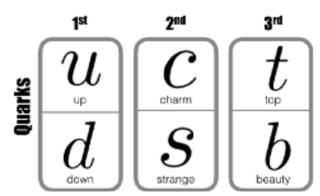
## Which is Cabibbo Favoured vs Cabibbo Suppressed?

*And why.*

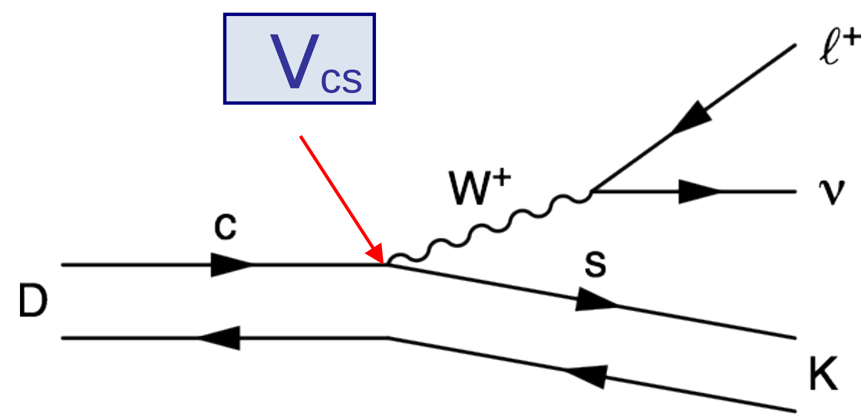
$D^0 = |c\bar{u}>$

$K^- = |s\bar{u}>$

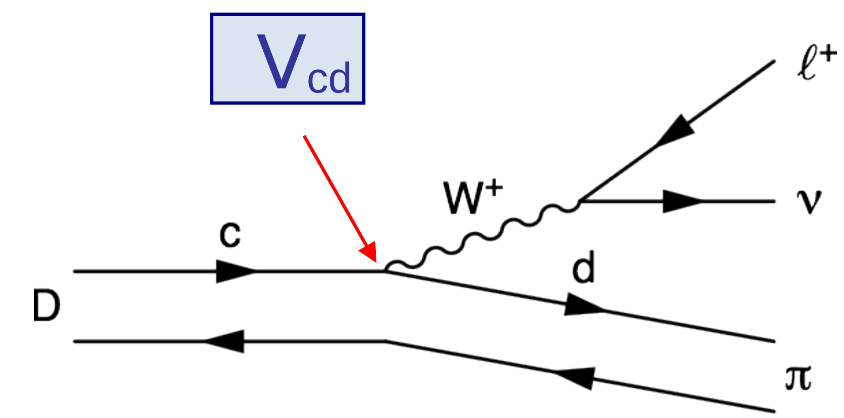
$\pi^- = |d\bar{u}>$




$D^0 \to K^- \ell^+ \nu$




$D^0 \to \pi^- \ell^+ \nu$




## Which is CKM Favoured vs CKM Suppressed?

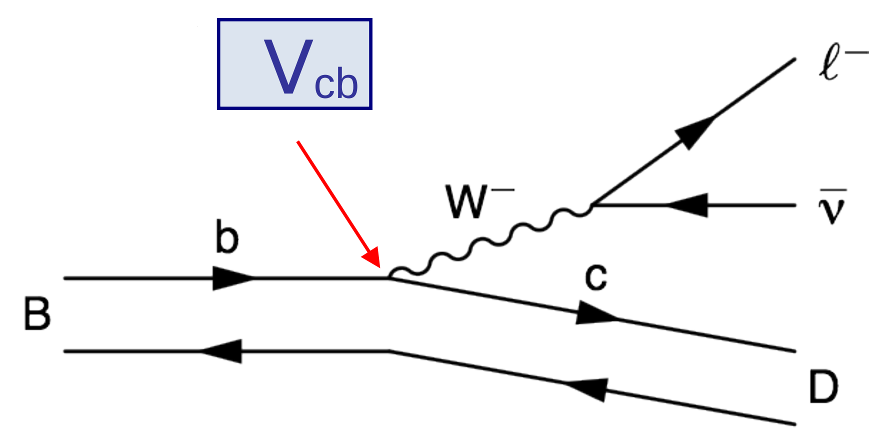
*And why.*

$B^- = |b\bar{u}>$

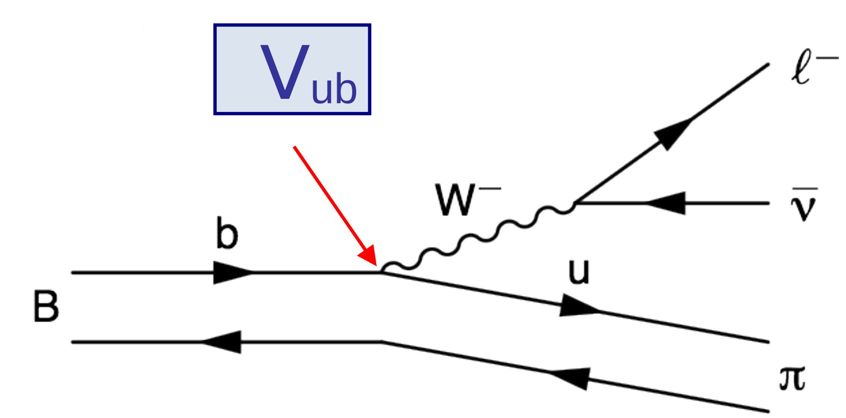
➠ Our case study.

Has gotten attention recently, as part of the "flavour anomalies".

$B \to D \ell \nu$



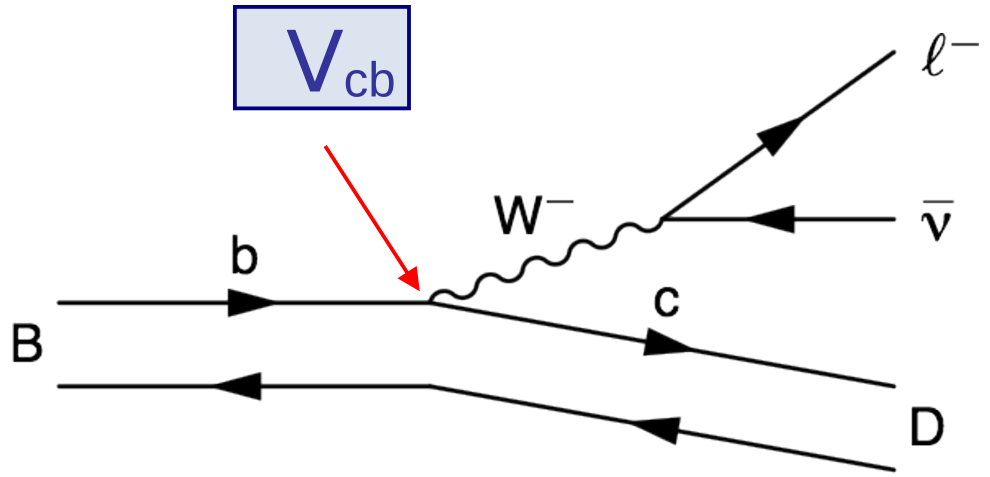

$B \to \pi \ell \nu$



# Starting Point for B→Dℓν: The **Spectator** Model

## Unlike B→ℓν, this is **not** an annihilation

Looks like a **weak decay of the heavy quark**, accompanied by a non-interacting **spectator**:




## Suggests a **simple** starting point for semi-leptonic decays**:**

Assume the quark(s) which accompany the heavy quark play **no role**.

$$\text{Spectator Model:}\quad \mathcal{M} = -\frac{G_F}{\sqrt{2}} V_{cb}\,[\bar{c}\gamma^\rho(1-\gamma^5)b]\,[\bar{\ell}\gamma_\rho(1-\gamma_5)\nu_\ell]$$

*Can give some insights (e.g., lepton spectrum) but not a* ***precision tool. How can we do better?***

# B→Dℓν with Hadronic Effects

**Can promote the spectator model's quark-level matrix element to a hadronic one by sandwiching it between initial and final hadronic states:**

$$\mathscr{M} = \frac{G_F}{\sqrt{2}} V_{cb} \left\langle D(p_D) \,|\, \bar{c}\gamma^\rho(1-\gamma^5)b \,|\, B(p_B) \right\rangle \, [\bar{\ell}\gamma_\rho(1-\gamma_5)\nu_\ell]$$

Both B and D are pseudoscalars. To construct a vector, must use L=1 ⇒ negative parity ⇒ Axial part does not contribute.

$$= \frac{G_F}{\sqrt{2}} V_{cb} \left\langle D(p_D) \,|\, \bar{c}\gamma^\rho b \,|\, B(p_B) \right\rangle \, [\bar{\ell}\gamma_\rho\nu_\ell]$$

Unlike for pion decay, we have two (independent) momenta here, $p_B$ and $p_D$ ⇒ a priori two Lorentz-covariant combinations

$$= \frac{G_F}{\sqrt{2}} V_{cb} \left[ f_+(q^2)(p_B+p_D)^\rho \; + f_-(q^2)(p_B-p_D)^\rho \right] \, [\bar{\ell}\gamma_\rho\nu_\ell]$$

$f_+$ and $f_-$ are called **Form Factors**

They depend on $q^2 = (p_B - p_D)^2 = p_W^2 = (p_\ell + p_{\bar{\nu}})^2$ = Momentum Transfer

# (Alternative Parameterisation)

We wrote:

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} V_{cb} \left[ f_+(q^2)(p_B + p_D)^\rho + f_-(q^2)(p_B - p_D)^\rho \right] [\bar{\ell}\gamma_\rho \nu_\ell]$$

Another common parametrisation [Wirbel, Stech, Bauer, Z.Phys. C29 (1985) 637] is to write in terms of a **"Transverse" $F_0$** and a **"Longitudinal" $F_1$** form factor:

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} V_{cb} \left[ F_0(q^2) \frac{m_B^2 - m_D^2}{q^2} q^\rho + F_1(q^2) \left( p_B + p_D - \frac{m_B^2 - m_D^2}{q^2} q \right)^\rho \right] [\bar{\ell}\gamma_\rho \nu_\ell]$$

with $q = p_B - p_D$ and $F_1(0) = F_0(0)$

Thus: $f_+ = F_1$

$f_- = (F_0 - F_1)(m_B^2 - m_D^2)/q^2$

Exercise: prove this

Note: for decays involving **vector mesons**, polarisations $\varepsilon^\mu$ ⇒ more form factors.

# Looks like we went from bad to worse?

**Our ignorance about non-perturbative physics is now cast as two whole functions.**

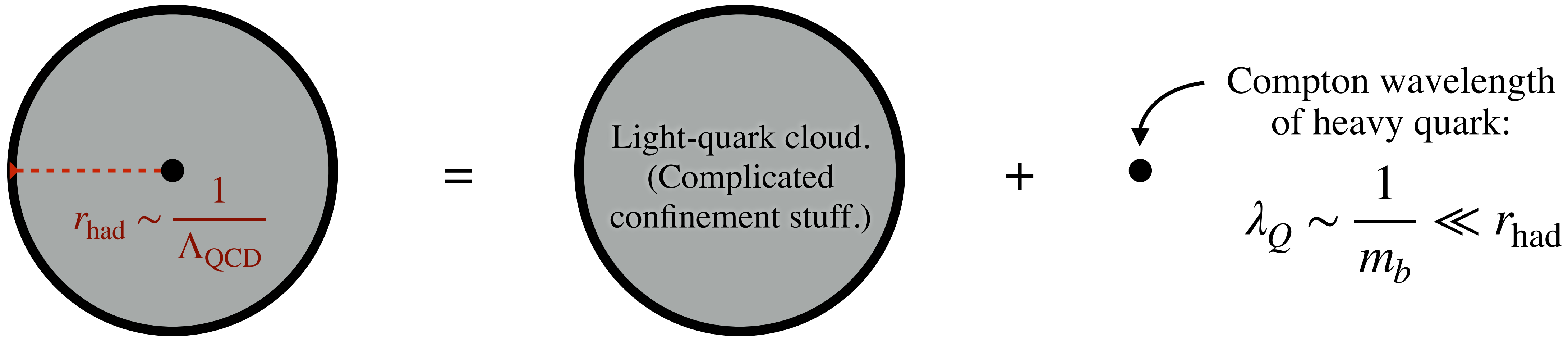
How can we learn anything (precise) from this?

Frustrating when the process *looks* so simple …

**Let's take a second look at the problem, physicist style:**

The B meson is a *heavy-light system;*

$m_b \sim 4$ GeV $\gg$ $\Lambda_{QCD}$ (confinement scale ~ 200 MeV)




➤ Large **separation of scales!**

# Heavy Quark Symmetry

Soft gluons exchanged between the heavy quark and the light constituent cloud can only resolve distances much larger than $\lambda_Q \sim 1/m_Q$

- **In limit $m_Q \to \infty$, the light degrees of freedom:**

  Are blind to the flavour (mass) and spin of the heavy quark.

  **Experience only the colour field of the heavy quark** (which extends over distances large compared with $1/m_Q$)

- **If we swap out the heavy quark Q by one with a different mass and/or spin, the light cloud would be the same.**

  $\Rightarrow$ Relations between B, D, $B^*$, and $D^*$, and between $\Lambda_b$ and $\Lambda_c$.

  **For finite $m_Q$**, these relations are only **approximate.**

  Deviations from exact heavy-quark symmetry: "*symmetry breaking corrections*"

  Can be organised systematically in powers of $\boldsymbol{\alpha_s(m_Q)}$ (perturbative) and $\boldsymbol{1/m_Q}$ (non-perturbative) in a formalism called **HQET** (heavy-quark effective theory).

# Physics of heavy-quark symmetry

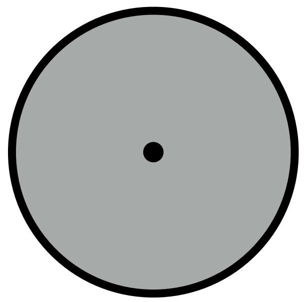
Isgur & Wise , Phys. Lett. B 232 (1989) 113; Phys. Lett. B 237 (1990) 527

**Before we consider decays, consider just elastic scattering* of a B meson**

Induced by giving a **kick** to the *b* quark at time $t_0$:

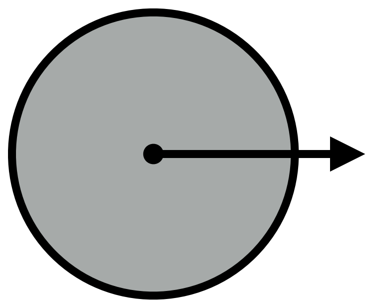
***Elastic Scattering:** means B meson does not break up.

*Illustrations and physics arguments inspired by the BaBar Physics Book.*

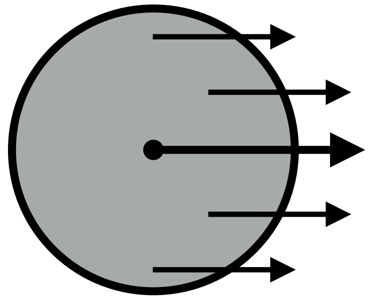


**Before $t_0$:** light degrees of freedom orbit around the heavy quark, which acts as a static source of colour.

On average, *b* quark and *B* meson have same velocity, $v$.



**At $t_0$:** instantaneously replace colour source by one moving at velocity $v'$ (possibly with a different spin).



**After $t_0$:** If $v=v'$ (spectator limit), nothing happens; light degrees of freedom have no way of knowing anything changed.

But if $v \neq v'$, *the light cloud will need to be rearranged (sped up), to form a new B meson moving at velocity v'.*

➤ Form-factor suppression. (Large $\Delta v \Rightarrow$ **elastic** transition less likely.)

# Elastic Form Factor of a Heavy Meson (Isgur-Wise Function)

Isgur & Wise , Phys. Lett. B 232 (1989) 113; Phys. Lett. B 237 (1990) 527

## In limit $m_b \to \infty$, form factor can only depend on the difference between v and v':

Lorentz invariance ☛ use the **relative boost** between the rest frames of the initial- and final-state mesons.

Using $v^\mu = \dfrac{p^\mu}{m_b}$ and $v'^\mu = \dfrac{p'^\mu}{m_b}$ the relative boost is $\gamma = v \cdot v' \geq 1$

Exercise: prove this

➤ In this limit, a dimensionless probability amplitude $\xi(\gamma)$ describes the transition amplitude. *($\xi$ is called the Isgur-Wise function.)*

➤ The hadronic matrix element can be written as:

$$\left\langle \bar{B}(p') \,|\, \bar{b}_{p'} \gamma^\mu b_p \,|\, \bar{B}(p) \right\rangle = \xi(\gamma)(p + p')^\mu$$

$\xi$ is the elastic form factor of a heavy meson. **Only depends on $\gamma = v.v'$**, not $m_B$.
Constraint: at $\gamma$=1 (zero momentum transfer), current conservation ⇒ $\xi(1)=1$

Question: why is $\xi(1)=1$ intuitive?

**Using heavy-quark symmetry, we can replace the b quark in the final-state meson by a c quark:**

$$\left\langle \bar{D}(v') \,|\, \bar{c}_{v'}\gamma^{\mu} b_v \,|\, \bar{B}(v) \right\rangle = \sqrt{m_B m_C}\, \xi(v \cdot v')\,(v + v')^{\mu}$$

Writing it terms of velocities, v and v', instead of momenta

(This corresponds to the field definitions in HQET)

**Same** Isgur-Wise functions!

**Compare with the general expression from before:**

$$\mathscr{M} = \frac{G_F}{\sqrt{2}} V_{cb} \left[ f_+(q^2)(p_B + p_D)^{\rho} + f_-(q^2)(p_B - p_D)^{\rho} \right] \left[ \bar{\ell}\gamma_{\rho}\nu_{\ell} \right]$$

$\Rightarrow$ the functions $f_+$ and $f_-$ are not independent. Both are related to $\xi$.

Assignment Problem 3: derive expressions for $f_+(\xi)$ and $f_-(\xi)$

**In the limit that $m_b, m_c \gg \Lambda_{QCD}$, the differential semileptonic decay rates become:**

$$\frac{d\Gamma(\overline{B} \to D\,\ell\,\overline{\nu})}{dw} = \frac{G_F^2 |V_{cb}|^2}{48\pi^3} (m_B + m_D)^2\, m_D^3\, (w^2 - 1)^{3/2}\, \xi^2(w)\,,$$

$$\frac{d\Gamma(\overline{B} \to D^*\ell\,\overline{\nu})}{dw} = \frac{G_F^2 |V_{cb}|^2}{48\pi^3} (m_B - m_{D^*})^2\, m_{D^*}^3\, \sqrt{w^2 - 1}\, (w+1)^2 \times \left(1 + \frac{4w}{w+1}\, \frac{m_B^2 - 2w\, m_B m_{D^*} + m_{D^*}^2}{(m_B - m_{D^*})^2}\right) \xi^2(w)$$

… in terms of the "recoil variable" $w = v \cdot v'$

(Similar expressions can be derived for semi-leptonic $\Lambda_b \to \Lambda_c \ell \bar{\nu}$ or $\bar{B} \to D^{**} \ell \bar{\nu}$
Different clouds so different Isgur-Wise functions $\xi$.)

Reminder: **corrections** from **finite $m_Q$** (breaking of heavy quark symmetry).

**Perturbative:** order $\alpha_s^n(m_Q)$

**Non-perturbative:** order $(\Lambda_{QCD}/m_Q)^n$ analysed in **HQET** (effective QFT with velocity-dependent Q fields, expansion in powers of $1/m_Q$ starting from $m_Q \to \infty$)

## An important result in HQET is "Luke's Theorem"

The leading **1/m$_Q$ correction** to $\bar{B} \to D^* \ell \bar{\nu}$ **vanishes at zero recoil** (not true for $\bar{B} \to D \ell \bar{\nu}$).

We write:

$$\frac{d\Gamma(\overline{B} \to D^* \ell \overline{\nu})}{dw} = \frac{G_F^2}{48\pi^3} (m_B - m_{D^*})^2 \, m_{D^*}^3 \sqrt{w^2 - 1} \, (w+1)^2 \times \left(1 + \frac{4w}{w+1} \frac{m_B^2 - 2w \, m_B m_{D^*} + m_{D^*}^2}{(m_B - m_{D^*})^2}\right) |V_{cb}|^2 \, \mathcal{F}^2(w)$$

Coincides with the Isgur-Wise function up to small symmetry-breaking corrections

Idea is to measure the product $|V_{cb}|\mathscr{F}(w)$ as a function of $w$ and then extrapolate to zero recoil, $w=1$ where the B and D* mesons have a common rest frame, and

$$\mathcal{F}(1) = \eta_A \, \eta_{\rm QED} \left(1 + 0 \times \frac{\Lambda_{\rm QCD}}{m_Q} + {\rm const} \times \frac{\Lambda_{\rm QCD}^2}{m_Q^2} + \ldots \right) \equiv \eta_A \, \eta_{\rm QED} \, (1 + \delta_{1/m^2})$$

QED $\eta \sim 1.007$

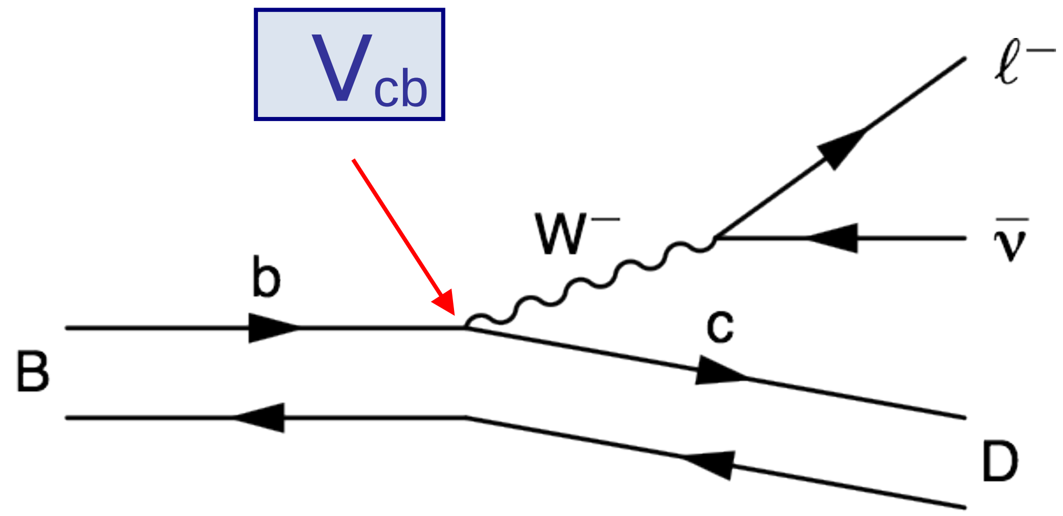
Luke's Theorem

Perturbative QCD: renomalization of flavour-changing axial current at zero recoil $\eta \sim 0.96$

## First approximation: the "spectator model"

The other quark is a pure "spectator";

Plays no role; ignore it.




## More realistic: Hadronic "form factors"

Embed quark-level amplitude inside hadronic one → Form factors

One form factor for each L.I. combination of relevant 4-vectors.

They parametrise the difference between spectator model (form factors =1) and real world.

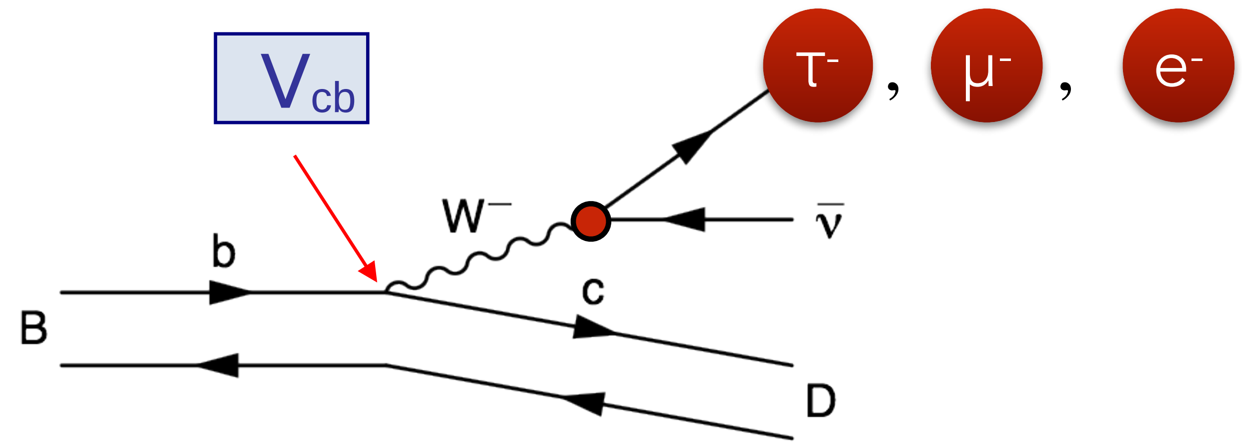
## Use **Heavy Quark Symmetry**: exploit $m_Q \gg \Lambda_{QCD}$

Light-quark cloud insensitive to mass (and spin) of heavy quark: $B^{(*)}$ cloud ~ $D^{(*)}$ cloud.

Physics depends only on velocity change, L.I.: $w = v \cdot v'$, reflected by **Isgur-Wise function** + HQ-symmetry-breaking corrections of order $(\alpha_s)^n$ and $(\Lambda/m_Q)^n$ ➡ **HQET**.

"Luke's Theorem": no $1/m_Q$ correction in $B \to D^* \ell \nu$ (but not so for $B \to D \ell \nu$)

## Apart from measuring $V_{cb}$, we can also use these decays to test "Lepton Universality"; compare different leptons:



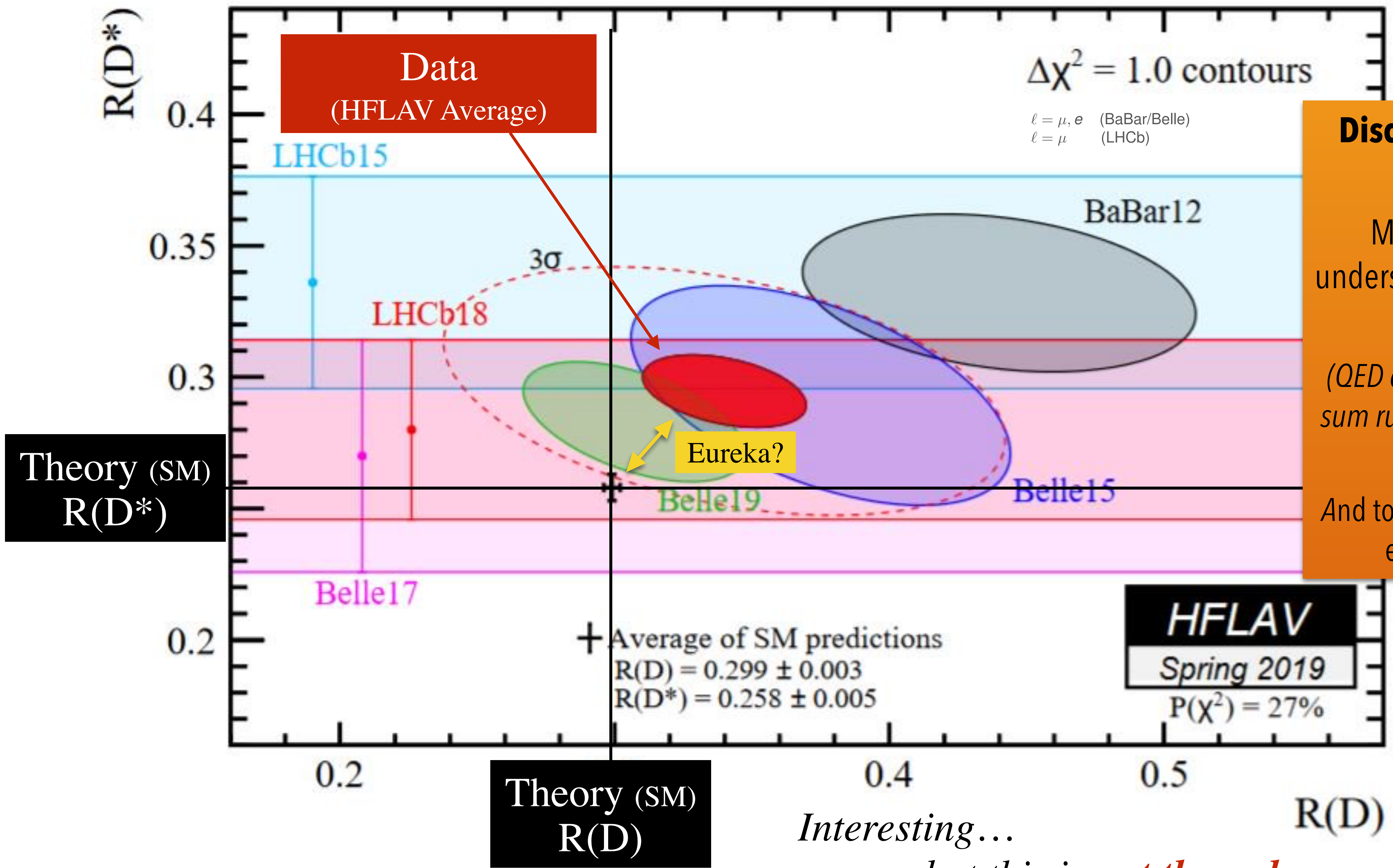

$m_B \sim 5.3\,\text{GeV}$ $\qquad$ $m_D \sim 1.9\,\text{GeV}$

The only difference are the lepton masses: $(m_\tau, m_\mu, m_e) \sim (\,1.8\,,\;\,0.1\,,0.0\,)\,\text{GeV}$

Form two ratios: $R(D) = \dfrac{\text{BR}(B \to D\tau\nu)}{\text{BR}(B \to D\ell\nu)}$ $\qquad$ $R(D^*) = \dfrac{\text{BR}(B \to D^*\tau\nu)}{\text{BR}(B \to D^*\ell\nu)}$

Different masses ⇒ **Expect R≠1** but should be **well approximated** by **calculable functions** of the lepton masses; see eg the dΓ expressions we wrote down previously

# What does the data say?




**Discrepancies with SM "only" ~ 2-3σ.**

Much activity now to understand if theory could be **that** wrong

*(QED effects? HQET expansion, sum rules, lattice all believed to be small)*

*And to provide complementary exp measurements.*

*Interesting…*

*…but this is **not the only anomaly**!*

# Summary of Problems and Exercises for Self Study

**Prove that $\gamma = v \cdot v'$**

**Prove the relation between ($f_+$,$f_-$) and ($F_0$,$F_1$)**

You will present your progress on these in the next lesson and we will discuss any questions / issues you encounter.

---

**Assignment Problem 1: $B \to \tau\nu$**

**Assignment Problem 2: $B \to \mu\nu$**

**Assignment Problem 3 : $B \to D\ell\nu$**