

Modified string tensions

in the context of hadronization

Javira Altmann - Monash University

- **Introduction** to the **Lund String Model**
- **Beyond Lund strings**
 - Time dependent string tensions
 - Excitations



MONASH
University



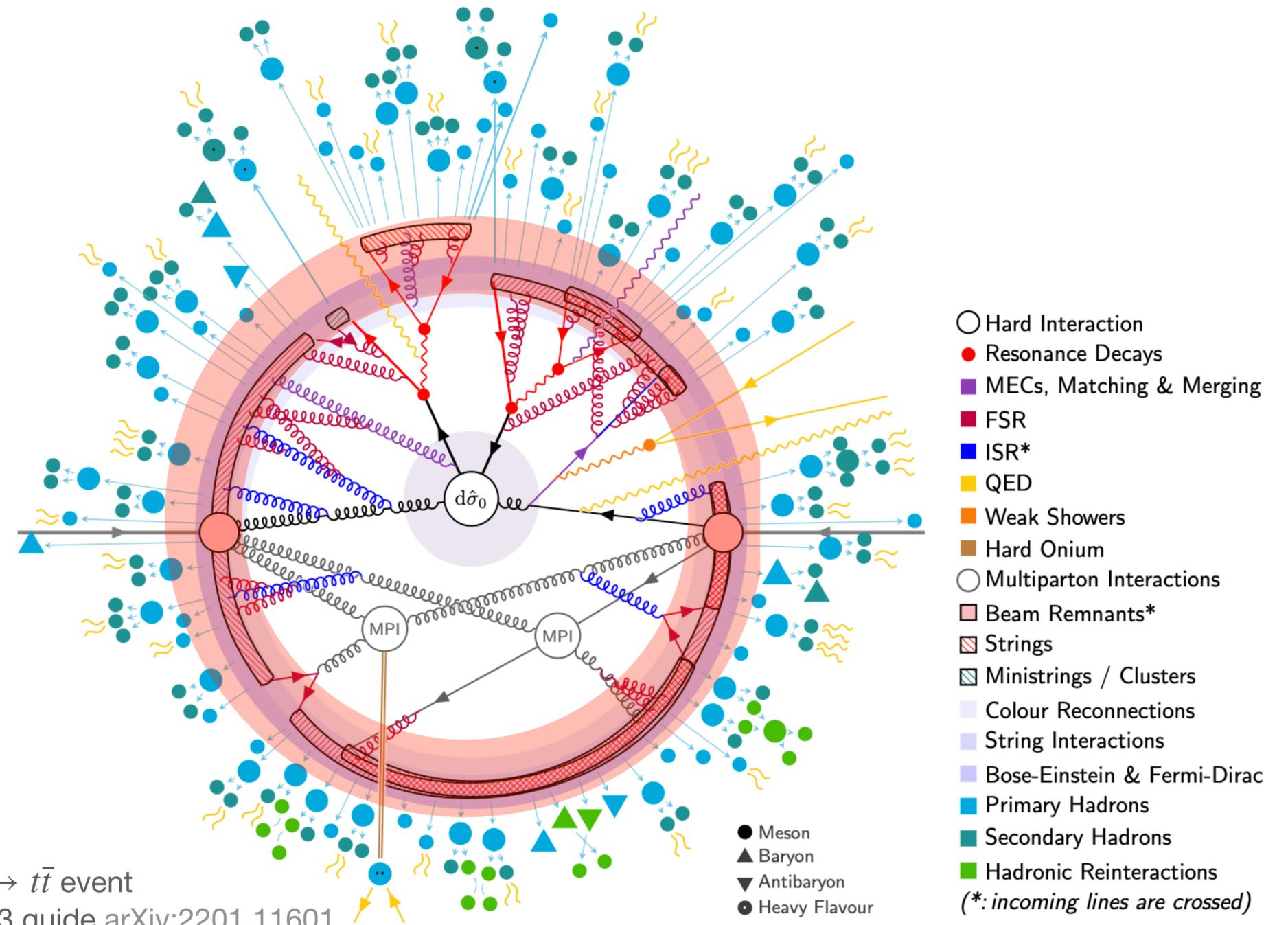
Confinement in high energy collisions

In high-energy collisions, such as proton-proton collisions at the LHC, need a dynamical process to ensure partons (**quarks and gluons**) become **confined** within hadrons

i.e. **non-perturbative**
parton → **hadron map**

Model requirements

- Confinement
- Dynamical mapping to on-shell hadrons



Example of $pp \rightarrow t\bar{t}$ event
From PYTHIA 8.3 guide arXiv:2201.11601

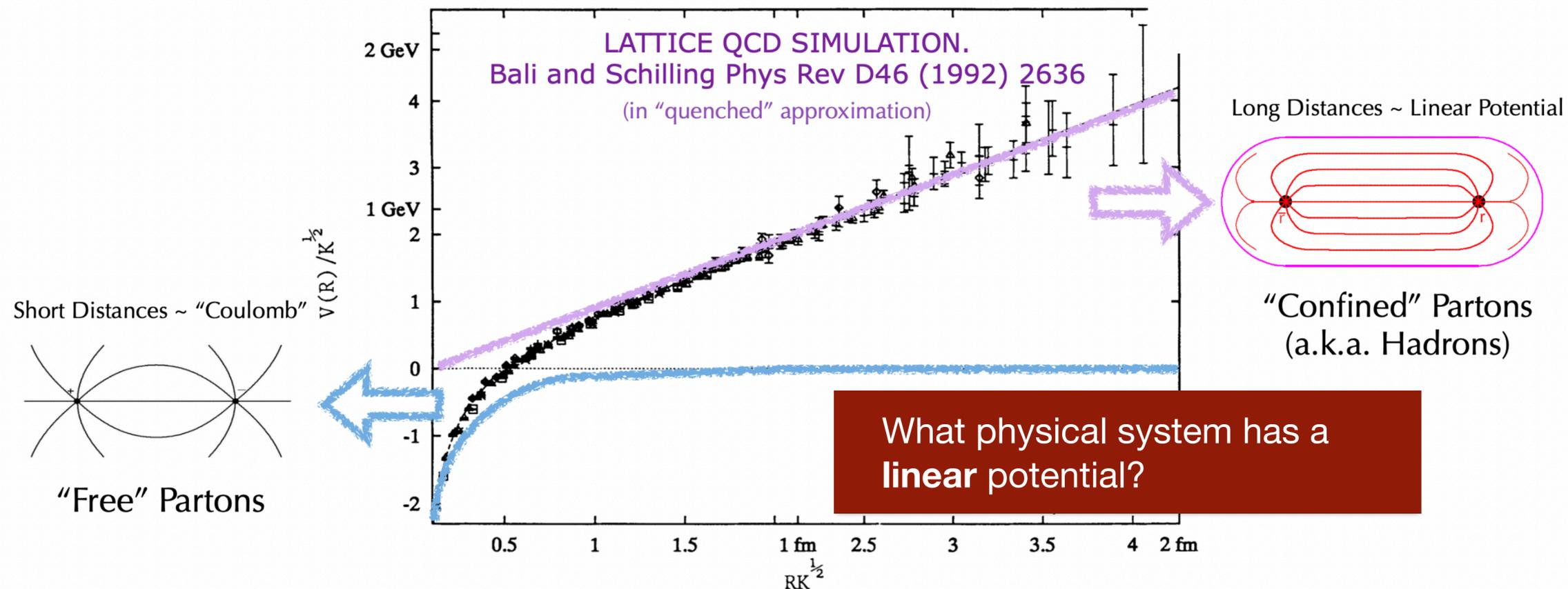
Confinement

Require colour neutralisation:

- The point of confinement is that partons are **coloured** → a physical model needs two or more partons to create **colour neutral** objects. Simplest example is a colour-anticolour $q\bar{q}$ pair

Lattice QCD **“Cornell potential”** $V(r) = -\frac{a}{r} + \kappa r$ with $\kappa \sim 1 \text{ GeV/fm}$

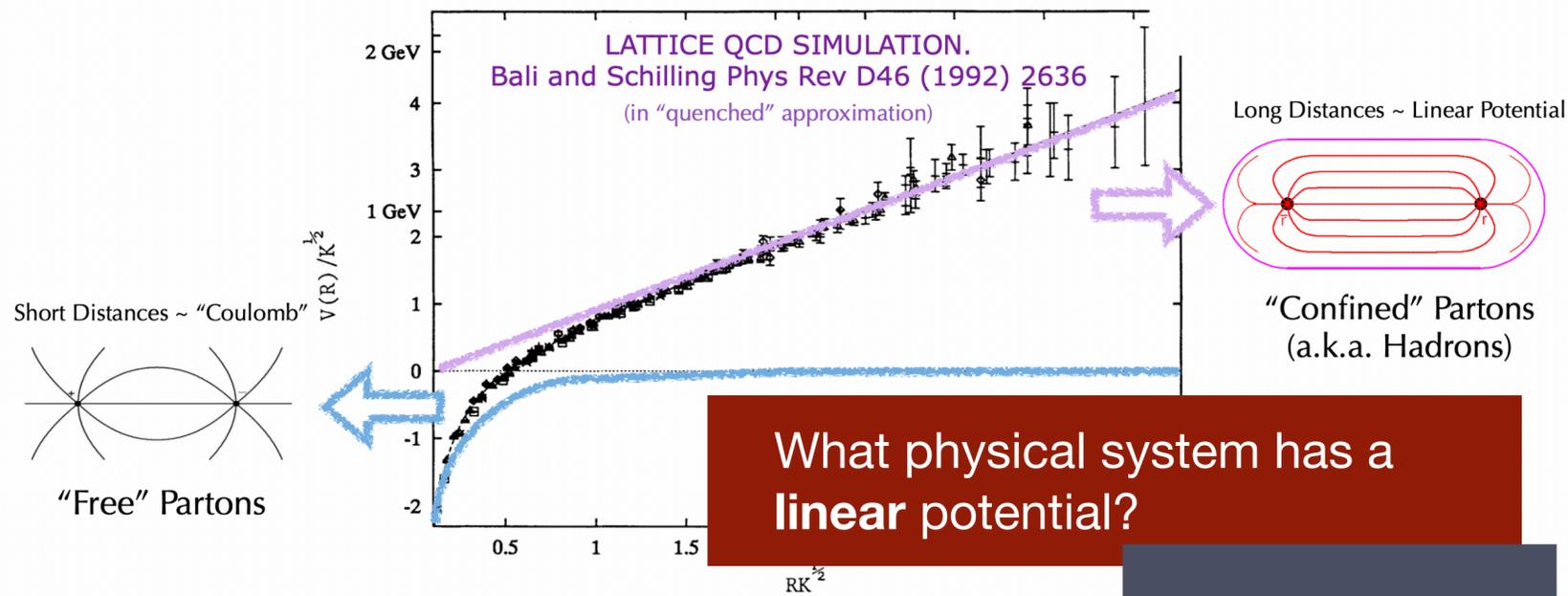
shows us the potential energy of a **static** colour singlet $q\bar{q}$ at separation distance r



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Lund string model

- model the **colour confinement field** as a **string** with a characteristic **constant tension** κ_0
- Strings form between partons that form overall **colour-singlet** states

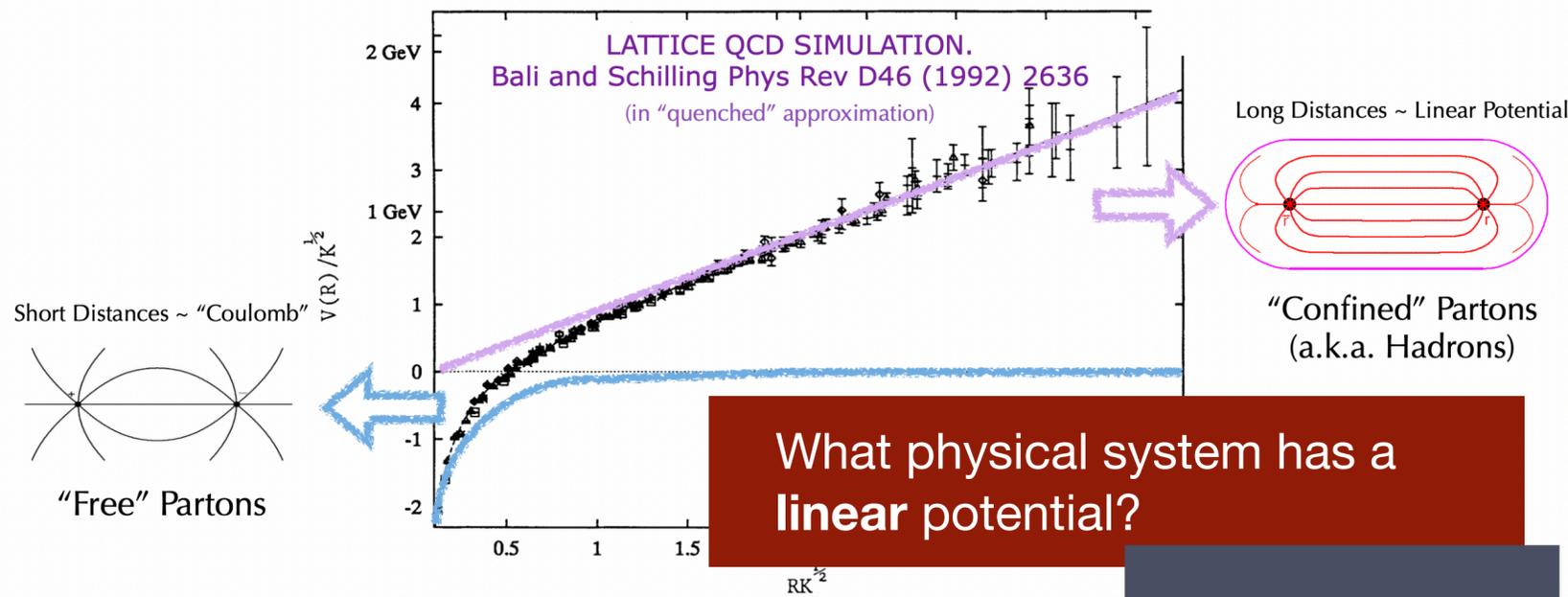


e.g. colour-anticolour singlet combination to make a "dipole" string

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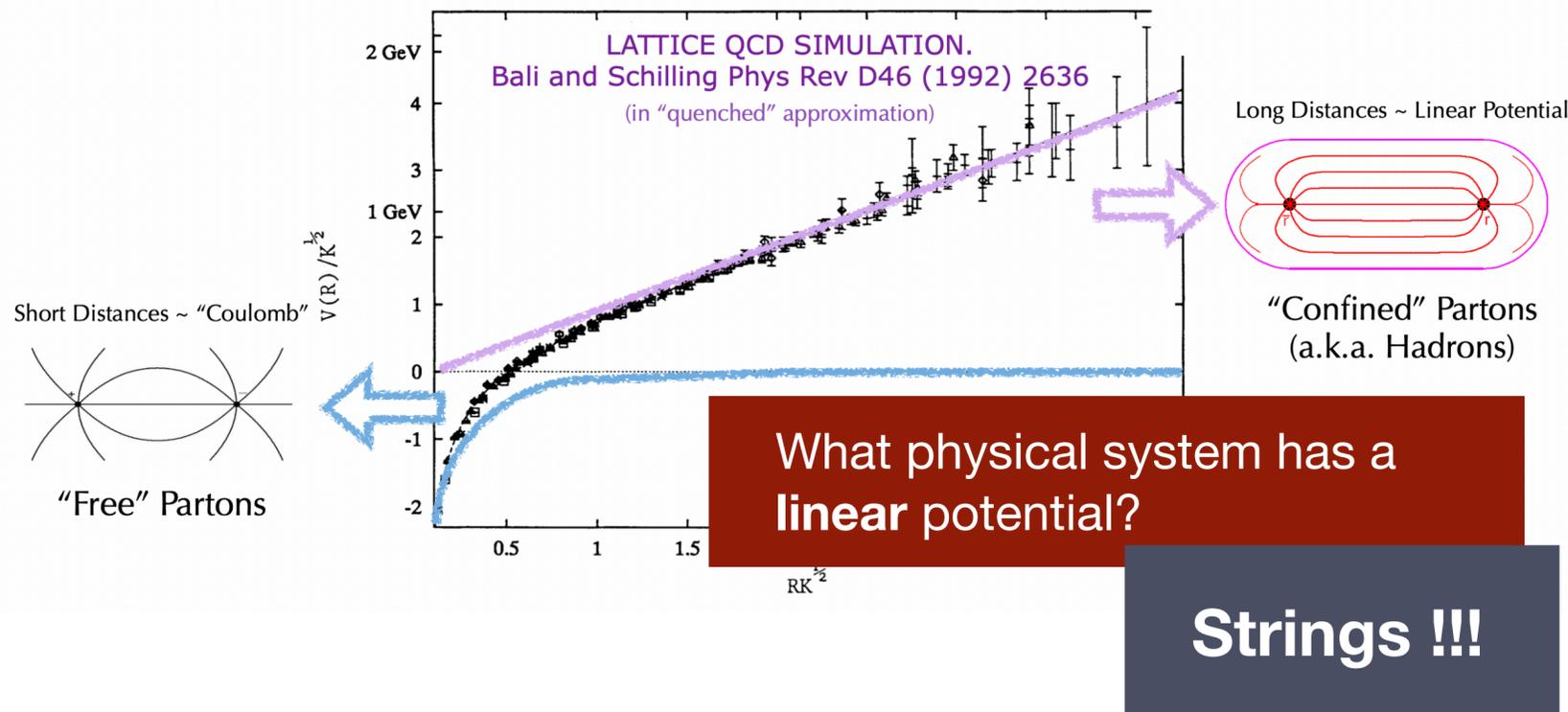
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High energy collisions → partons move apart at high energies

Confinement

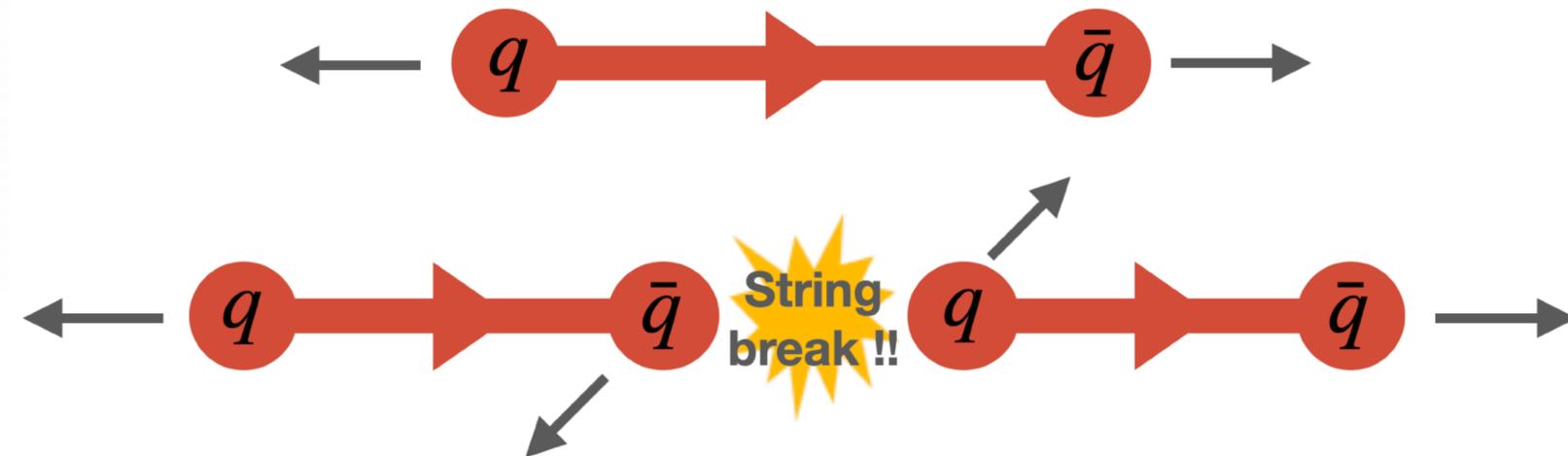
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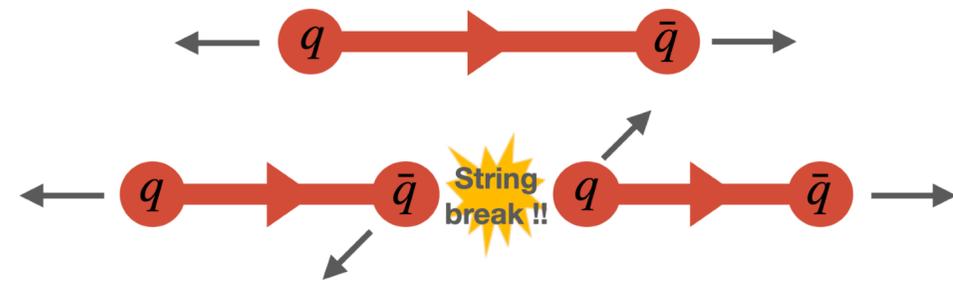


Partons \rightarrow Hadrons

Hadronization:

Partons move apart and stretch the string \rightarrow **string breaks**

\rightarrow creates **quark-antiquark pairs** with some momentum



Two things we need to describe the momentum of the produced hadrons

\rightarrow transverse component (quark p_{\perp} and mass/flavour)

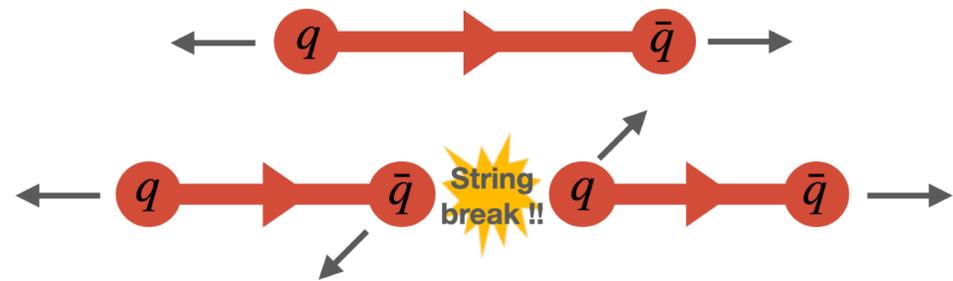
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Flavour and p_{\perp} selection

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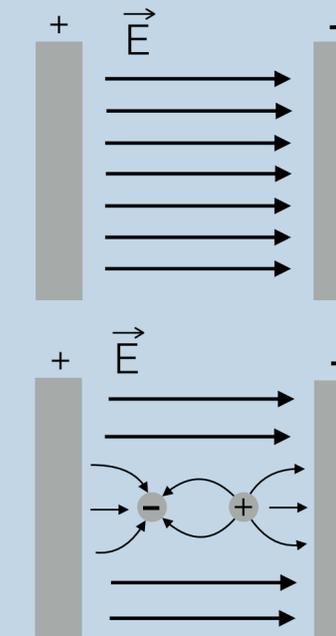


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Schwinger mechanism QED



Non-perturbative creation of e^+e^- pairs in a strong electric field

Probability from tunnelling factor

$$\mathcal{P} \propto \exp\left(\frac{-m^2 - p_{\perp}^2}{\kappa/\pi}\right)$$

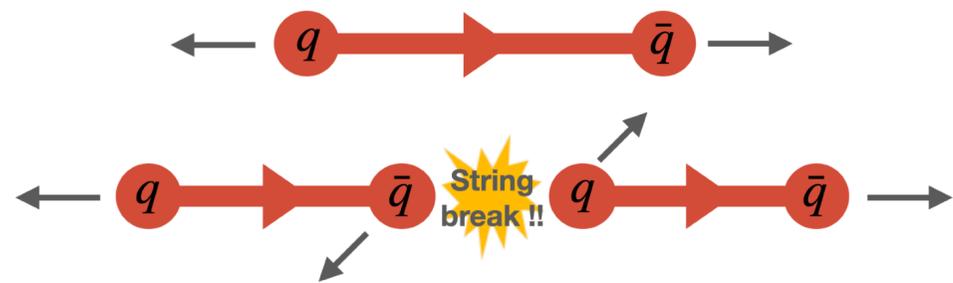
Gaussian suppression of high $m_{\perp} = \sqrt{m_q^2 + p_{\perp}^2}$

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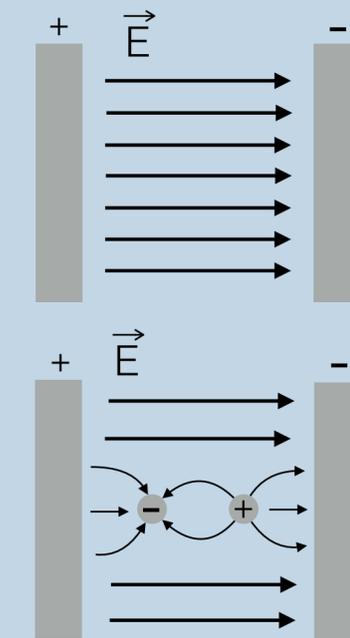


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Schwinger mechanism

- \rightarrow Gaussian p_{\perp} spectrum
- \rightarrow Heavy flavour suppression

constant flavour probability along the string

Prob(u:d:s) \approx 1 : 1 : 0.2

Prob(q:qq) \approx 1 : 0.081

Baryon formation: **diquark-antidiquark** pair creation

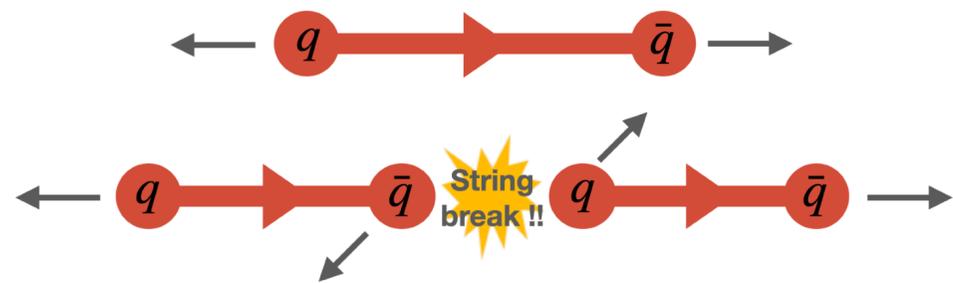


Lund Symmetric Fragmentation Function

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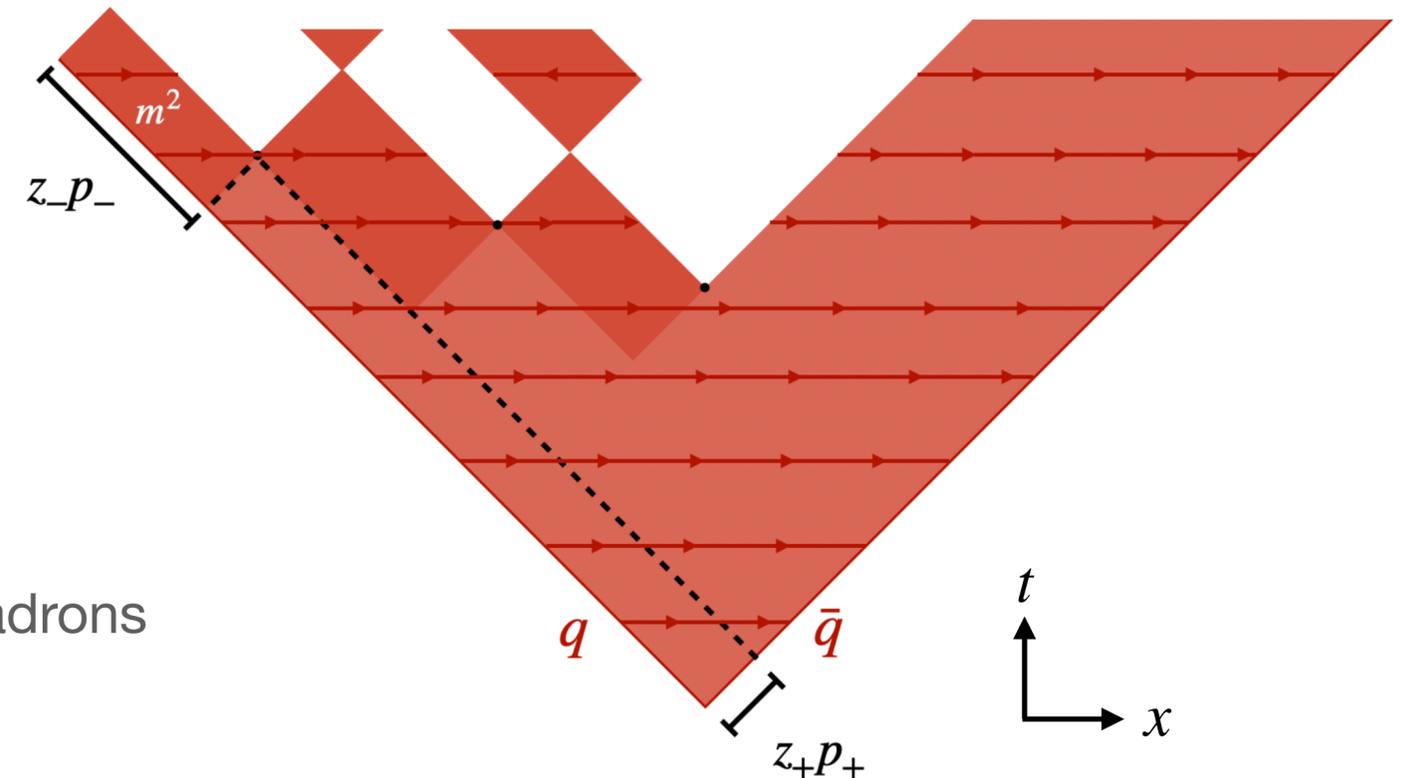
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$$f(z) \propto \frac{1}{z} (1-z)^a \exp\left(\frac{-b(m_h^2 + p_{\perp h}^2)}{z}\right)$$

Free tuneable parameters a and b



Fragment off hadrons from either **string end**

→ easier to obey hadronic mass constraints

Probability distribution for the **fraction of quark**

momenta, z , the hadron will take

Lund string model assumptions

Assumptions made by the Lund string model

- String fragmentation in **dense string environment** is treated the same as vacuum string fragmentation

→ closepacking/ropes

- Treatment of **diquarks** as forming directly from Schwinger-type breaks

→ popcorn

- Beyond just colour-anticolour singlet states, what about **red-green-blue singlets**

→ junctions

- **Constant string tension** from motivated from the Cornell potential i.e. potential between **static** colour charges

→ beyond a constant string tension

- time dependent string tensions
- excitations on the string

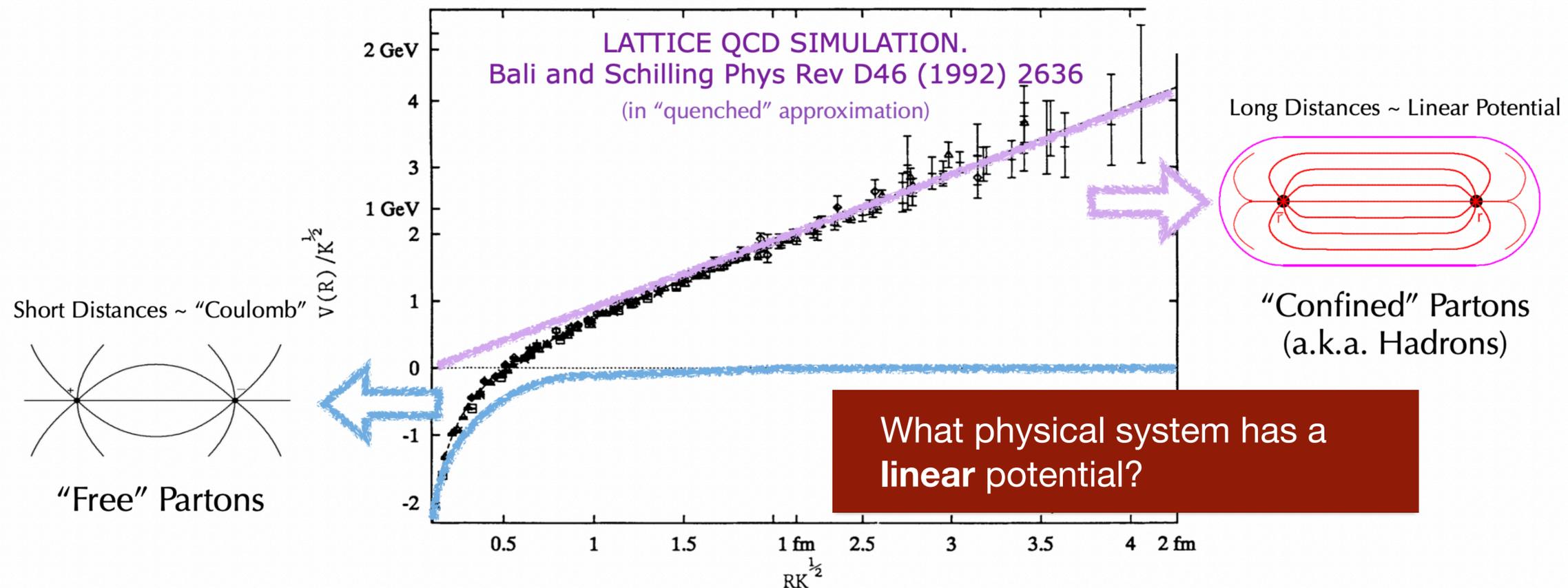
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- effect on Schwinger mechanism
- fragmentation procedure
- coordinates along a string

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Schwinger Mechanism

Schwinger mechanism:

→ Gaussian **suppression of masses**

Parameterise flavour probabilities in Pythia as ratios e.g. $P_{s:u/d}$, $P_{qq:q}$, etc.

Here κ_0 is the constant string tension

$$P_{s:u/d} = \frac{P(m_s^2)}{P(m_{u/d}^2)} = \frac{\exp\left(\frac{-\pi m_s^2}{\kappa_0}\right)}{\exp\left(\frac{-\pi m_{u/d}^2}{\kappa_0}\right)} = \exp\left(\frac{-\pi(m_s^2 - m_{u/d}^2)}{\kappa_0}\right)$$

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Schwinger mechanism:

→ Gaussian **suppression of masses**

Parameterise flavour probabilities in Pythia as ratios e.g. $P_{s:uld}$, $P_{qq:q}$, etc.

Introduce some effective tension, $\kappa_{eff}(\tau, \sigma, \dots)$
which can be a function of coordinates on the string,
or the surrounding environment, etc.

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Larger tension reduces mass suppression!!!

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$$\sigma(\kappa_{eff})^2 = \frac{\kappa_{eff}}{\pi} = \frac{\kappa_0}{\pi} \frac{\kappa_{eff}}{\kappa_0} = \sigma^2 \frac{\kappa_{eff}}{\kappa_0}$$

Diquark probabilities

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- coordinates along a string

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Fragmentation procedure

How can we put a **non-constant tension**, κ_{eff} , on the string? e.g. $\kappa(\tau)$ or $\kappa(y)$

Standard fragmentation procedure

- 1) Select **flavour and p_{\perp}**
- 2) Select **z -fraction** according to fragmentation function $f(z, m_{\perp h}^2)$

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→ need m_{\perp} to sample $f(z, m_{\perp}^2)$, **BUT κ_{eff} is needed to calculate m_{\perp}**

Stuck in a loop !!!

Modified fragmentation procedure

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Solution: **overestimate** m_{\perp} **distribution** and **accept/reject** string breaks to correct overestimation

Overestimate

$$\mathcal{P}_s(\kappa_{eff}) = P_{acc}(\kappa_{eff}) \mathcal{P}_s(\kappa_{max})$$

Probability we need
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$$\mathcal{P}_s(\kappa_{eff}) = \frac{P_{s:uld}(\kappa_{eff})}{2 + P_{s:uld}(\kappa_{eff})}$$

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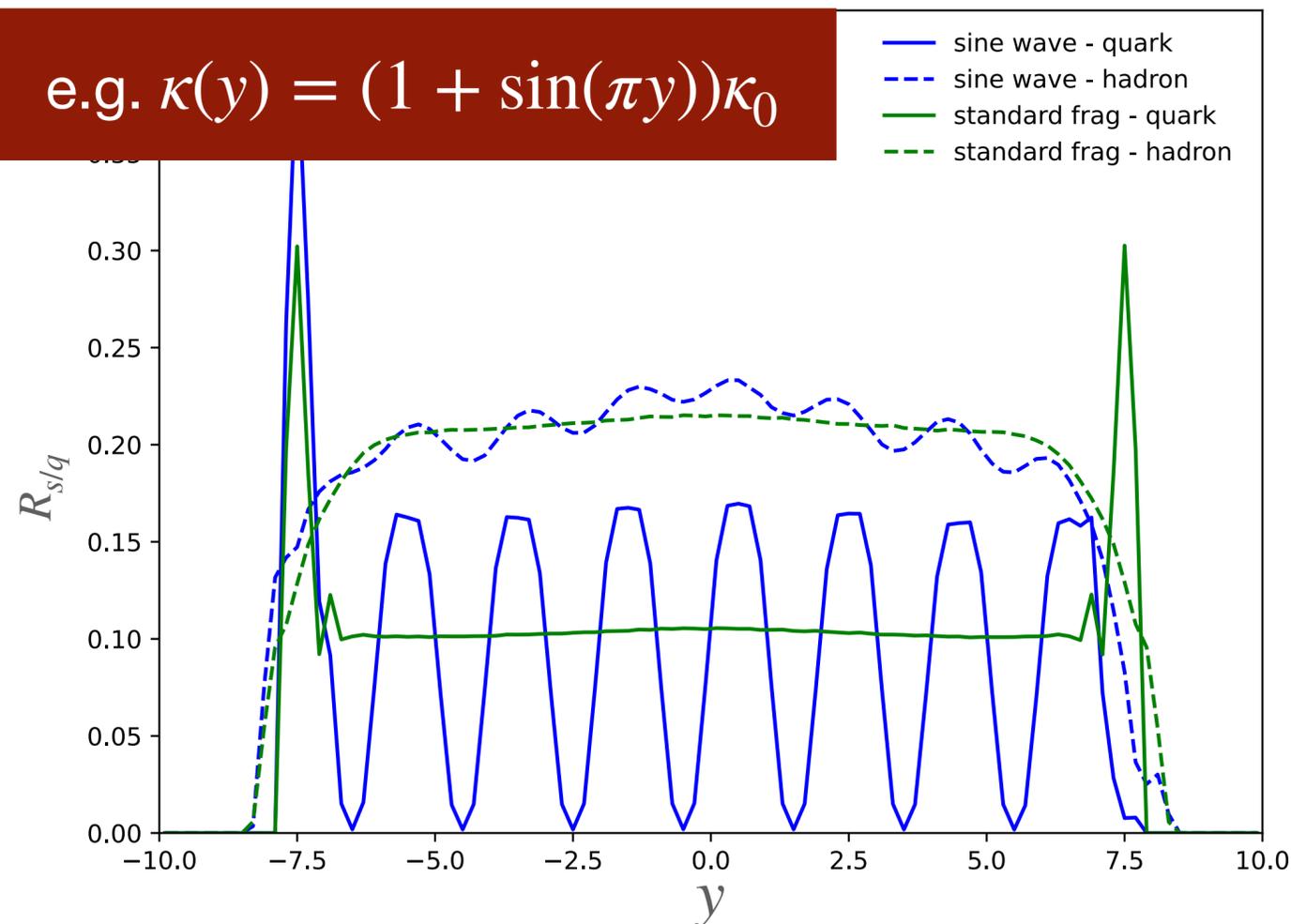
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e.g. $\kappa(y) = (1 + \sin(\pi y))\kappa_0$



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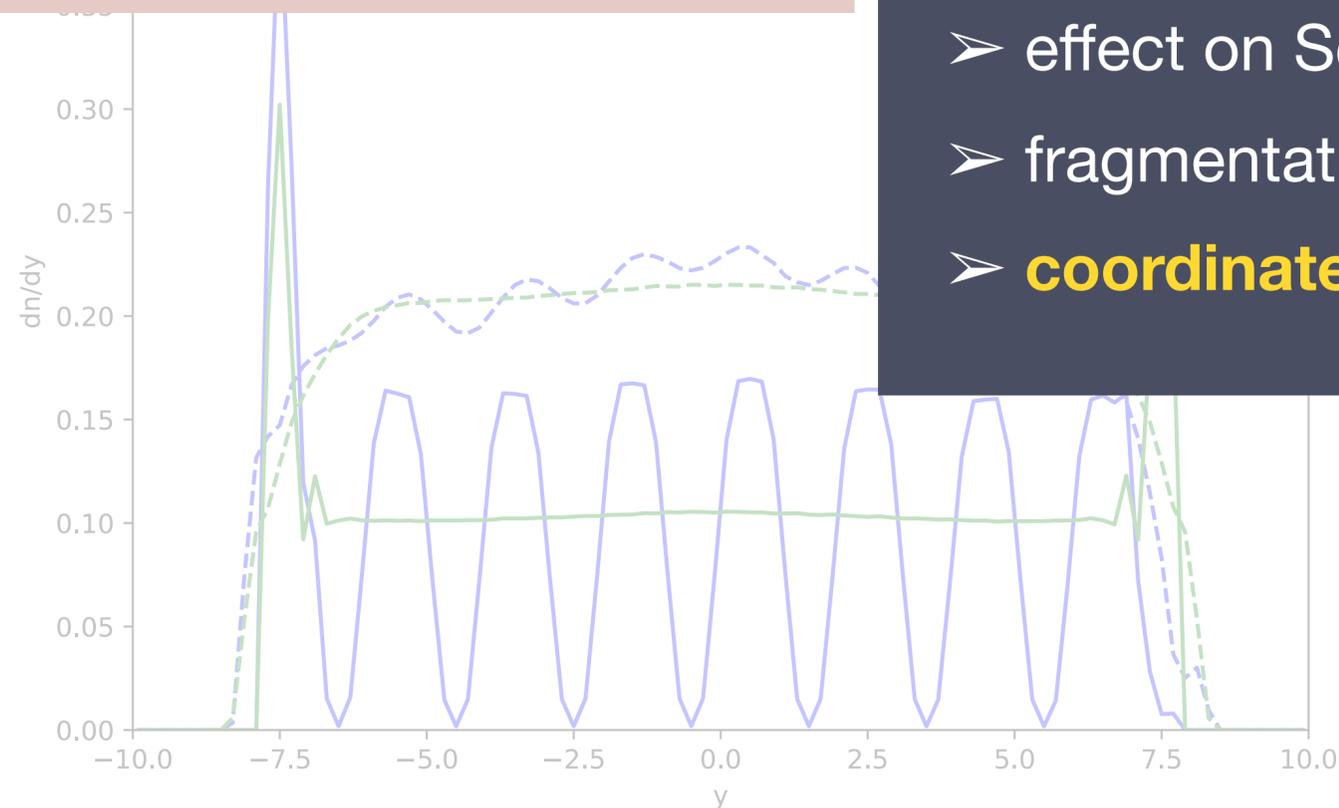
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- effect on Schwinger mechanism ✓
- fragmentation procedure ✓
- **coordinates along a string**

varying tension

break **flavour with overestimate**

according to $f(z, m_{\perp})$

coordinates and therefore κ_{eff}

string break

What about space-time coordinates for e.g. $\kappa(\tau)$?

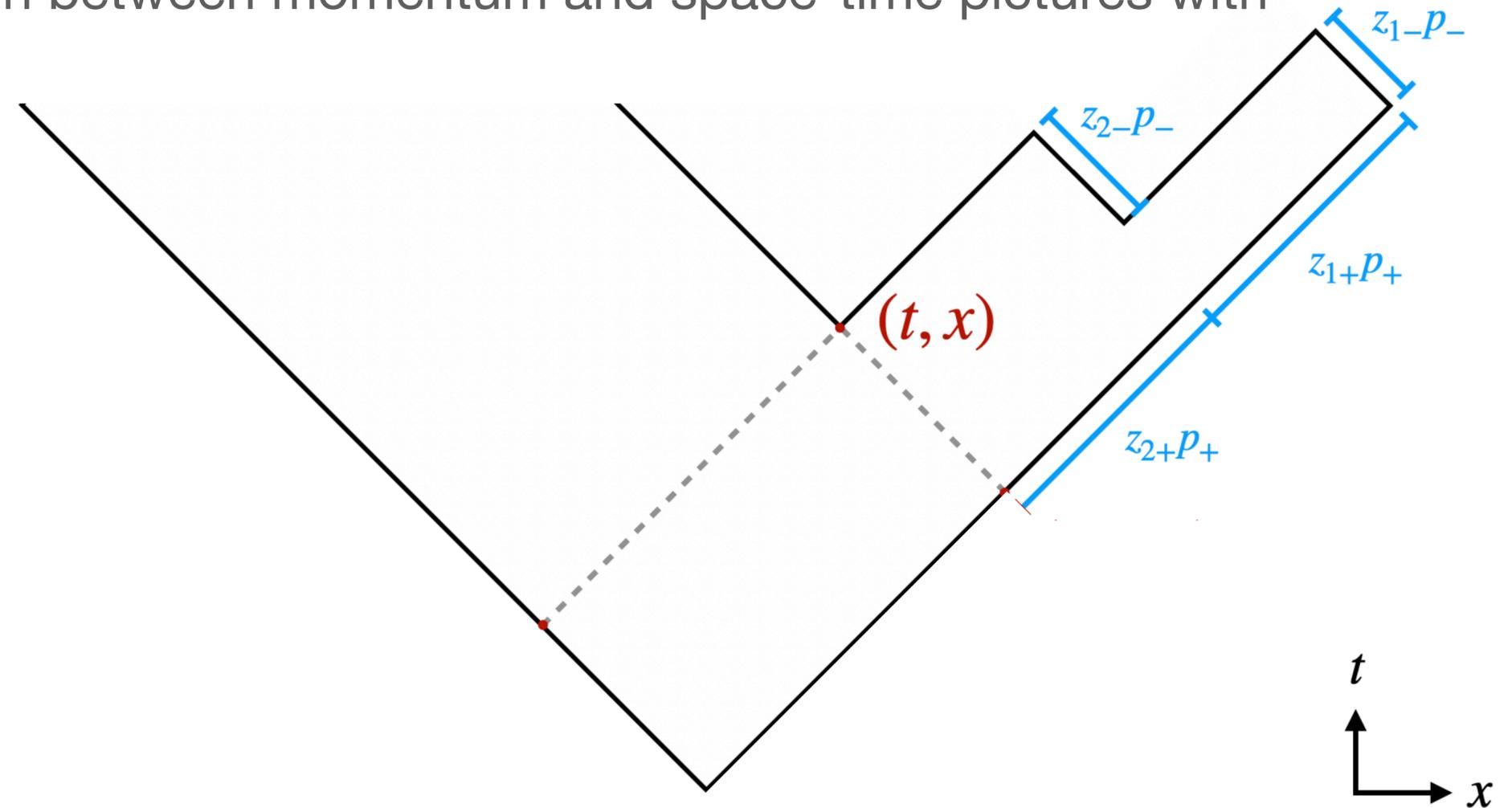
$$\Gamma_{acc}(\kappa_{eff}) = \frac{1}{\mathcal{P}_s(\kappa_{max})}$$

{t, x} coordinate calculation

Fragmentation procedure is a fully **momentum-space** picture

Constant string tension — conversion between momentum and space-time pictures with **factors of κ** , e.g. $\Gamma = \kappa_0^2 \tau^2$

What if we have a $\kappa(\tau)$?

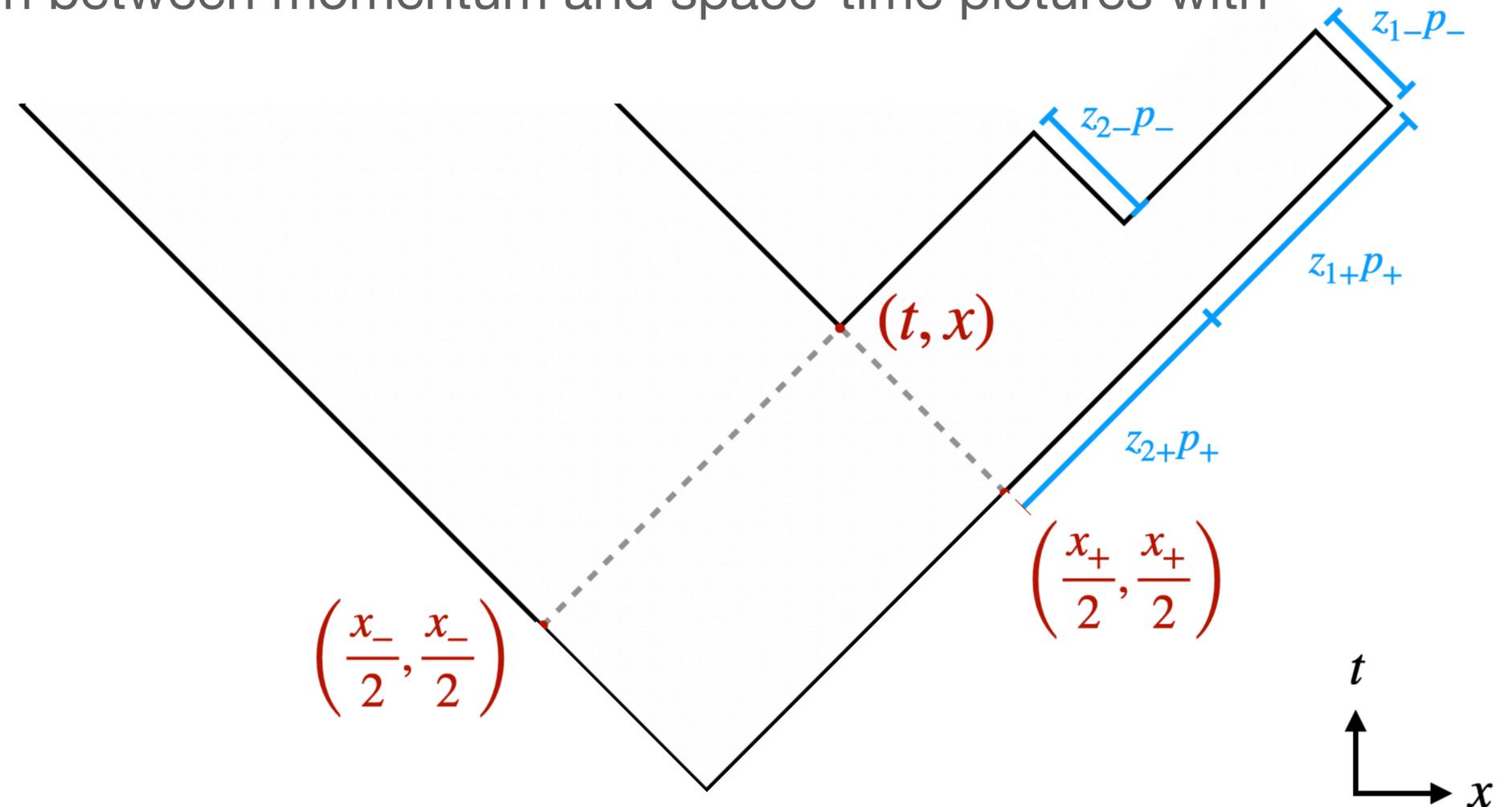


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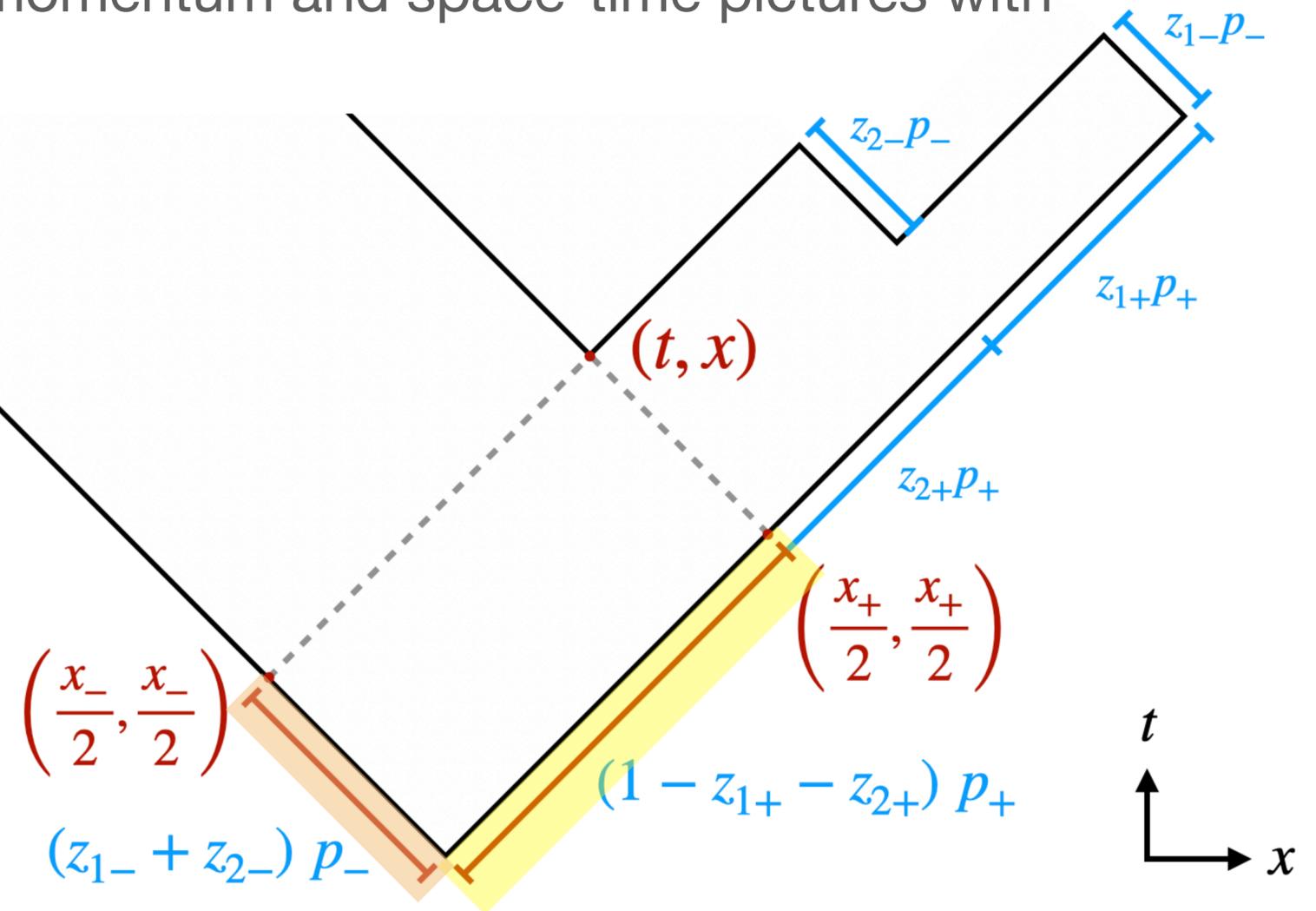


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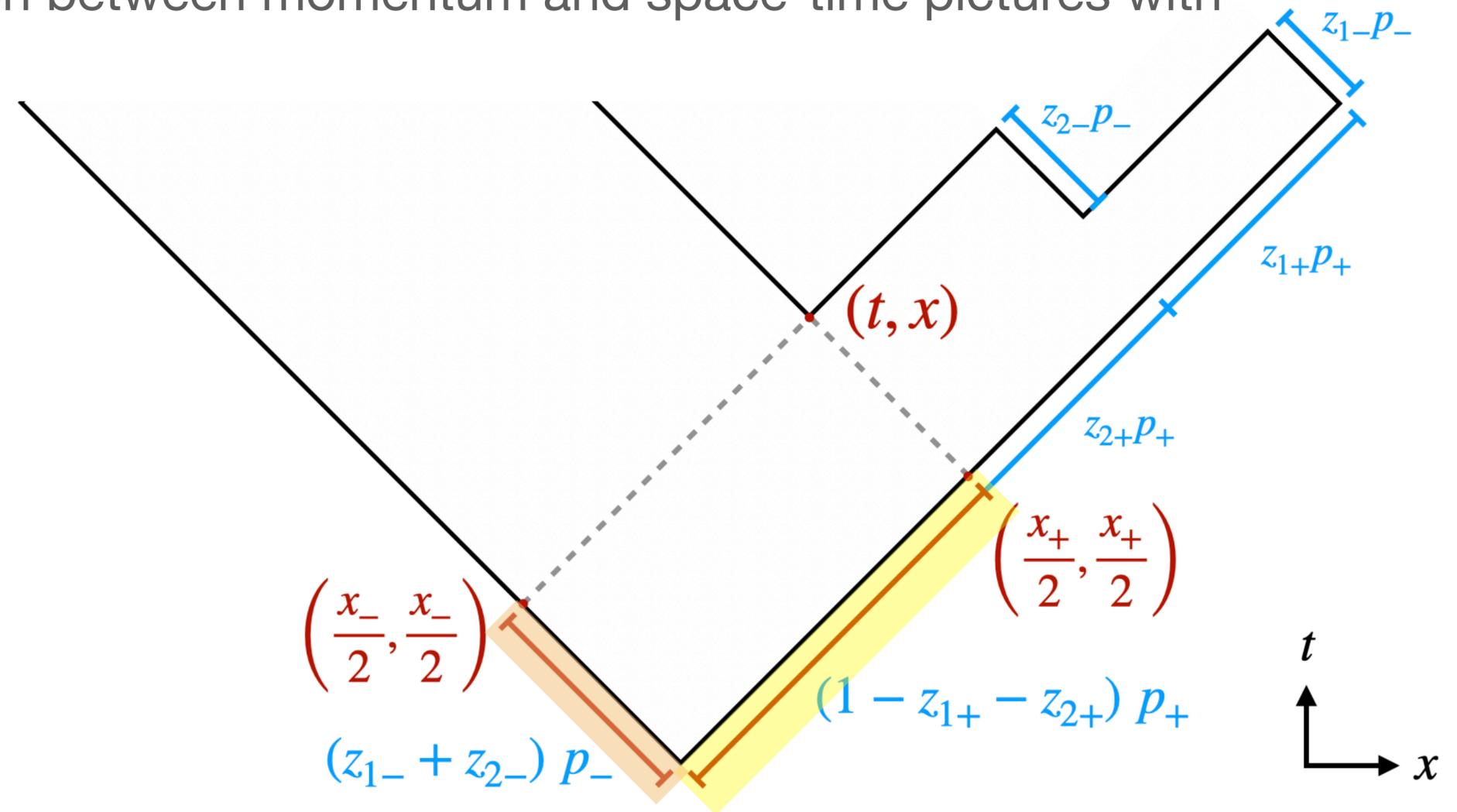
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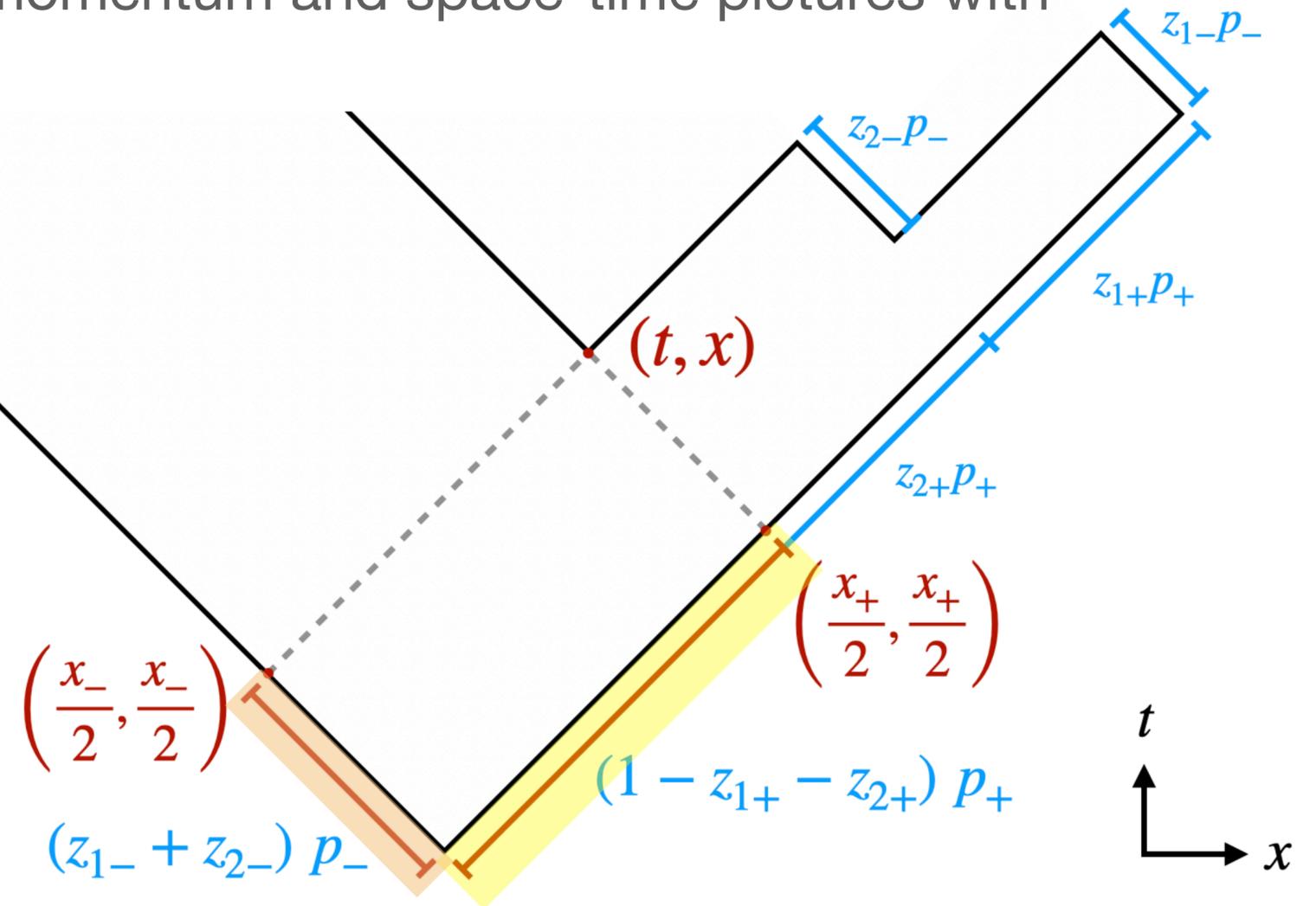
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$$\frac{dE_q}{dt} = \frac{dp_q}{dt} = -\frac{1}{2} \frac{dV(t)}{dt} = -\frac{1}{2} \frac{d}{dt} \int_{-t}^t \kappa(t, x) dx$$



Work in progress: generalisation to gluon kinks

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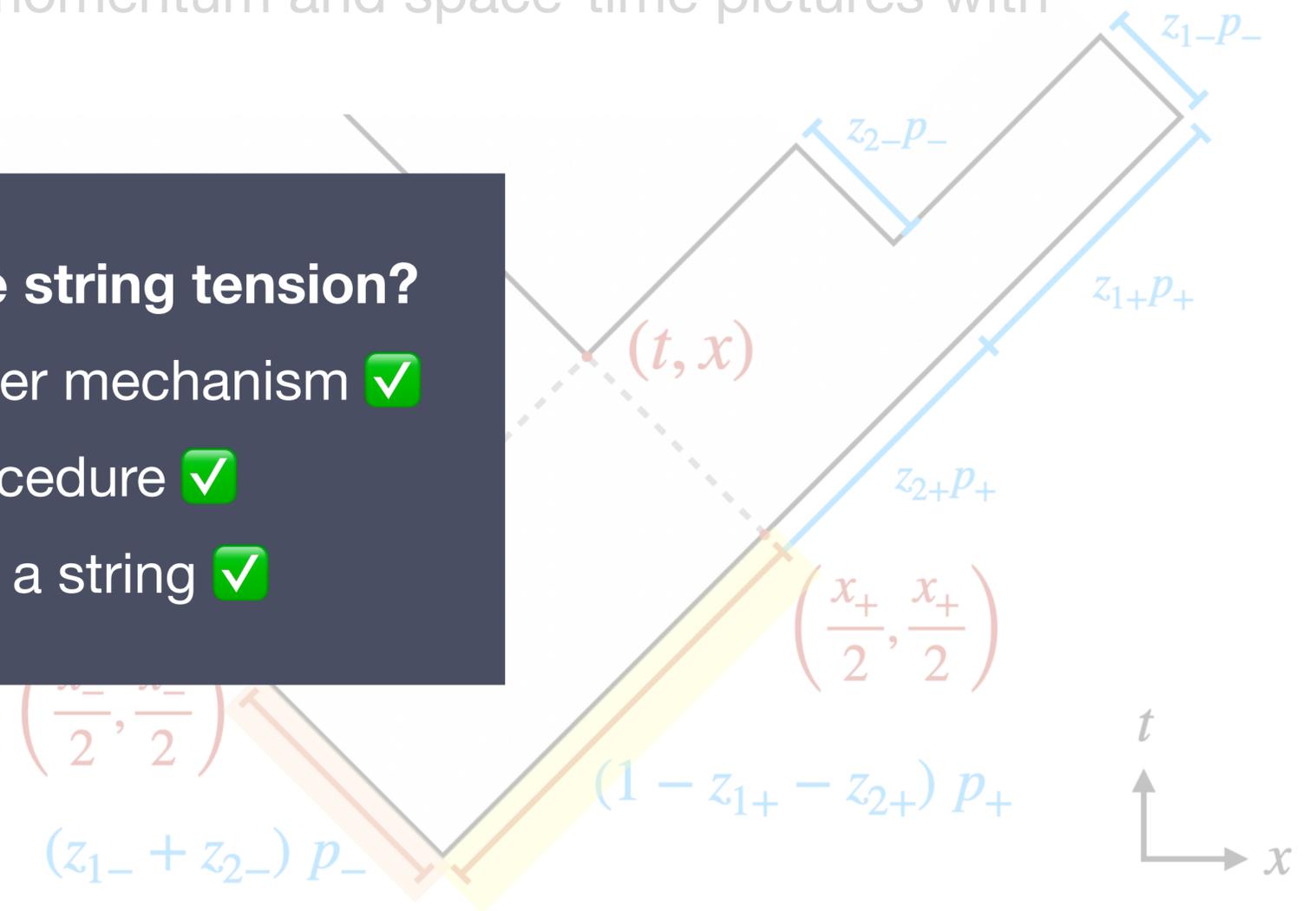
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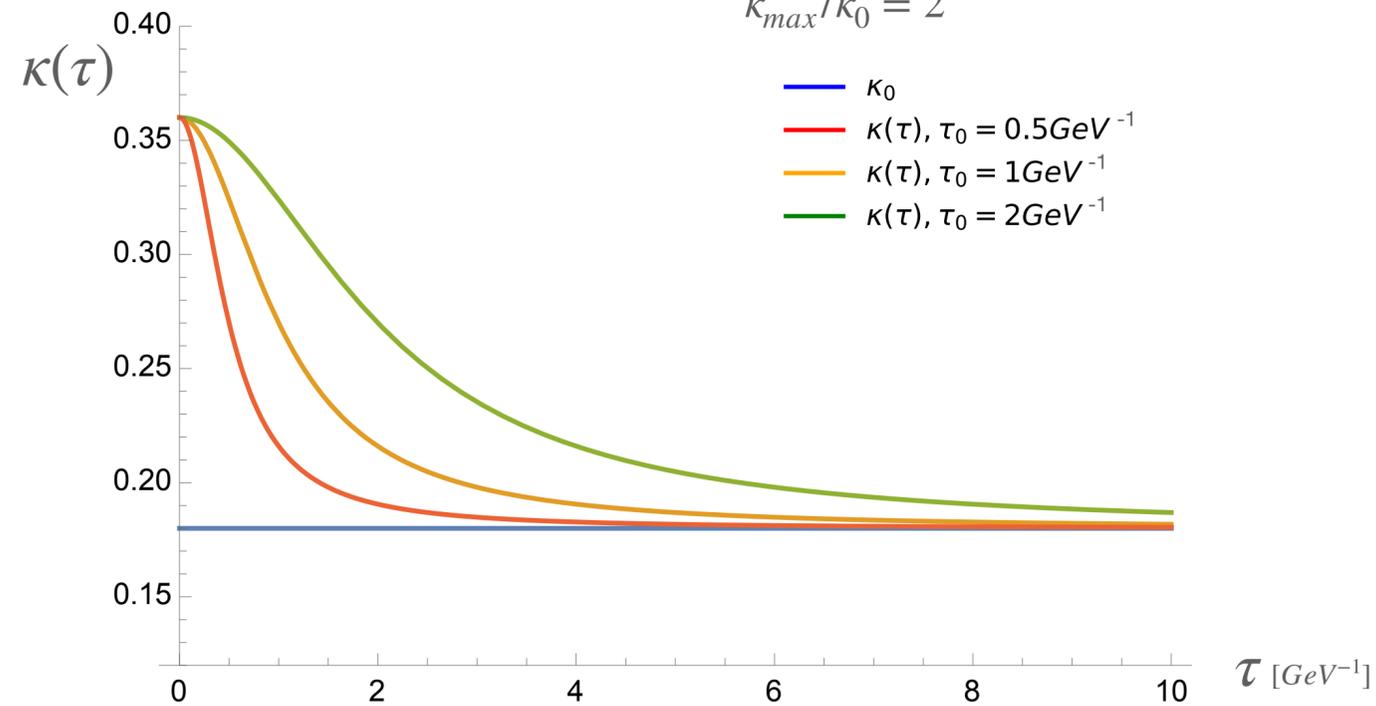
Tau-dependent example

e.g. higher tensions at early times

$$\kappa(\tau) = \kappa_0 \left(1 + \frac{\left(\frac{\kappa_{max}}{\kappa_0} - 1 \right)}{1 + \frac{\tau^2}{\tau_0^2}} \right)$$

$$\tau^2 = t^2 - x^2$$

$$\kappa_{max}/\kappa_0 = 2$$



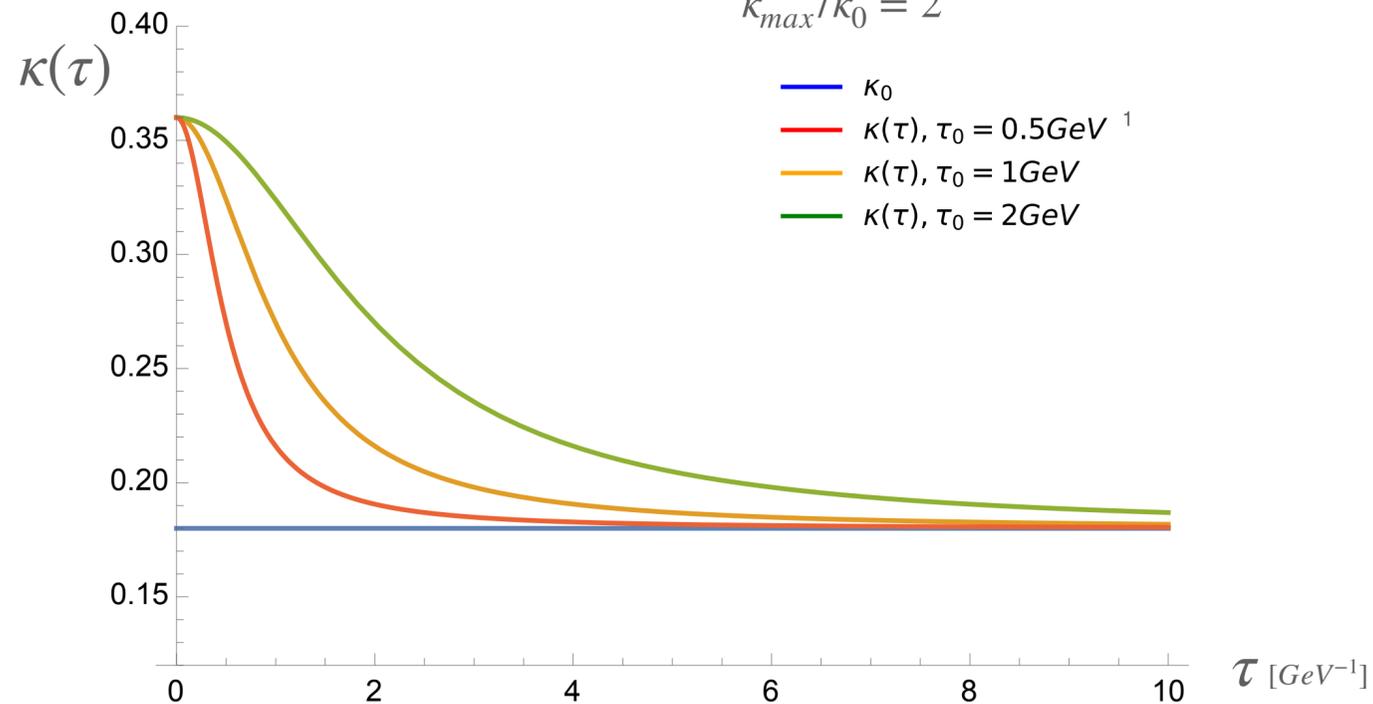
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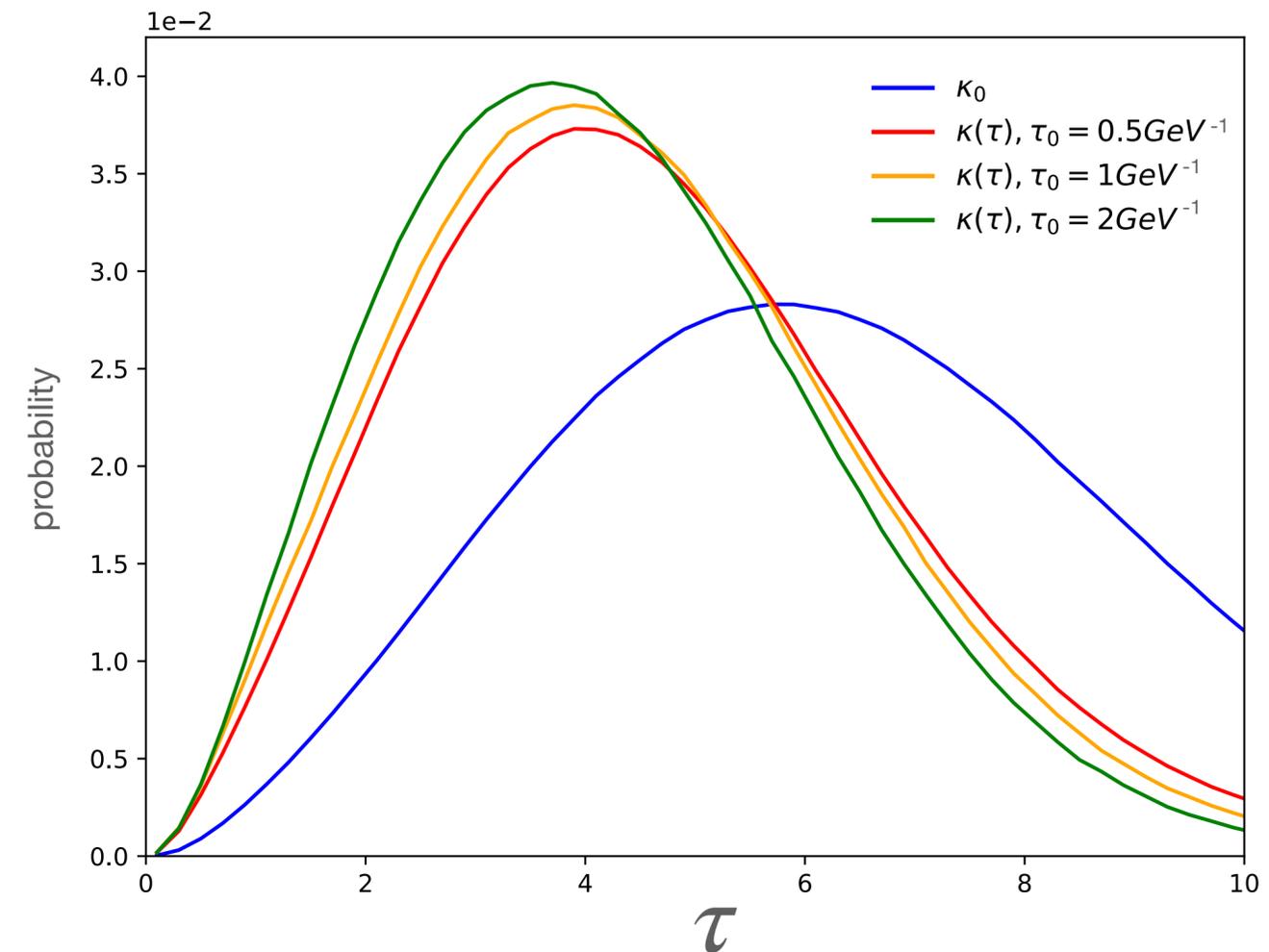
$$\tau^2 = t^2 - x^2$$

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Expected consequences of an increased string tension

➤ earlier string breaks



Massless up quark endpoints on a $q\bar{q}$ string with 1000 GeV

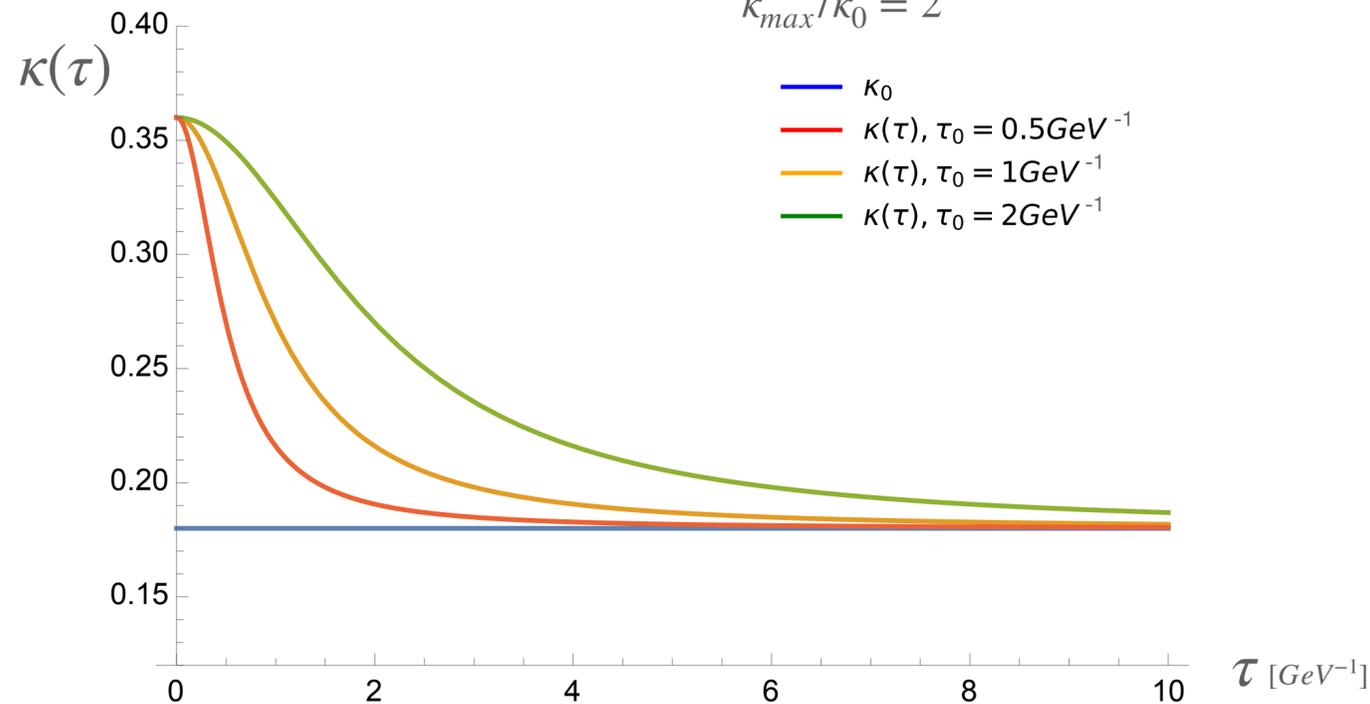
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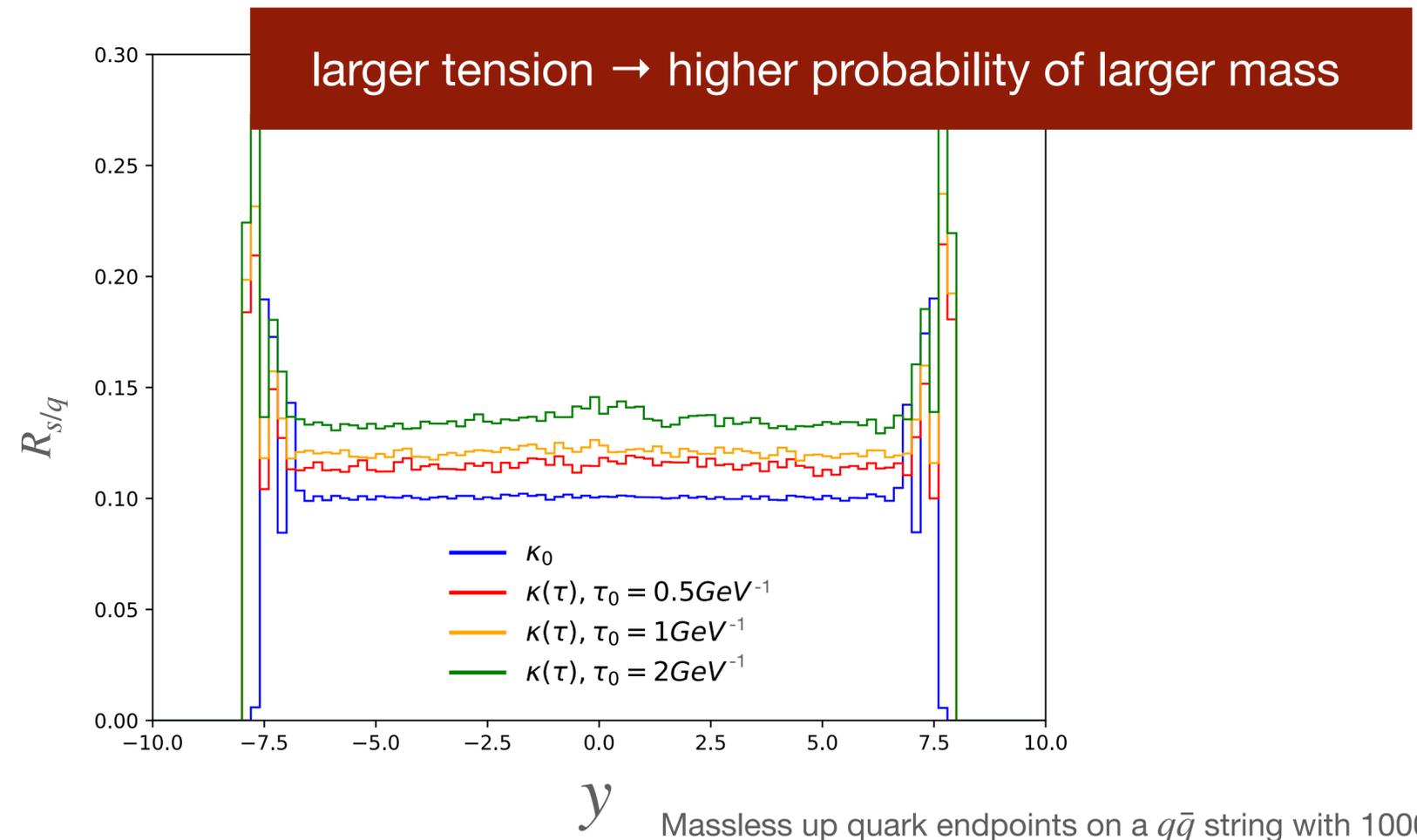
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Expected consequences of an increased string tension

- earlier string breaks
- higher strange / diquark probabilities



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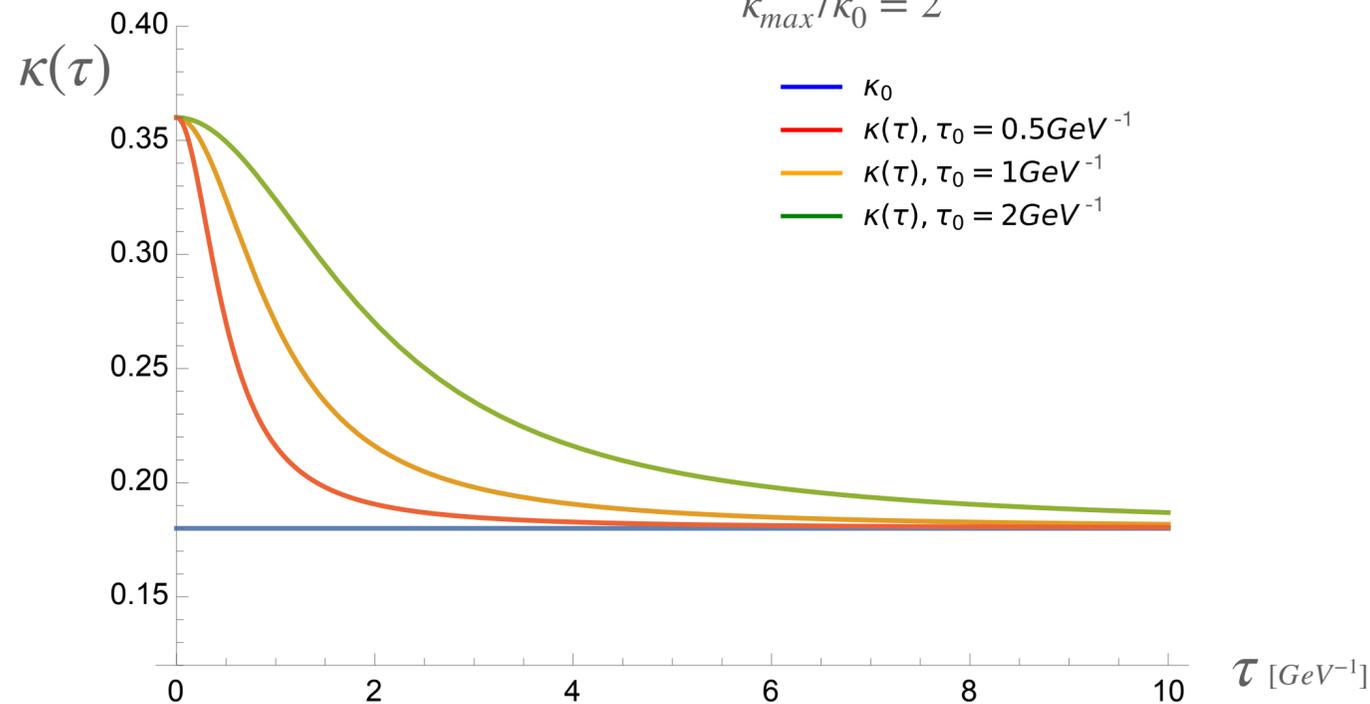
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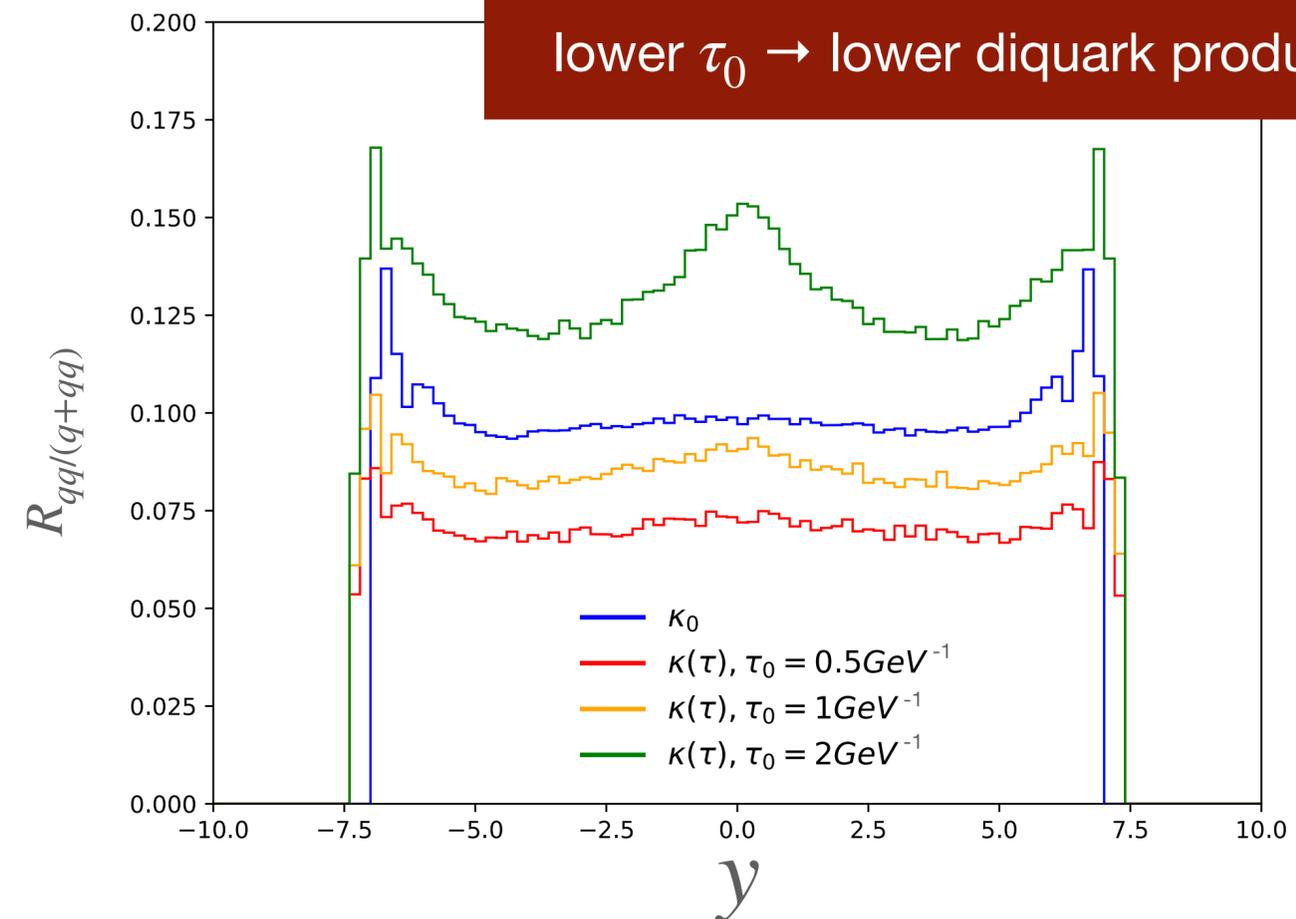
$$\tau^2 = t^2 - x^2$$

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Expected consequences of an increased string tension

- earlier string breaks
- higher strange / diquark probabilities



lower $\tau_0 \rightarrow$ lower diquark production ratios?

Massless up quark endpoints on a $q\bar{q}$ string with 1000 GeV

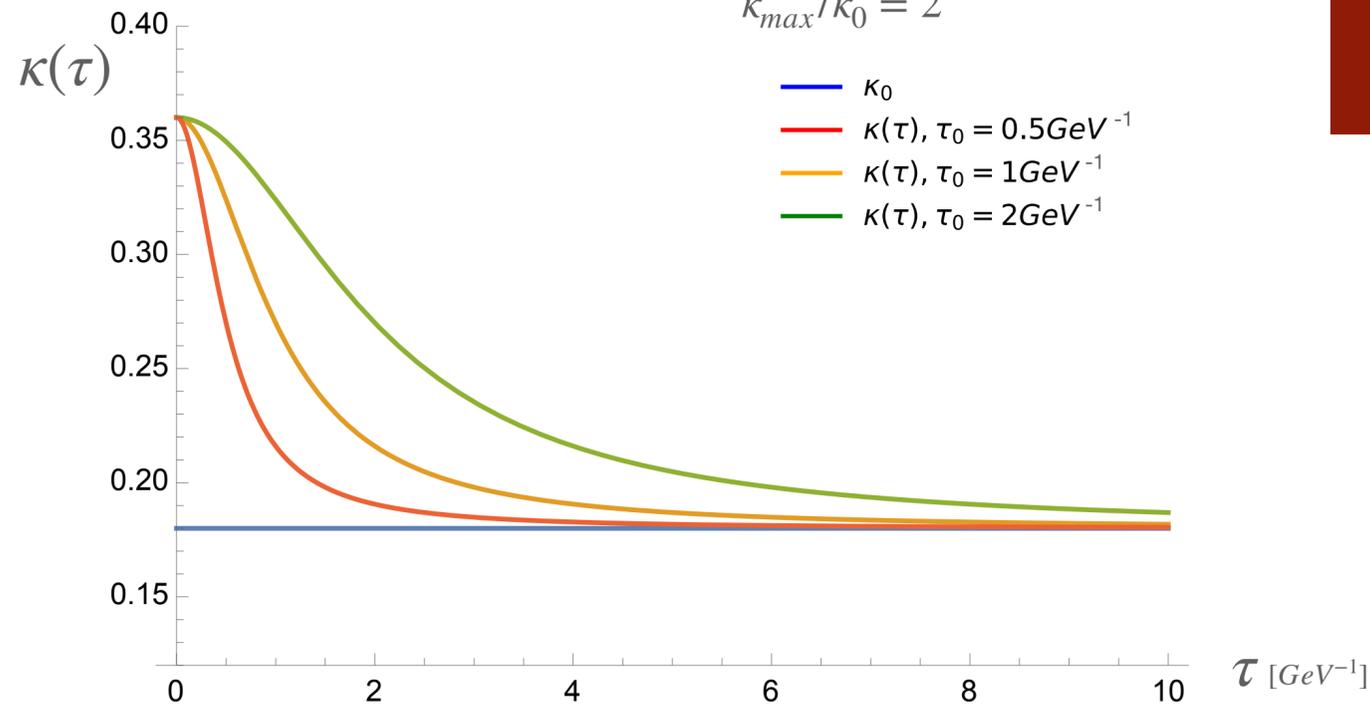
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Expected consequences of an increased string tension

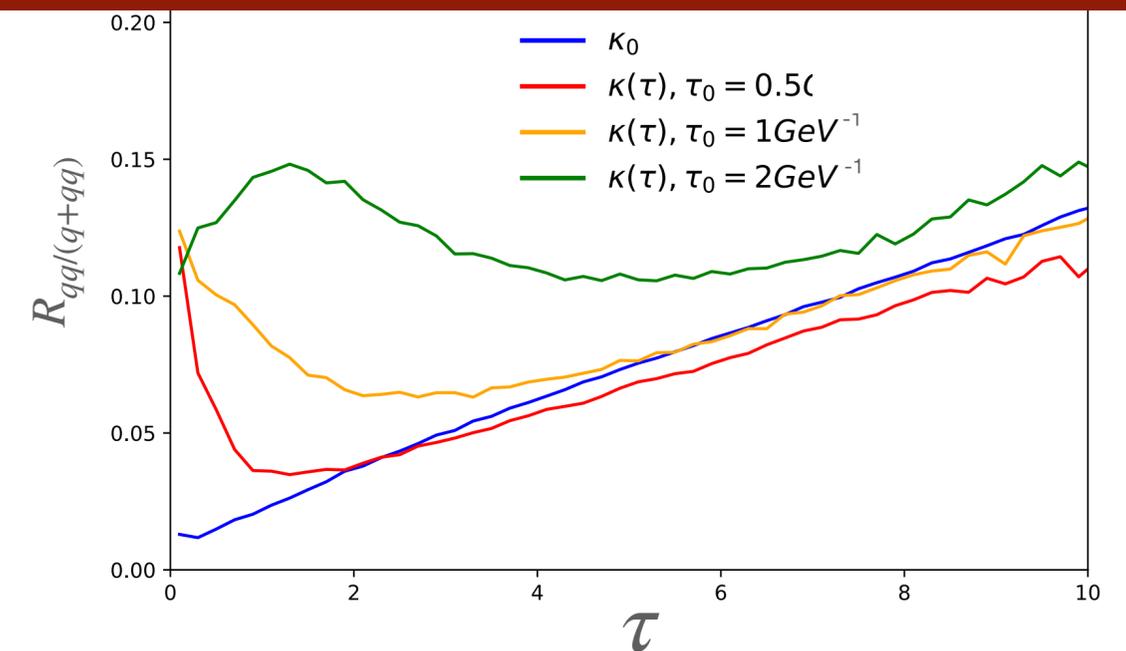
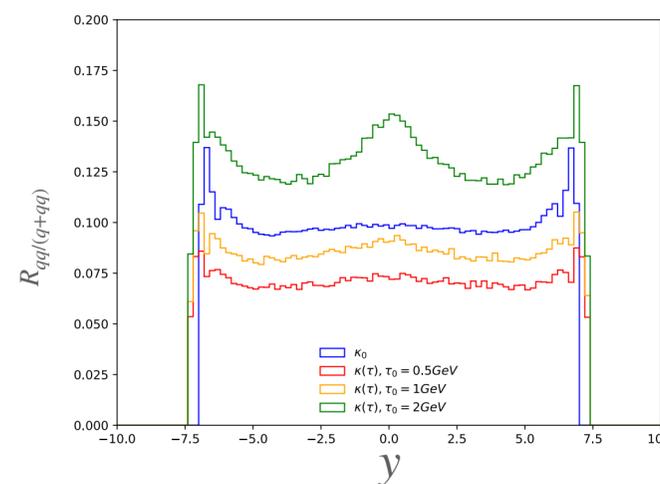
- earlier string breaks
- higher strange / diquark probabilities

diquark mass → phase space constraints lead to typically larger z_+ and z_- values

→ later invariant times

BUT

larger tension → earlier string breaks → less diquark production overall



Beyond Lund Strings

Lund string model assumes **classical string dynamics** — using only first term of the Nambu-Goto action

Work from our Oxford collaborators:

Excitations on an expanding string by considering further terms in the action, i.e. **Nambu-Goldstone** degrees of freedom

$$S_{NG} = \int dt \int_{L_-(t)}^{L_+(t)} dx \left[-\kappa + \frac{1}{2} \partial_t X^i \partial_t X^i - \frac{1}{2} \partial_x X^i \partial_x X^i + \mathcal{O} \left(\frac{(\partial X)^4}{\kappa} \right) \right]$$

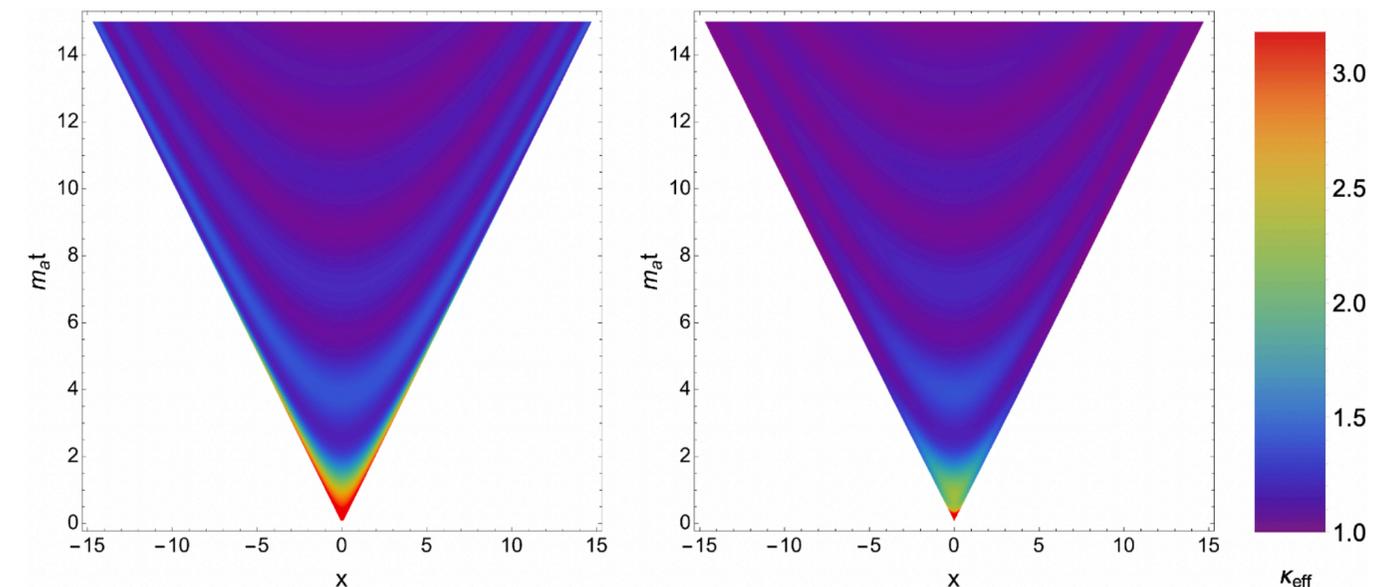
$x \in [L_-(t), L_+(t)]$

X^i are massless Nambu-Goldstone degrees of freedom

Axions

Lattice data strongly suggests that the lightest massive mode that exists on the worldsheet of a long static QCD string is the so-called "worldsheet axion", a pseudoscalar particle of mass $m_a \approx 1.85\sqrt{\kappa}$

$$S_{eff} = \int dt d\sigma \sqrt{-\det(h)} \left(-\kappa + \frac{1}{2} h^{\alpha\beta} \partial_\alpha a \partial_\beta a - \frac{1}{2} m_a^2 a^2 \right) + S_q + S_a$$



(left) Neumann and (right) Dirichlet boundary conditions

Beyond Lund Strings

Lund string model assumes **classical string dynamics** — using only first term of the Nambu-Goto action

Work from our Oxford collaborators:

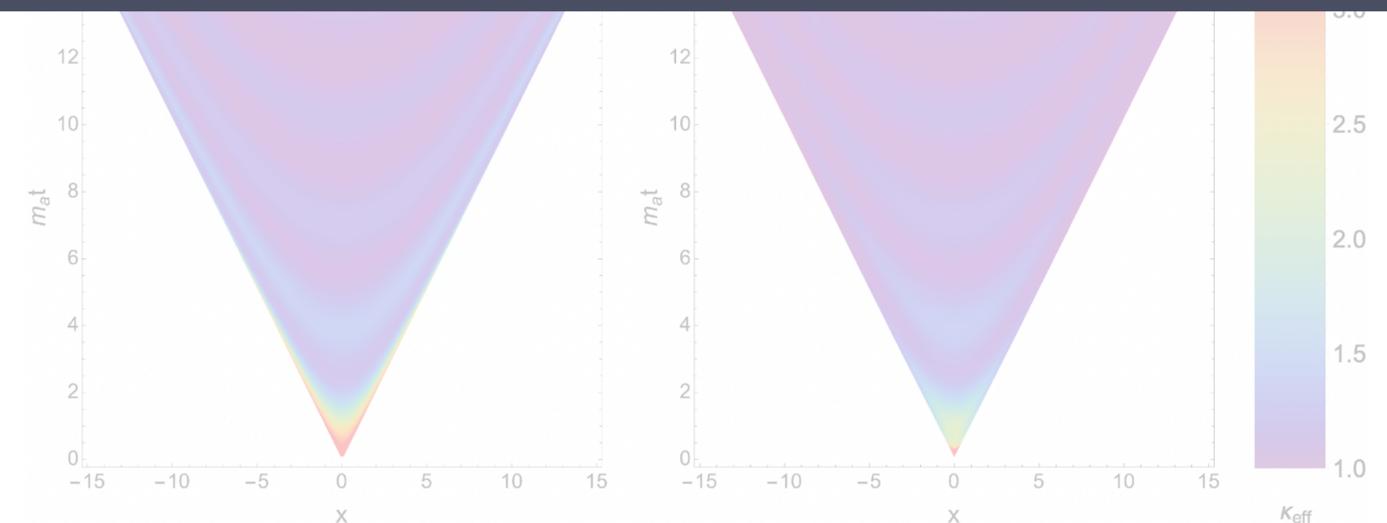
Excitations on an expanding string by considering further terms in the action, i.e. **Nambu-Goldstone** degrees of freedom

- Study effects on string tensions for **lowest lying NGB/axion modes**
- Look at **strange/diquark** production rates, **particle correlations**, etc. for string excitations and τ -dependent string tensions
- Study observables for e^+e^- **collision data**

Axions

Lattice data strongly suggests that the lightest massive mode that exists on the worldsheet of a long static QCD string is the so-called "worldsheet axion", a pseudoscalar particle of mass $m_a \approx 1.85\sqrt{\kappa}$

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(left) Neumann and (right) Dirichlet boundary conditions

Future studies

Projects planned for my PhD

Sunshine

- Validation tests — $2 \rightarrow 3$ and power showers working, but interplay between $2 \rightarrow 4$ and $2 \rightarrow 3$ is yet to work

Closepacking tuning

- Theory side complete, awaiting tuning which has been delayed due to bug in fragmentation code in PYTHIA, but procedure is outlined and tested

Beyond Lund Strings

- Implementation of string excitations according to Nambu-Goldstone and axion modes

Future work / Aspirational projects

Colour reconnections

- New method to better describe colour algebra → better model for junction formation

Modified string tensions

- Extensions to gluon kinks on the string — for both tau-dependent tensions and string excitations

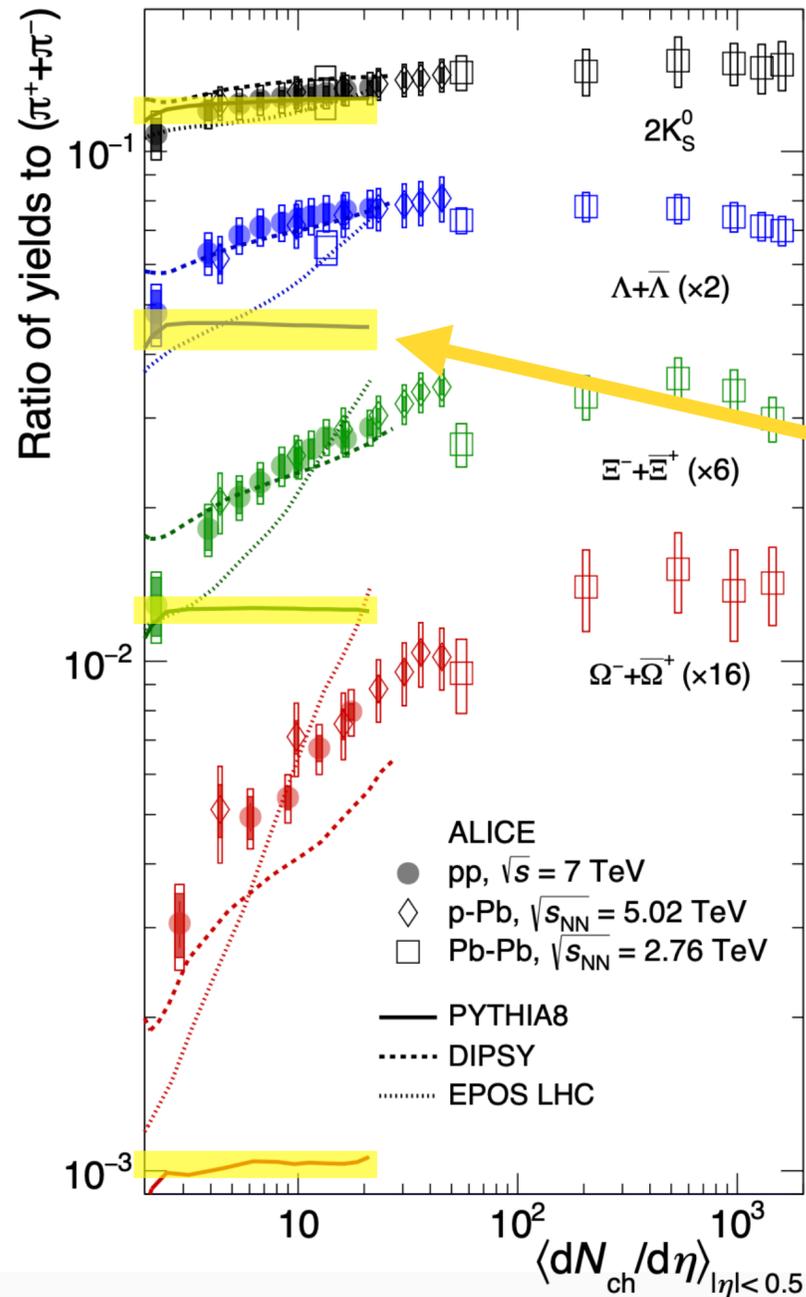
Strangeness

- Closepacking in jets (for e^+e^- studies)
- Ξ_c ratio under-predictions

Thank you for listening!

Backup slides

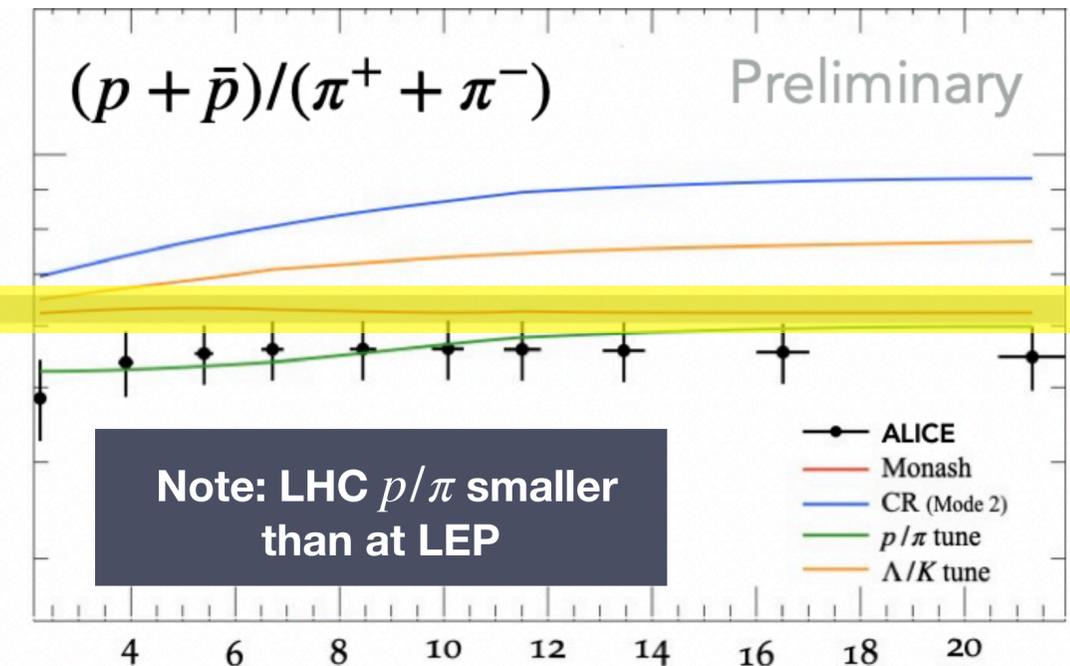
Dense string systems



ALICE data shows increase in **strange hadron production** with **multiplicity**
 high multiplicity is correlated with more partons
 → more dense string environments

Baseline Lund string model shows constant strange quark production along a string

Proton-to-pion ratio is overpredicted in **pp collisions** given diquark production rate value (StringFlav::ProbQQtoQ ($P_{qq:q}$)) tuned to e^+e^- data



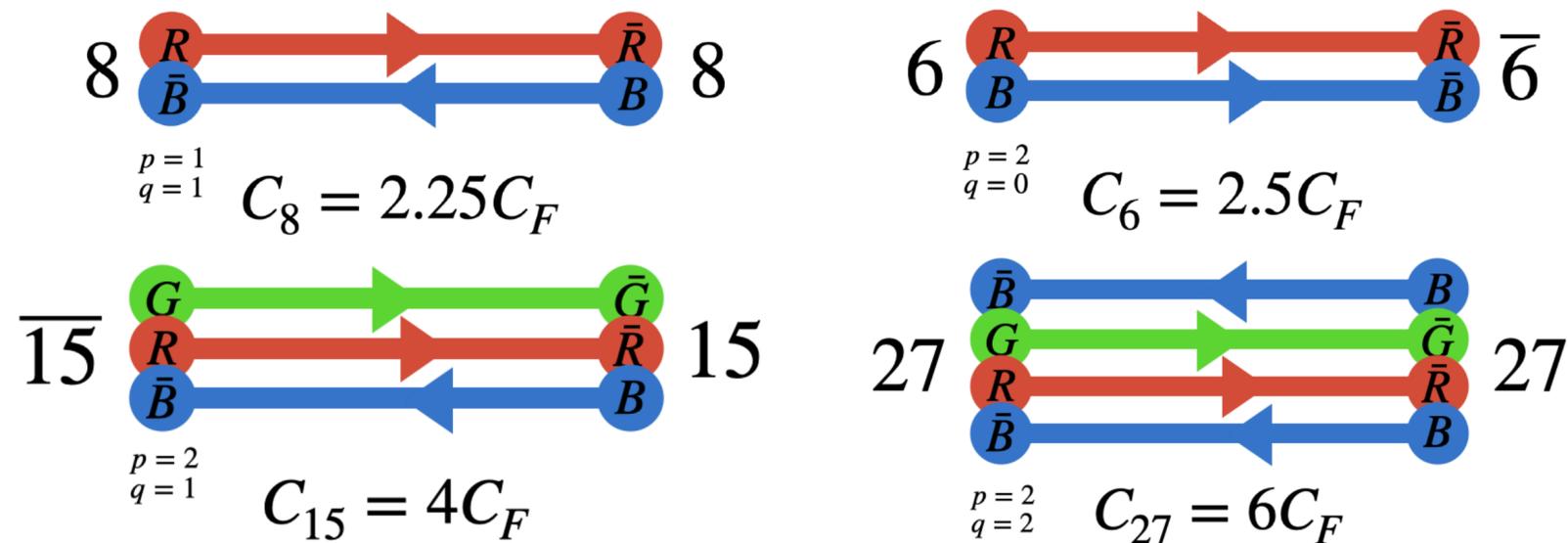
Overprediction of proton-to-pion ratio

Strange to non-strange hadron ratios

Dense string systems

Closepacking

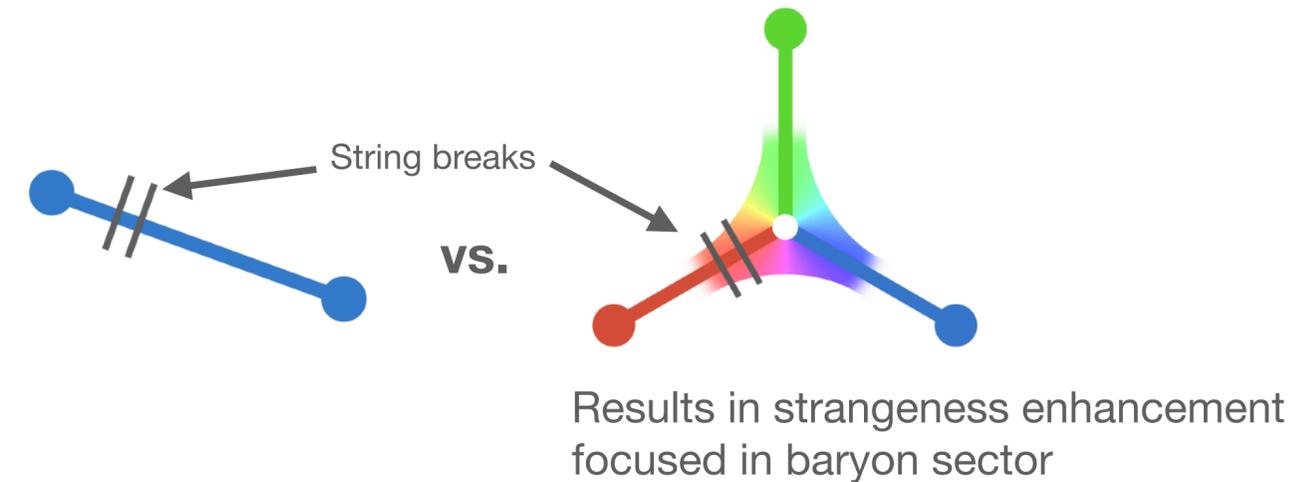
Enhance string tension for higher multiplets according to **Casimir scaling**



High multiplicity is correlated with more partons
 → more dense string environments

Strange Junctions

String tension could be different from the vacuum case compared to near a junction



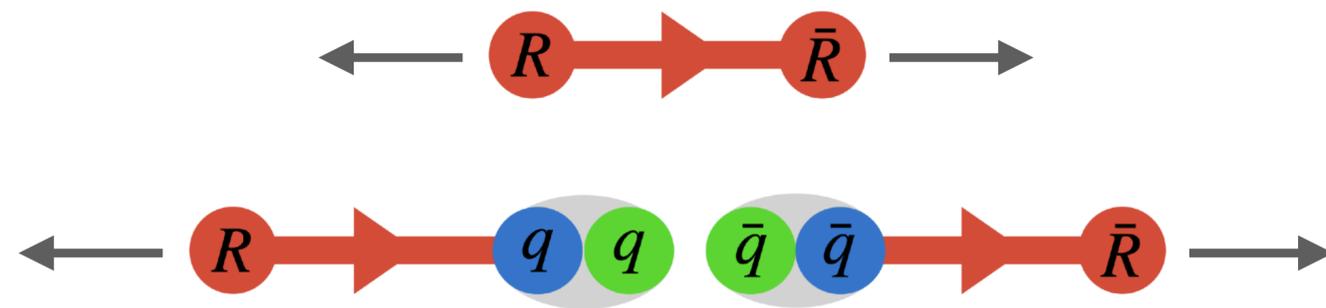
Junction formation is correlated with density of string systems

Diquark production

Proton-to-pion ratio is overpredicted in pp collisions given diquark production rates tuned to e^+e^- data

Junctions needed to describe other baryon-to-meson ratios \rightarrow examine baryon formation via **diquark production**

Schwinger – direct tunnelling from vacuum



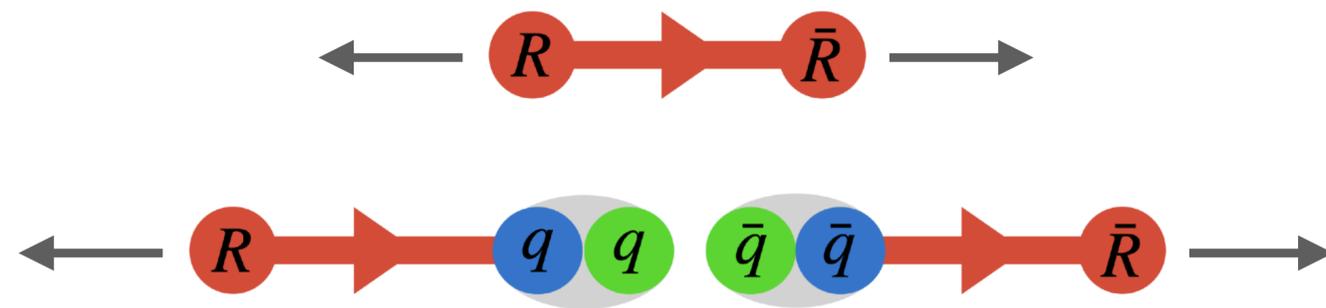
$$\mathcal{P} \propto \exp\left(\frac{-m_{qq}^2 - p_{\perp}^2}{\kappa/\pi}\right)$$

Diquark production

Proton-to-pion ratio is overpredicted in pp collisions given diquark production rates tuned to e^+e^- data

Junctions needed to describe other baryon-to-meson ratios \rightarrow examine baryon formation via **diquark production**

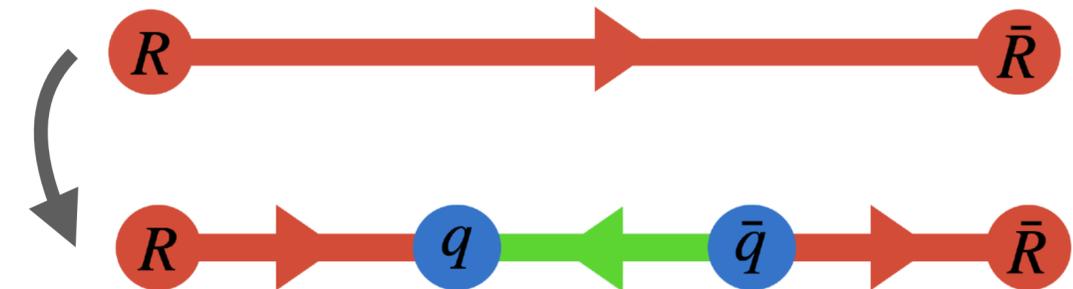
Schwinger – direct tunnelling from vacuum



$$\mathcal{P} \propto \exp\left(\frac{-m_{qq}^2 - p_{\perp}^2}{\kappa/\pi}\right)$$

Popcorn mechanism for diquark production

Diquark formation via **successive colour fluctuations** – popcorn mechanism

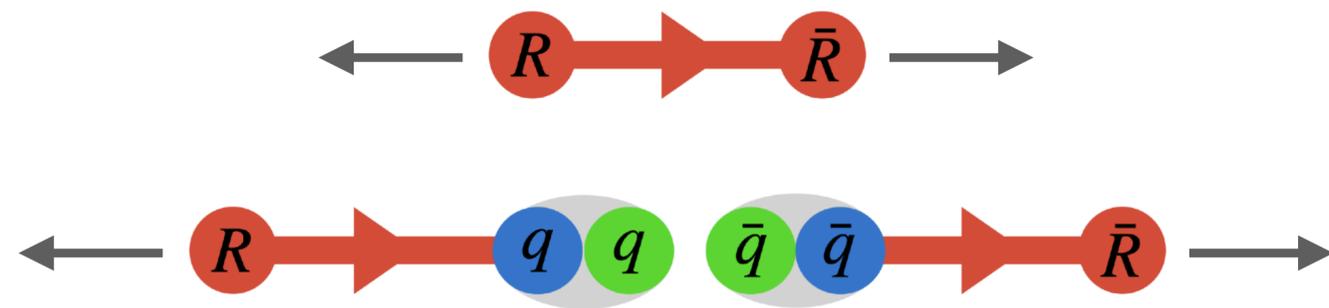


blue $q\bar{q}$ fluctuation on the string

Diquark production

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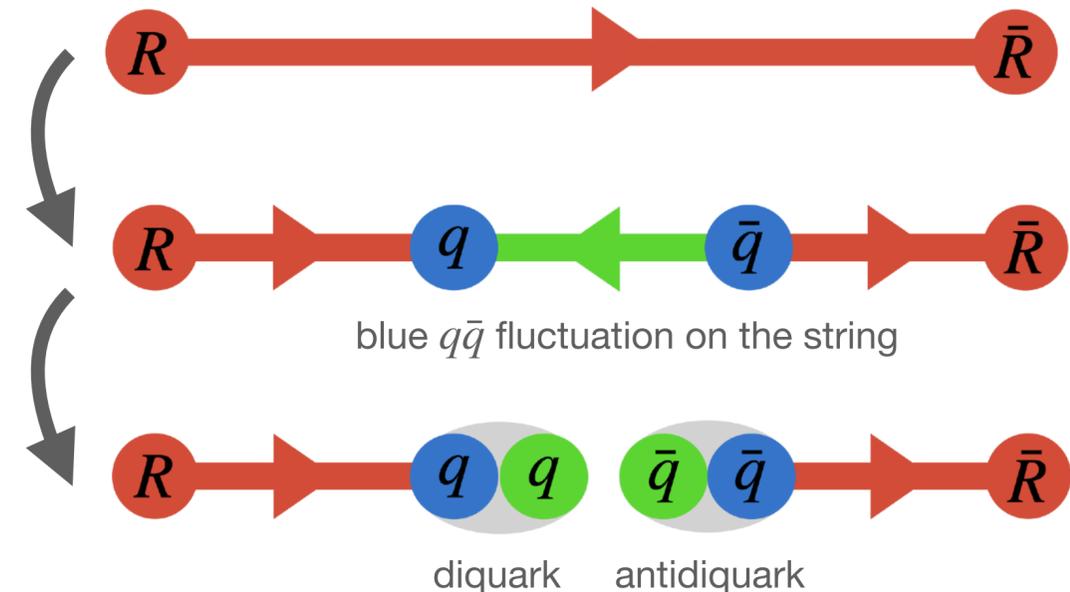
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Diquark formation via **successive colour fluctuations** – popcorn mechanism



What would happen if we put this **red string** next to another string? e.g. a **blue string**

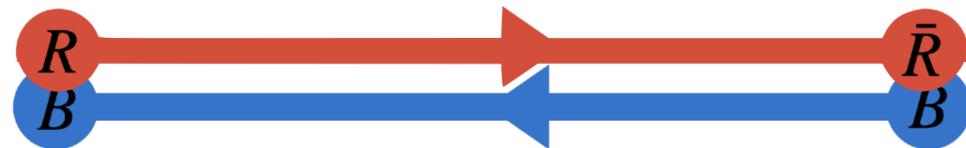
Popcorn destructive interference

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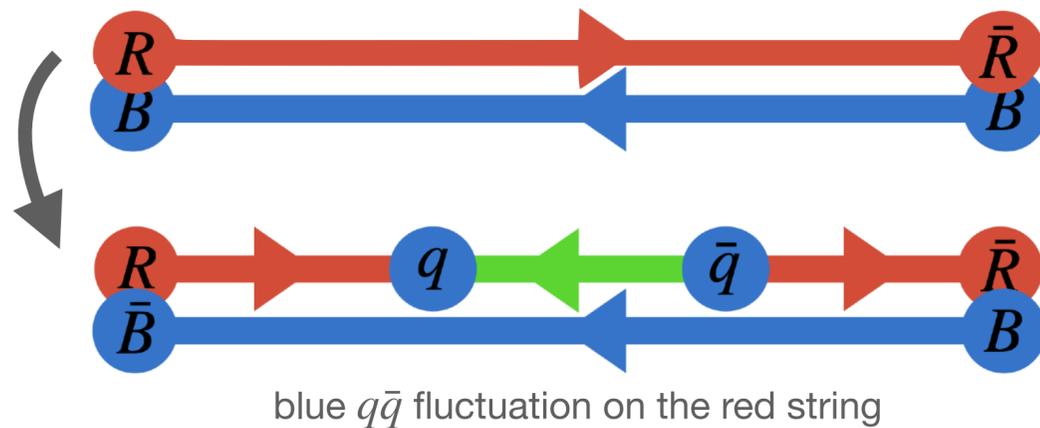
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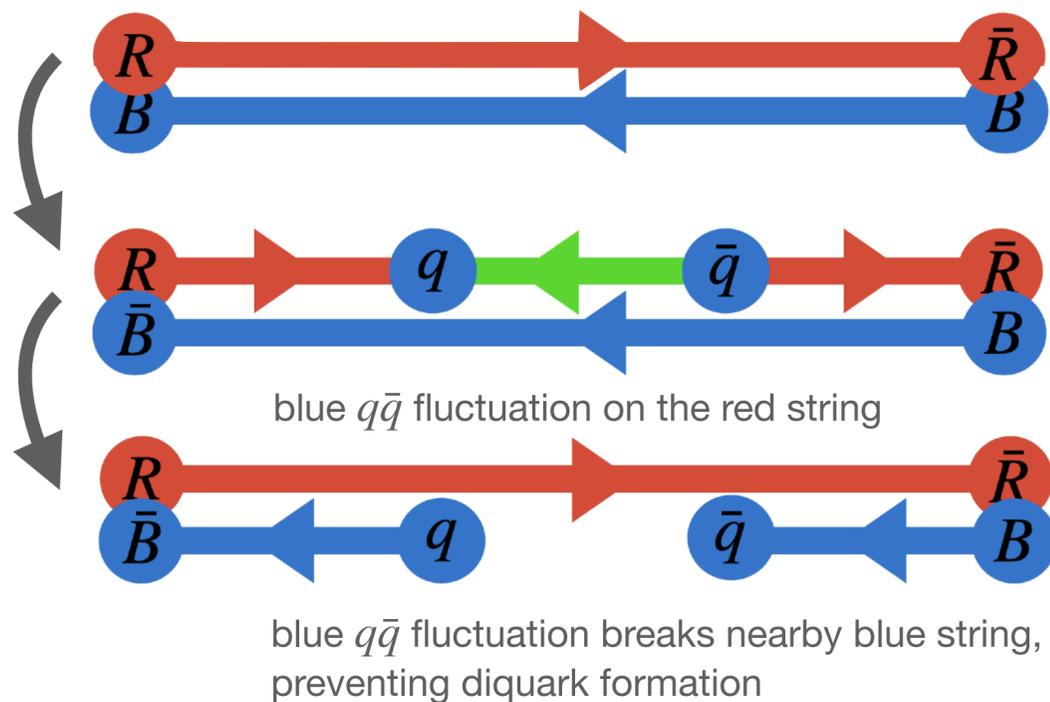
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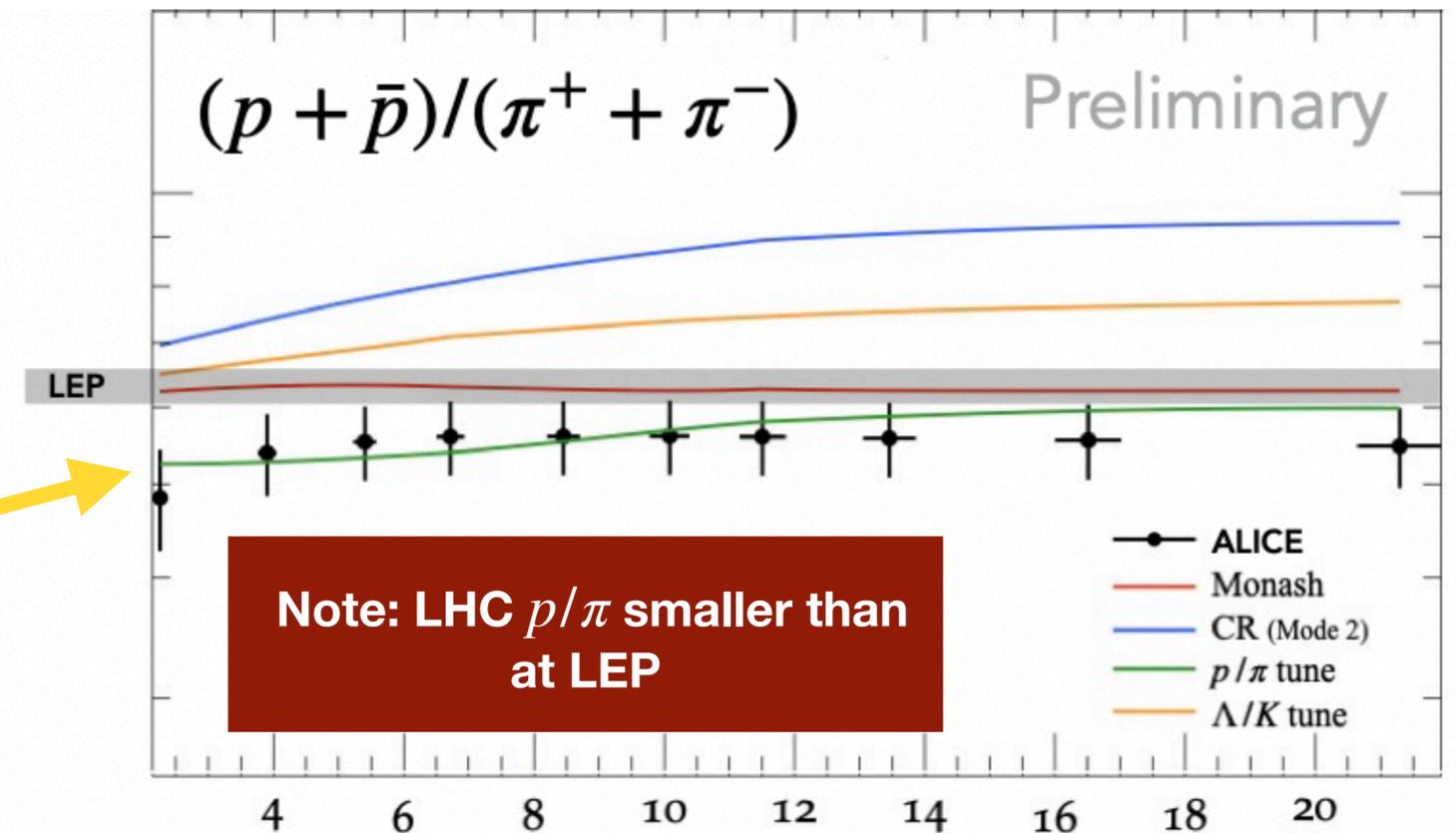
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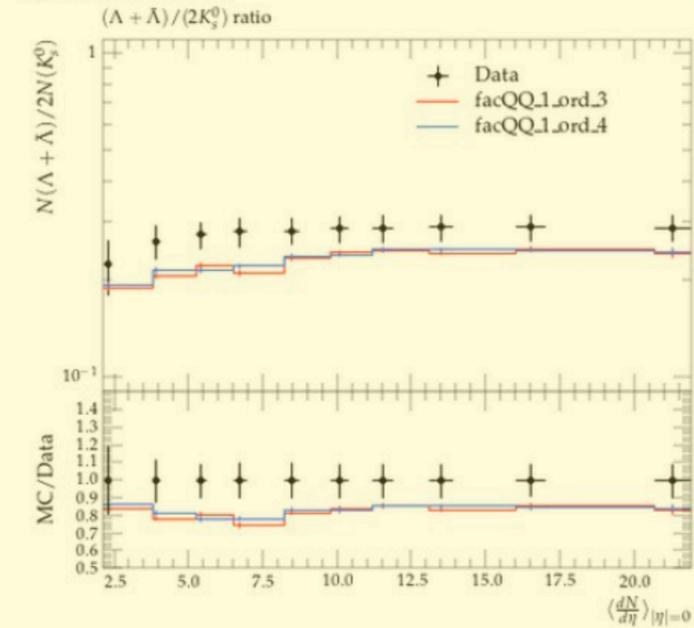


Popcorn destructive interference

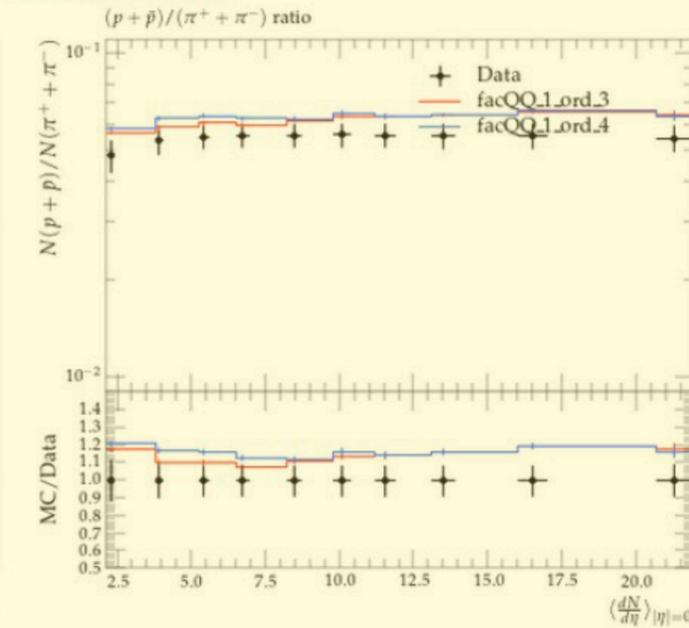


Results — ongoing

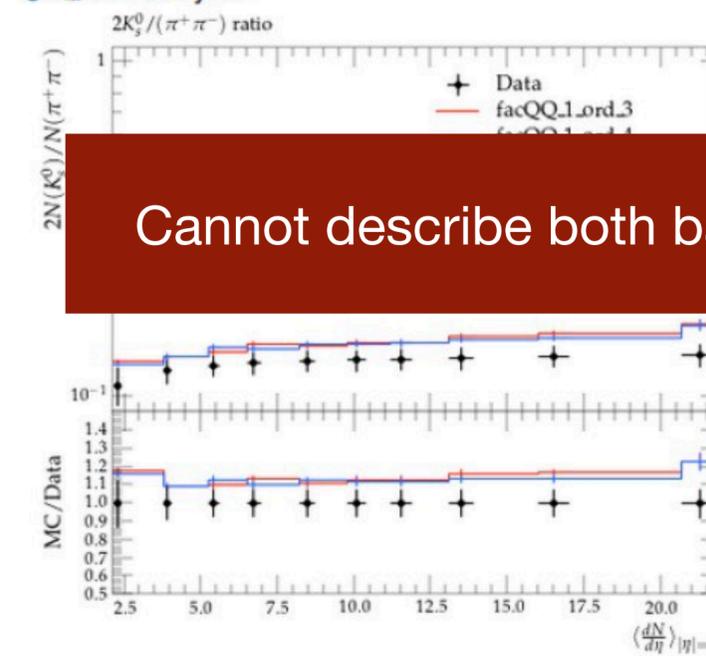
d46-x01-y01:



d47-x01-y01:

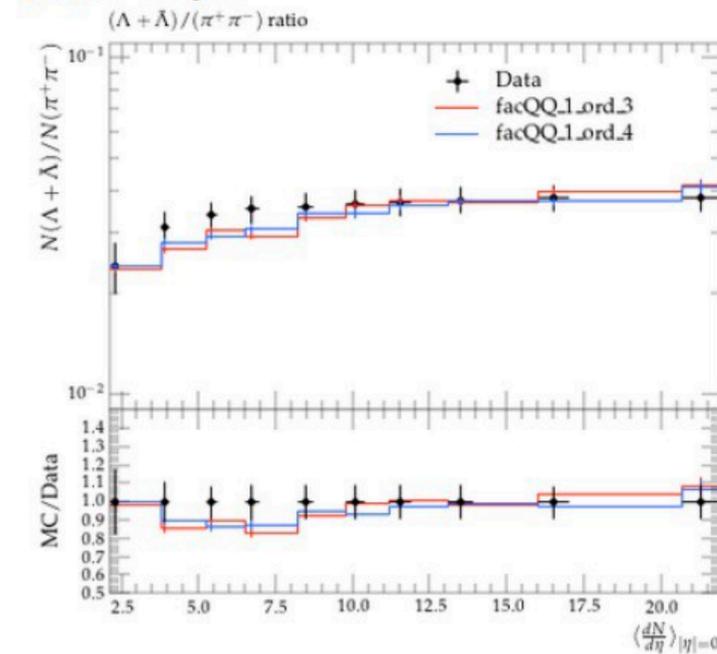


d36-x01-y01:

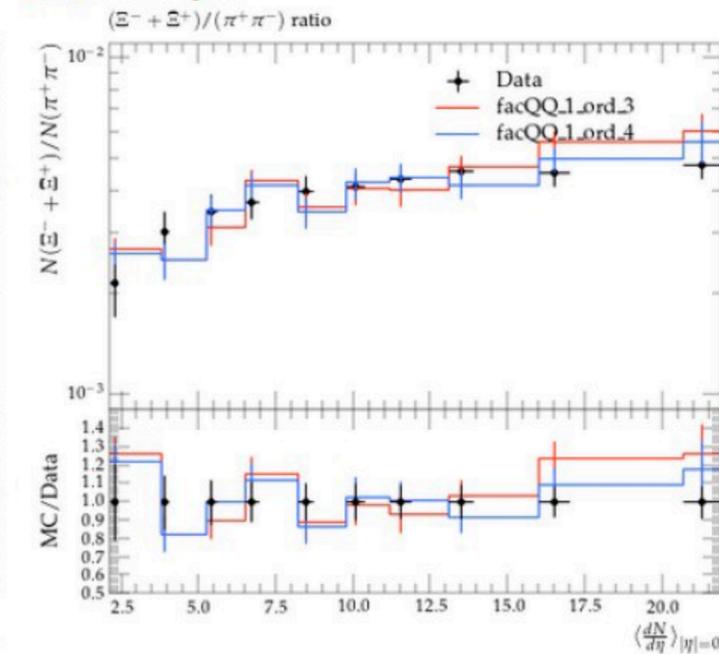


Cannot describe both baryon-to-meson ratios simultaneously

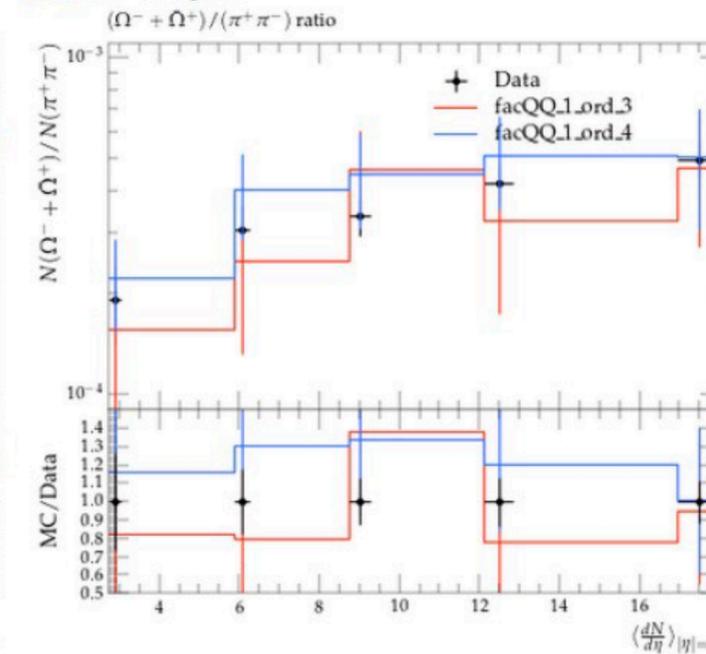
d37-x01-y01:



d38-x01-y01:



d39-x01-y01:



Taken from slide by Lorenzo Bernadinis:
PhD student currently in Trieste
undertaking tuning project with the model

String tension modifications

Diquark production:

$$P_{qq:q} = \frac{\sum_{qq_s} P_{qq_s}}{\sum_q P_q} = \alpha \frac{P_{ud0}}{P_u}$$

$$P_{qq:q}(\kappa_{eff}) = \tilde{\alpha} \left(\frac{P_{qq:q}}{\alpha} \right)^{\kappa_0/\kappa_{eff}}$$

p_{\perp} spectrum:

$$\exp\left(\frac{-\pi p_{\perp}^2}{\kappa_0}\right) = \exp\left(\frac{-p_{\perp}^2}{\sigma^2}\right) \quad \sigma^2 = \frac{\kappa_0}{\pi}$$

$$\sigma'^2 = \frac{\kappa_{eff}}{\pi} = \frac{\kappa_0}{\pi} \frac{\kappa_{eff}}{\kappa_0} = \sigma^2 \frac{\kappa_{eff}}{\kappa_0}$$

Increased width of p_{\perp} spectrum
→ higher probability of higher p_{\perp}

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$x \in [L_-(t), L_+(t)]$

$$X(t, \sigma) = \left(t, \frac{L_+(t) + L_-(t)}{2} + \sigma \frac{L_+(t) - L_-(t)}{2}, \frac{X^2(t, \sigma)}{\sqrt{\kappa}}, \frac{X^3(t, \sigma)}{\sqrt{\kappa}} \right) \quad \sigma \in [-1, 1]$$

X^i are massless Nambu-Goldstone degrees of freedom

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Modelling Colour

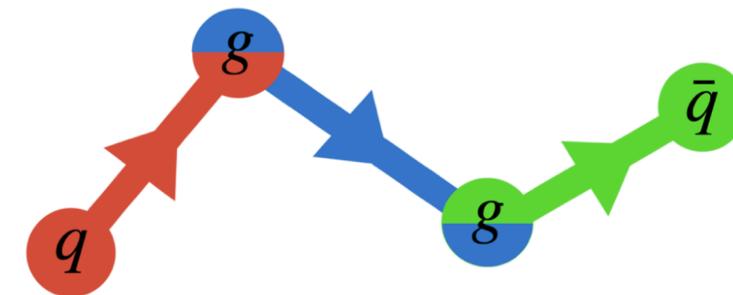
Leading Colour limit:

Starting point for Monte Carlo event generators $N_C \rightarrow \infty$

- Each **colour is unique** \rightarrow only one way to make colour singlets
- Only **dipole** strings
- Used by PYTHIA in the default (Monash 2013) tune

In e^+e^- collisions :

- Corrections suppressed by $1/N_C^2 \sim 10\%$
- Not much overlap in phase space



e.g. a dipole string configuration which make use of the **colour-anticolour** singlet state

But high-energy pp collisions involve **very many** coloured partons with significant **phase space overlaps**

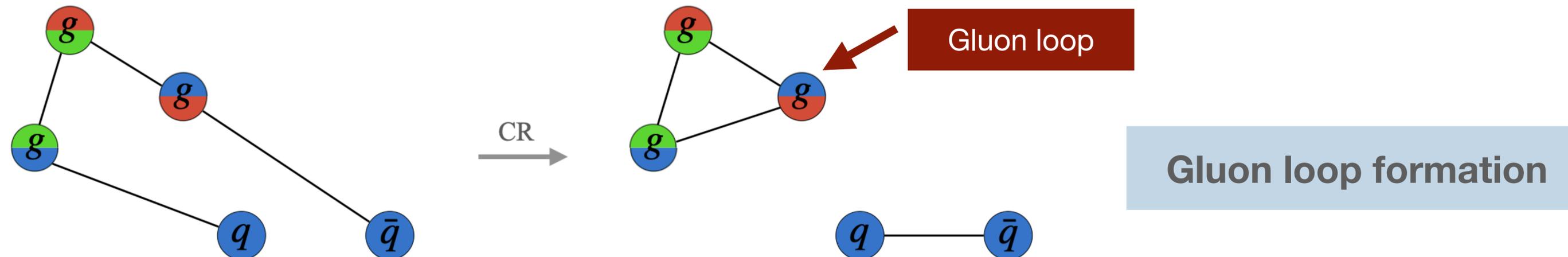
QCD Colour Reconnection (CR) model

QCD Colour Reconnections

Stochastically restores colour-space ambiguities according to **SU(3) algebra**

- Allows for reconnections to **minimise string lengths**

Colour - anticolour singlet state

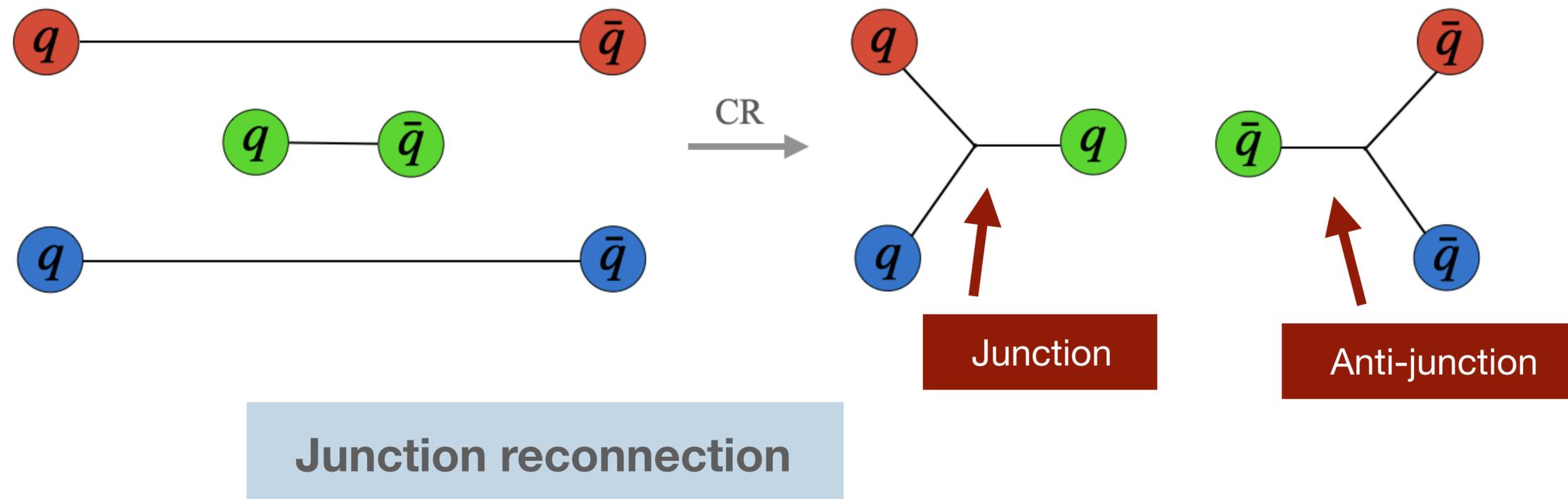


QCD Colour Reconnections

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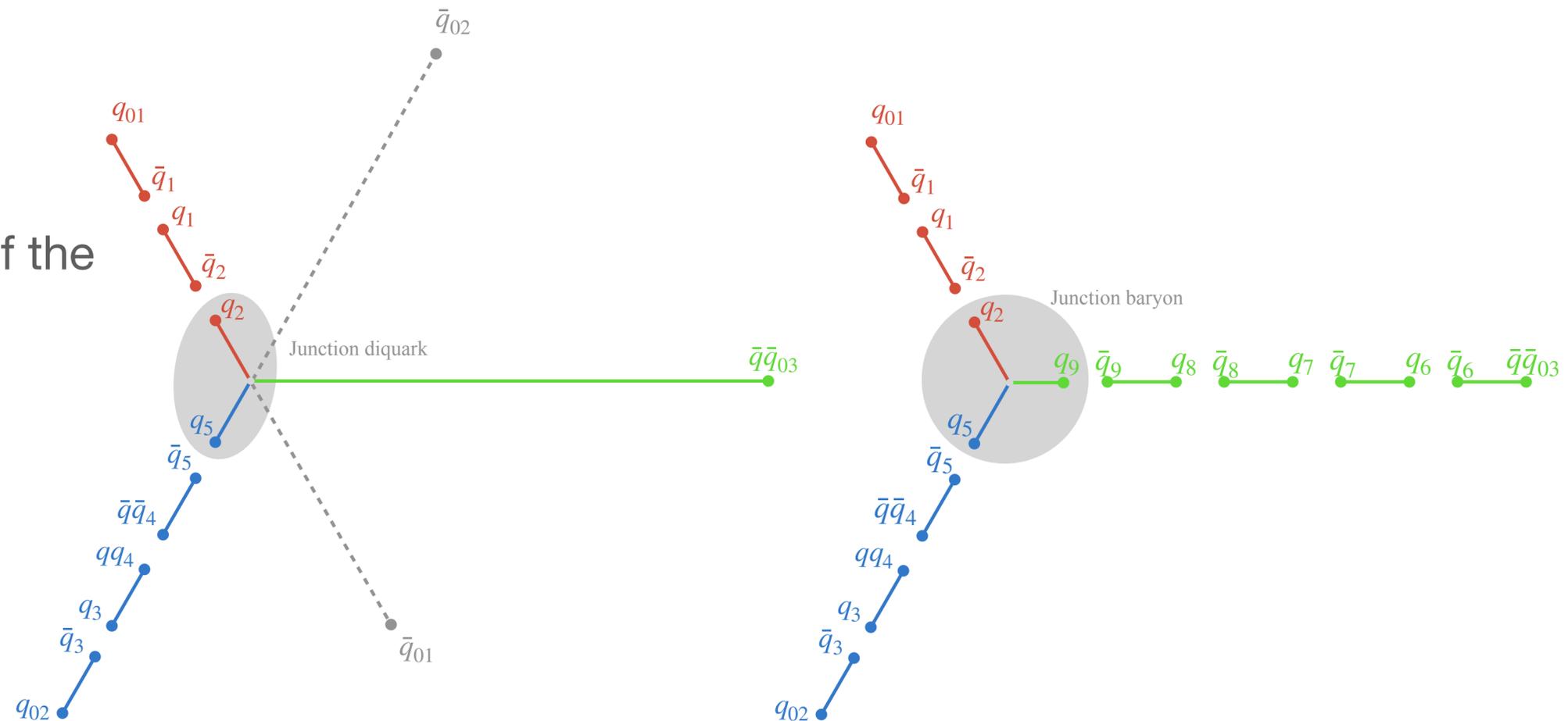
What about the **red-green-blue** colour singlet state?



Junction fragmentation

Junction fragmentation

- **Go to JRF**
- Fragment two softest strings first
 - Reflect each leg on the other side of the junction (“fictitious leg”) to form a dipole string
- Form junction diquark
- Fragment last leg by fragmenting diquark — endpoint string



Partons \rightarrow Hadrons

Baryon formation:

Diquark endpoints (e.g. beam remnants)

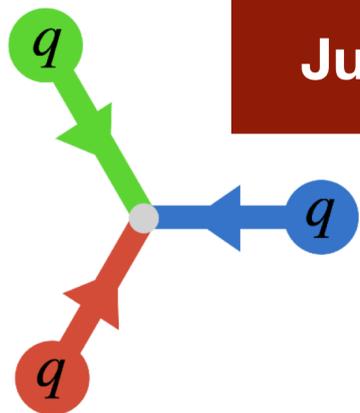


Diquark-antidiquark string breaks

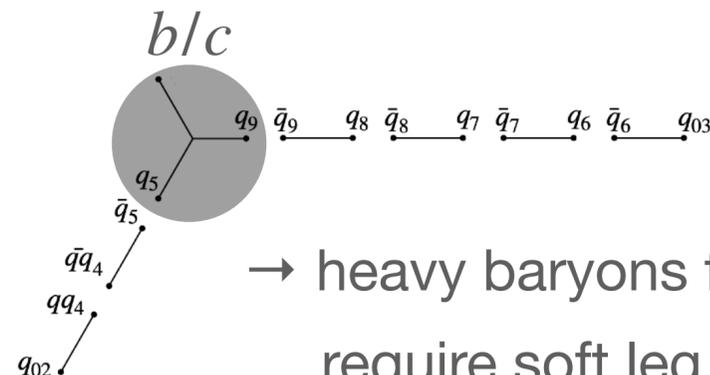
➤ constant probability suppressed by Schwinger mechanism



What about the **red-green-blue** colour singlet state?



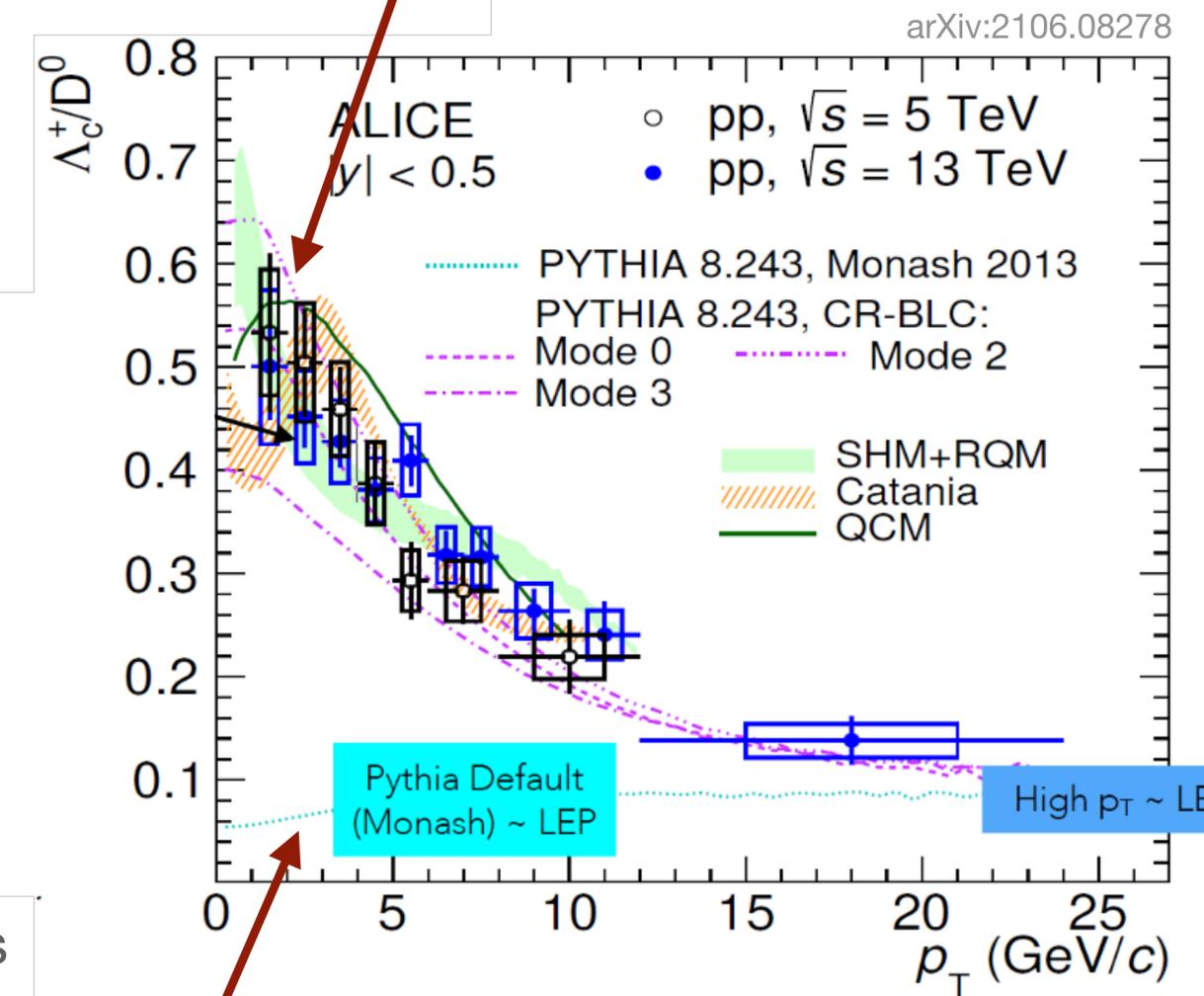
Junctions!



\rightarrow heavy baryons from junctions
require soft leg treatment

Heavy baryon-to-meson ratio

WITH JUNCTIONS

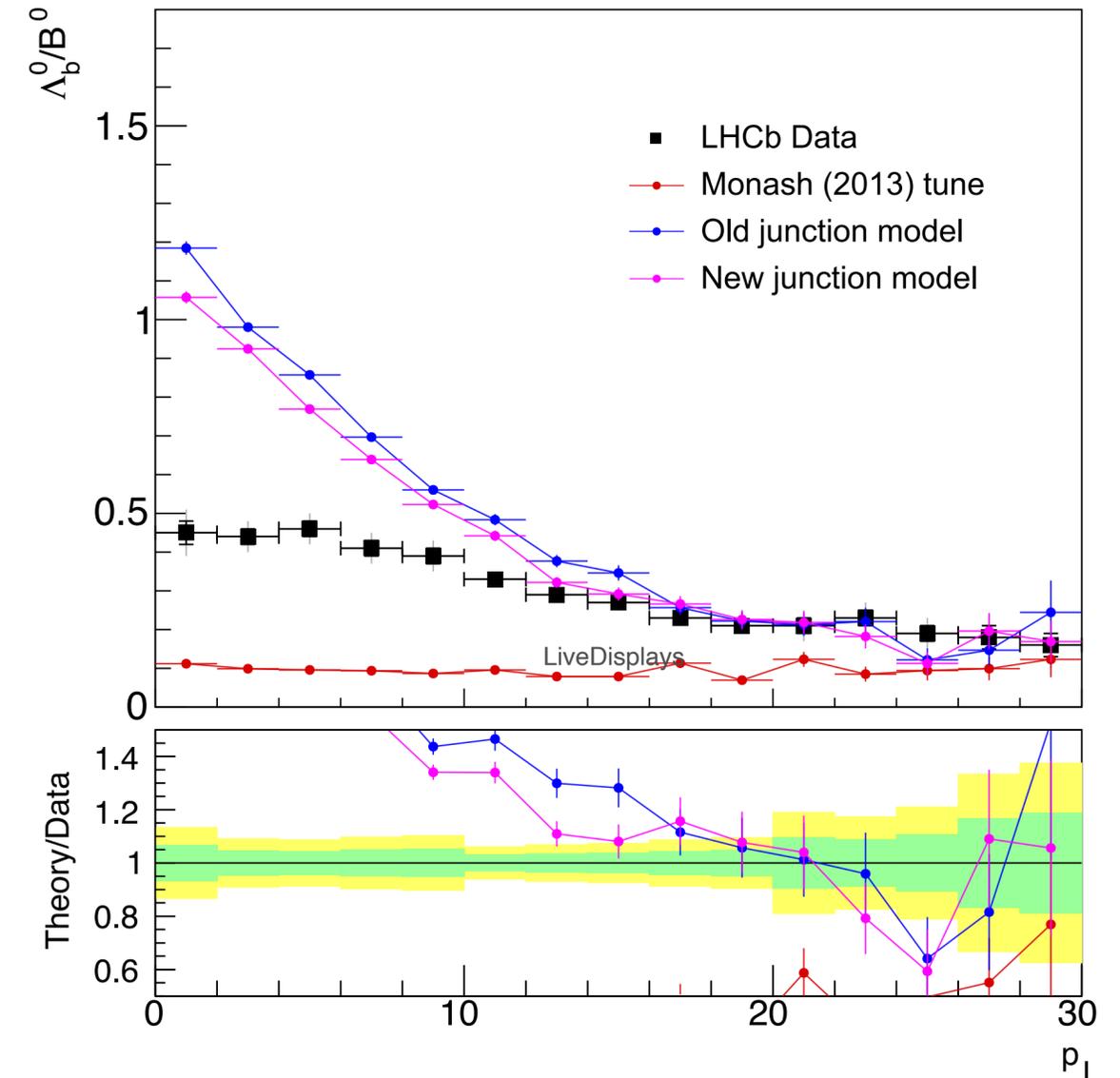
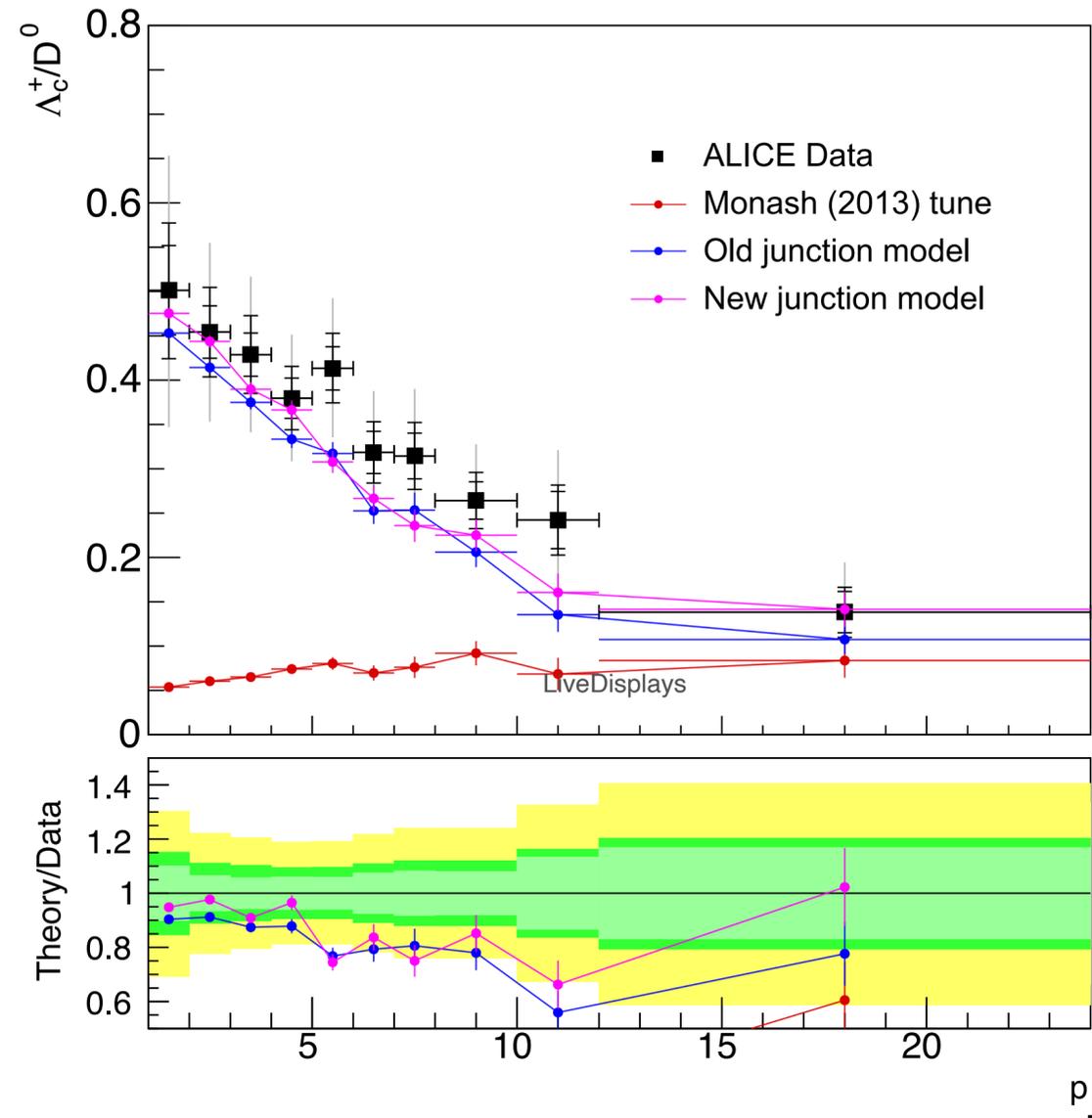


NO JUNCTIONS

Heavy baryon ratios

Λ_b/B^0 overprediction

- study of Λ_b vs Λ_c production
- other heavy flavour ratios such as Λ_b/Λ_c and B^0/D^0
- general study of what portion of each baryon comes from junctions



*Note Λ_c/D^0 is lower than typically as probQQ1toQQ0join was reverted to its default values and left untuned. Value should be slightly lower than default value and this ratio should increase