

Heavy Flavour Hadronisation in Pythia

Peter Skands (Monash University)

1. Heavy-Flavour Hadronisation in the Lund Model
2. Constraints
3. From ee to pp
4. New Theory Models in Pythia
5. Some Suggestions for New Measurements
6. Multiply heavy hadrons?



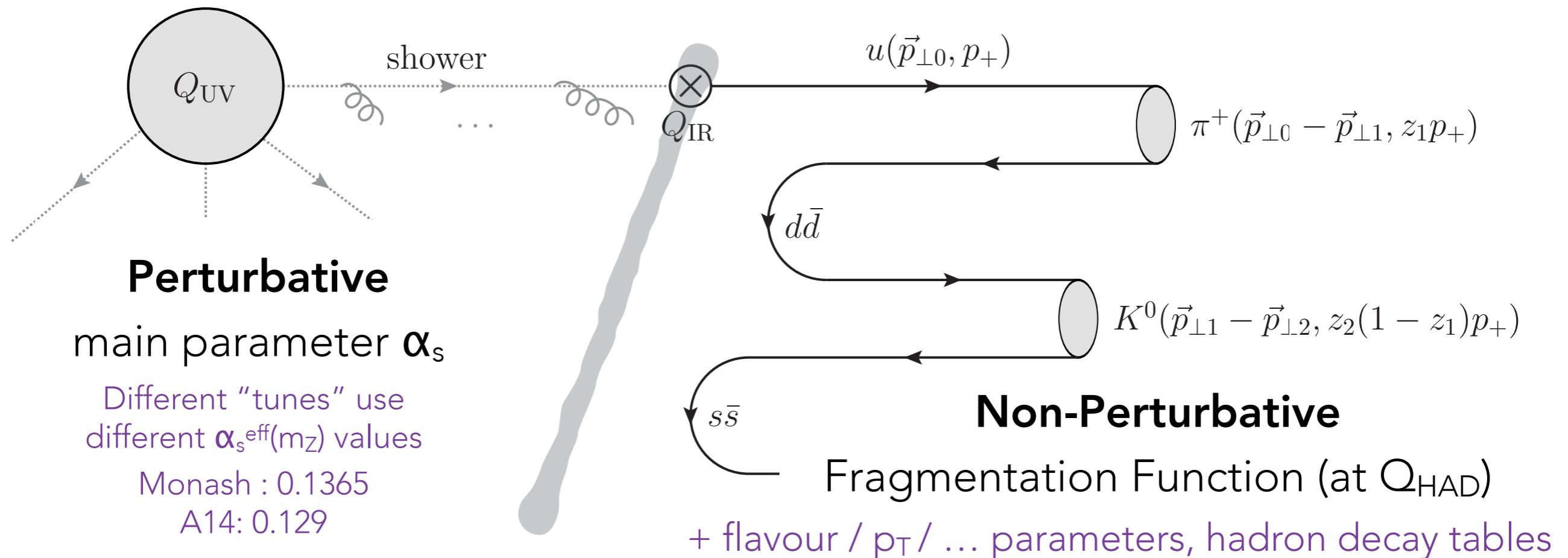
*Heavy-Flavour Hadronization in pp
& HI Collisions, CERN, March 2020*

Reminder: Fragmentation Models

Hard process (e.g., dijets) \blacktriangleright hard factorisation scale $Q_{UV} \sim p_{Tjet}$

Parton Showers: perturbative bremsstrahlung down to $Q_{IR} \sim 1 \text{ GeV}$

Hadronisation: confinement (+ hadron decays) at $Q_{HAD} \sim Q_{IR}$



Spectrum = combination of α_s choice & non-perturbative parameters

Flavour Composition in the Lund Model

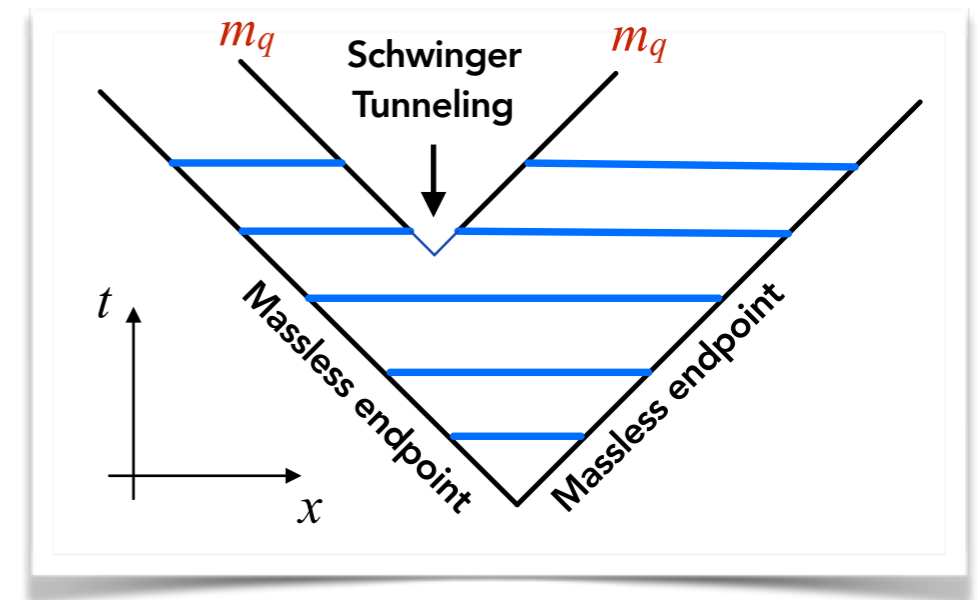
Starting point: **isolated** string in 1+1 dimensions

Tension $\kappa \sim 1 \text{ GeV/fm} \sim 0.2 \text{ GeV}^2$

String breaks by Schwinger mechanism

→ **Suppression** of strange quarks (and diquarks) $\exp\left(-\frac{m_q^2 + p_\perp^2}{\kappa}\right)$

→ StringFlav:probStoUD = 0.217



+ Spin-splitting in hadron multiplets $V/P \neq 3$

ρ/π StringFlav:mesonUDvector = 0.50

D^*/D StringFlav:mesonCvector = 0.88

K^*/K StringFlav:mesonSvector = 0.55

B^*/B StringFlav:mesonBvector = 2.2

Note: model parameters are for **primary** hadrons \neq measured ratios (feed-down)

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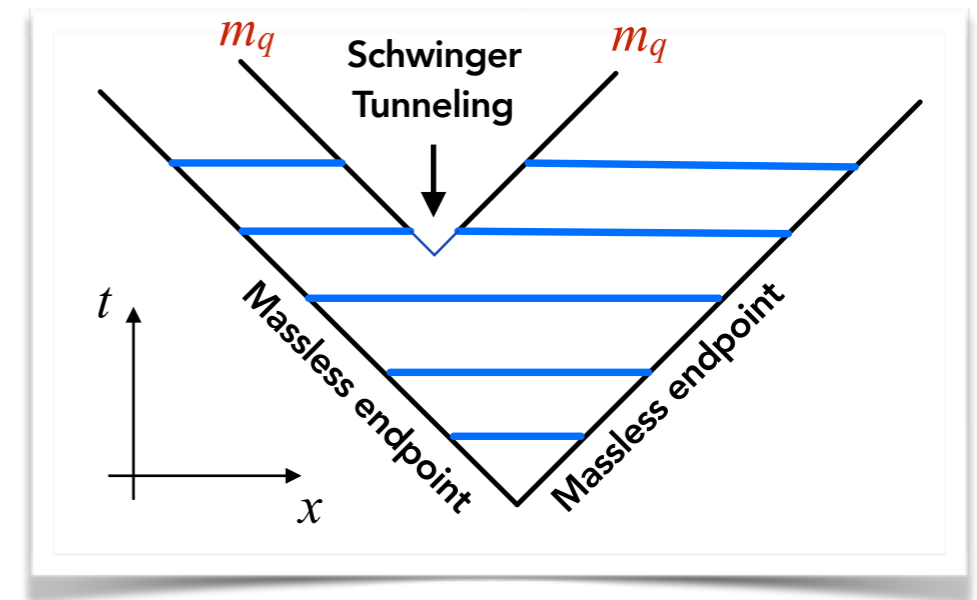
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Rookie Mistake: for D^*/D in the Monash tune [arXiv:1404.5630](https://arxiv.org/abs/1404.5630)

I took the D and D^* rates from separate sources ➤ wrong ratio
Should be higher $\sim 1.25 - 1.5$ to agree with measured values

Thanks to D. Bardhan for pointing to this

Heavy-Flavour Endpoint Quarks

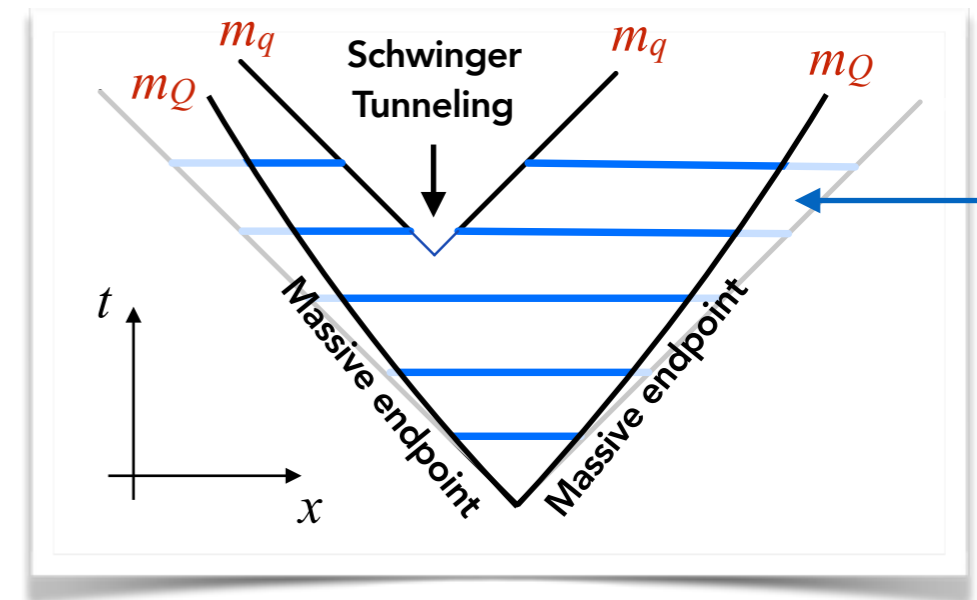
Same starting point as for massless endpoints

Tension $\kappa \sim 1 \text{ GeV/fm} \sim 0.2 \text{ GeV}^2$

String breaks by Schwinger mechanism

→ **Suppression** of strange quarks (and diquarks) $\exp\left(-\frac{m_q^2 + p_\perp^2}{\kappa}\right)$

→ **StringFlav:probStoUD = 0.217**



! Same parameters govern $\mathbf{D_s/D, B_s/B, \Lambda_c/D, \Lambda_b/B} \rightarrow$ Interesting to check if $D_s/D, B_s/B$ affected in same way in same environments where we see *strangeness enhancements* in light-quark sector: **multiplicity** dependence

Massive endpoints have $v < c \rightarrow$ smaller string space-time area:

→ Modified ("Lund-Bowler") FF:

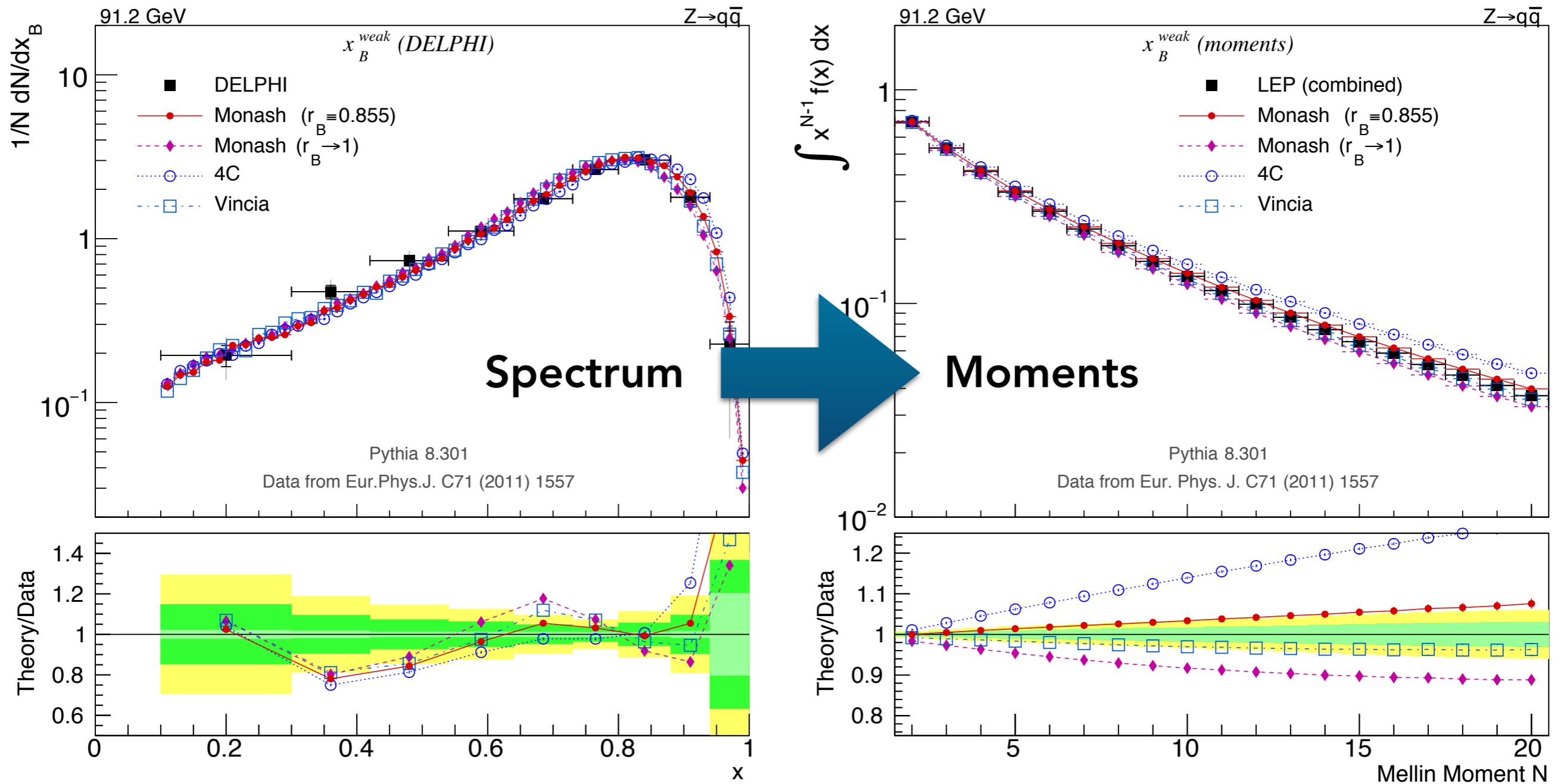
$$\frac{(1-z)^a}{z^{1+r_Q} b m_Q^2} \exp\left(\frac{-b m_{\perp,h}^2}{z}\right) \quad \text{with } r_b \sim r_c \sim 1$$

(Note: Peterson etc strictly speaking incompatible with causality in string picture)

StringZ:rFactB = 0.855

Constraints : B Spectra

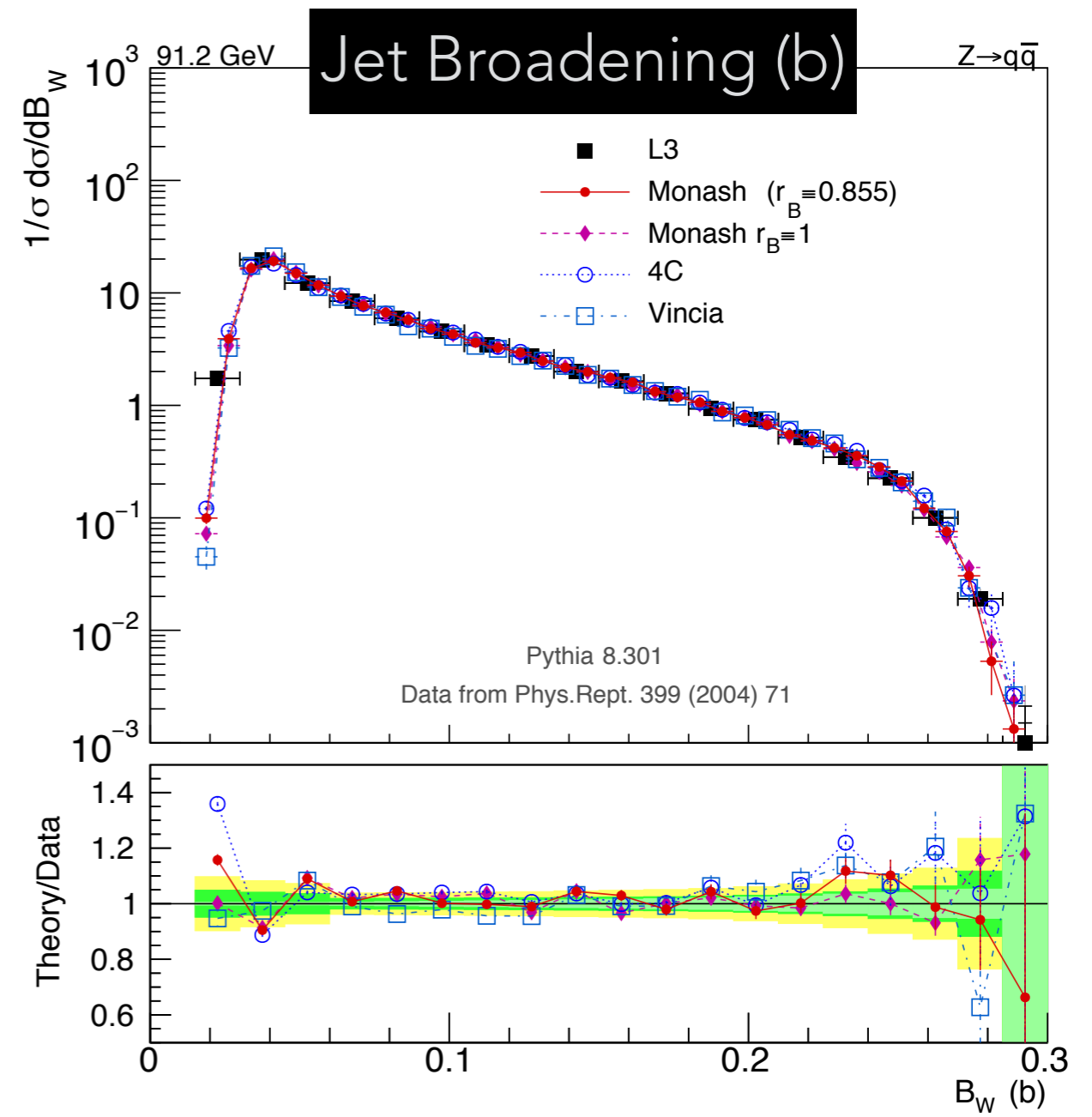
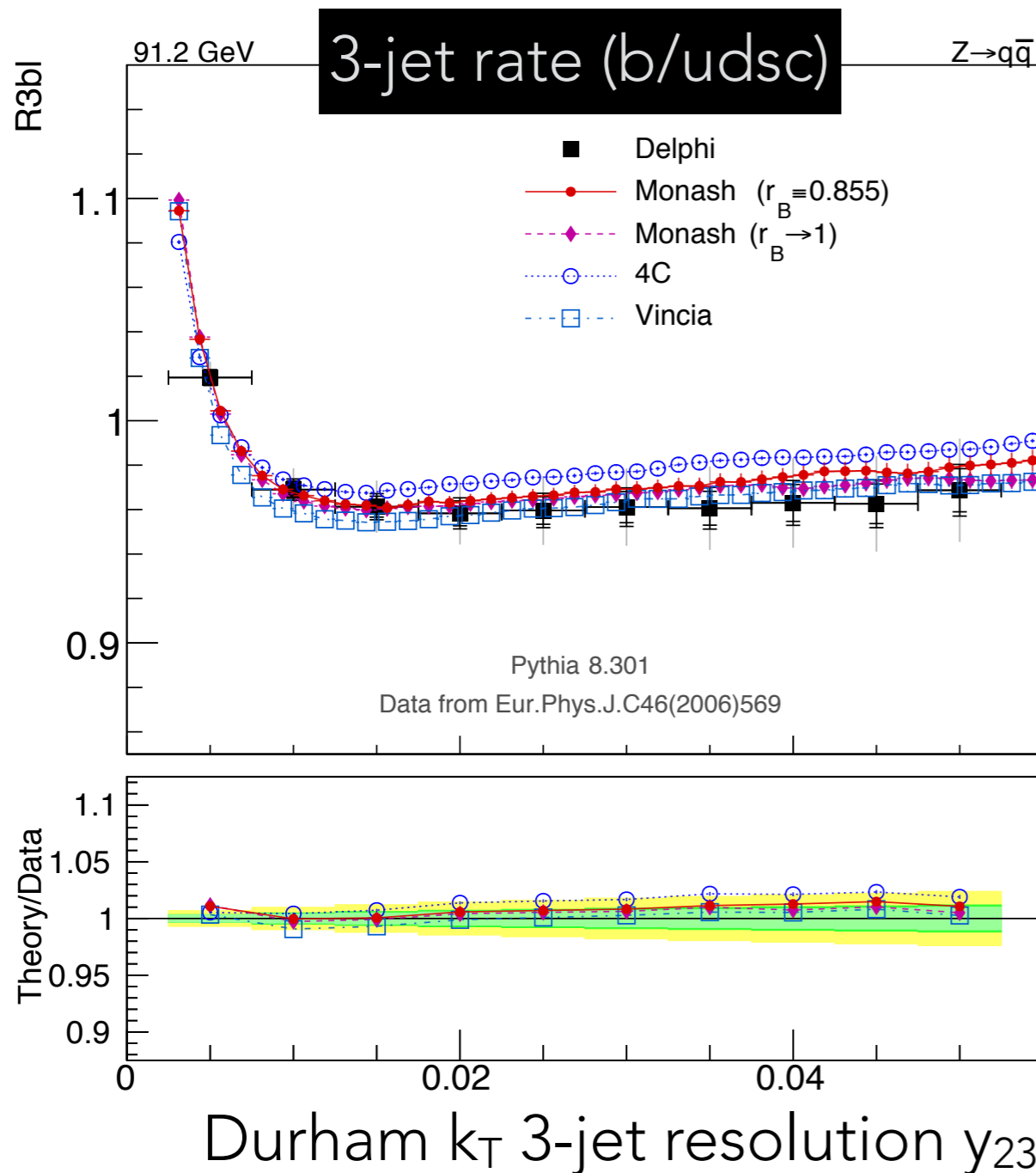
Main constraint: x_B spectra of weakly decaying B hadrons in Z decays



for details see arXiv:1404.5630 (section 2.3)

+ B-tagged Event Shapes & Jet Rates

IR safe: sensitive to α_s and b mass effects in shower + hadronisation



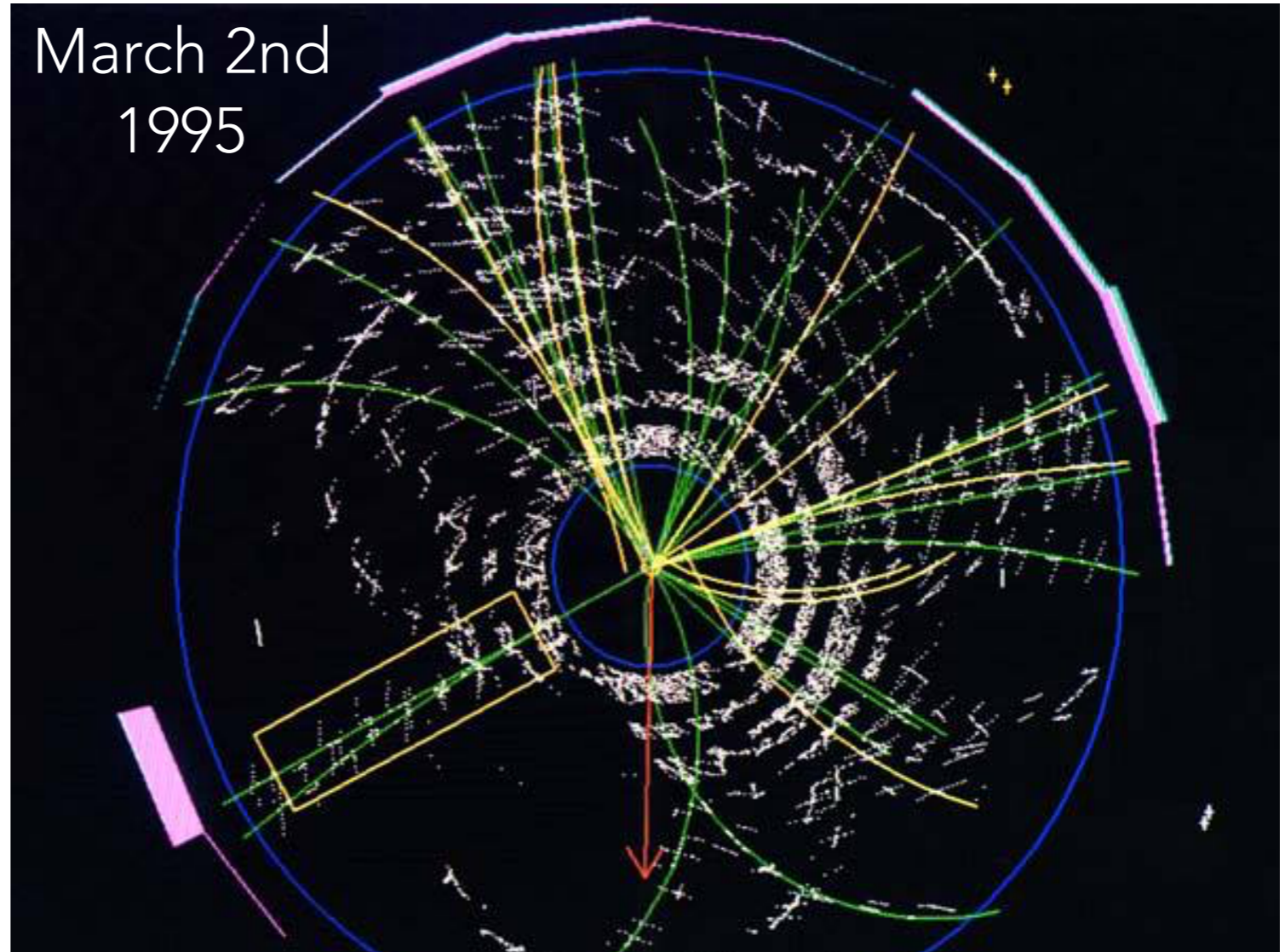
for details see arXiv:1404.5630 (section 2.3)

LHC: Top Decays ➤ In-situ controlled B-Jet Sample?

Yesterday: **25th anniversary of the top quark discovery**



$t \rightarrow bW$ provides a clean high-statistics reference sample, with a well-defined initial b-quark energy (in top CM) very similar to $Z \rightarrow bb$.



Compare B FF(x) and B hadron flavour ratios to those for inclusive b-jets, incl. any dependence on UE level (measured away from the top jets)

Note: finite top width ➔ "collective effects" may be suppressed in top
("early" vs "late" resonance decays)

Some Comments on b fragmentation "tuning"

Note: Monash uses "large" `TimeShower:alphaSvalue = 0.1365`

Regarded at least in part as making up for **NLO** K-factor for **ee → 3 jets** (baseline Pythia only accurate to LO for 3 jets).

Consistent with 3-flavour $\Lambda_{\text{QCD}} \sim 0.35$ GeV (since we use 1-loop running)

Not guaranteed to be universal.

LHC studies tend to prefer lower effective values of α_s

E.g., A14 uses `TimeShower:alphaSvalue = 0.129` (could be reinterpreted via CMW to $\overline{\text{MS}}$ $\alpha_s(m_Z) \sim 0.12$ so consistent with world average.)

(but I would then also change to 2-loop running to preserve Λ_{QCD} value)

E.g., a lower $\alpha_s \rightarrow$ less perturbative radiation \rightarrow harder $x_b(Q_{\text{IR}})$

\rightarrow Would need to retune non-perturbative parameters (e.g., r_b) at LEP

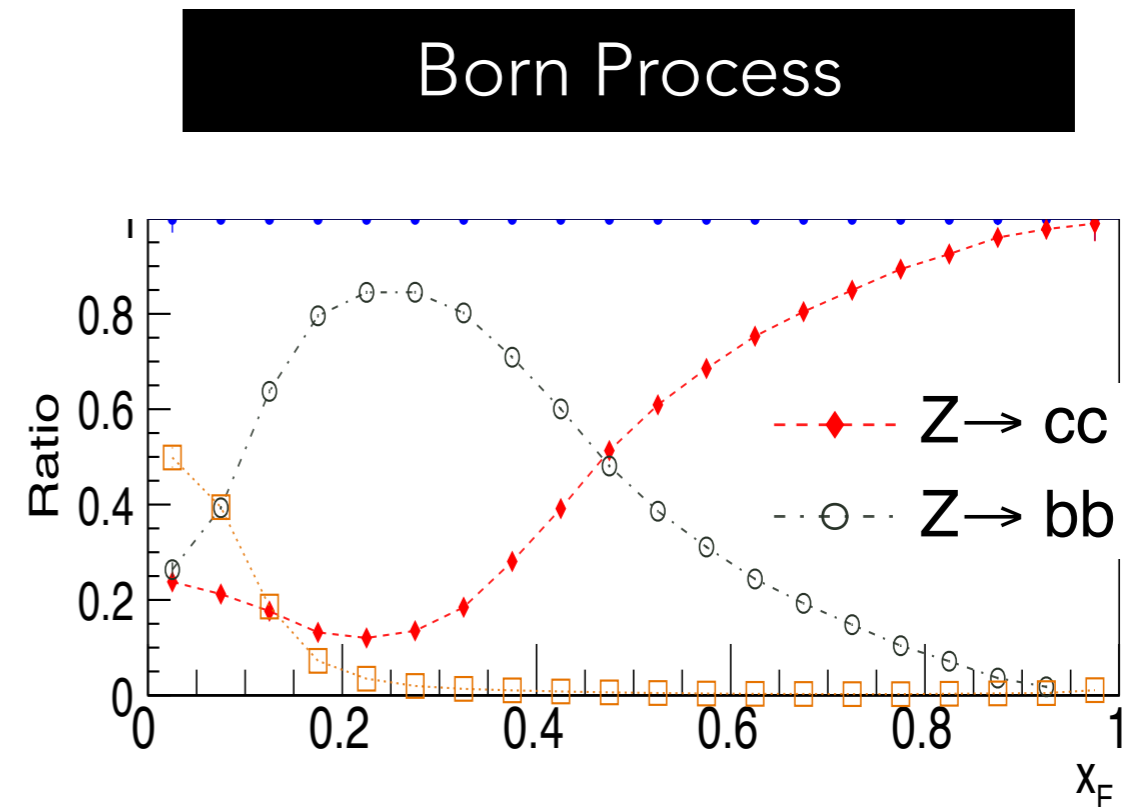
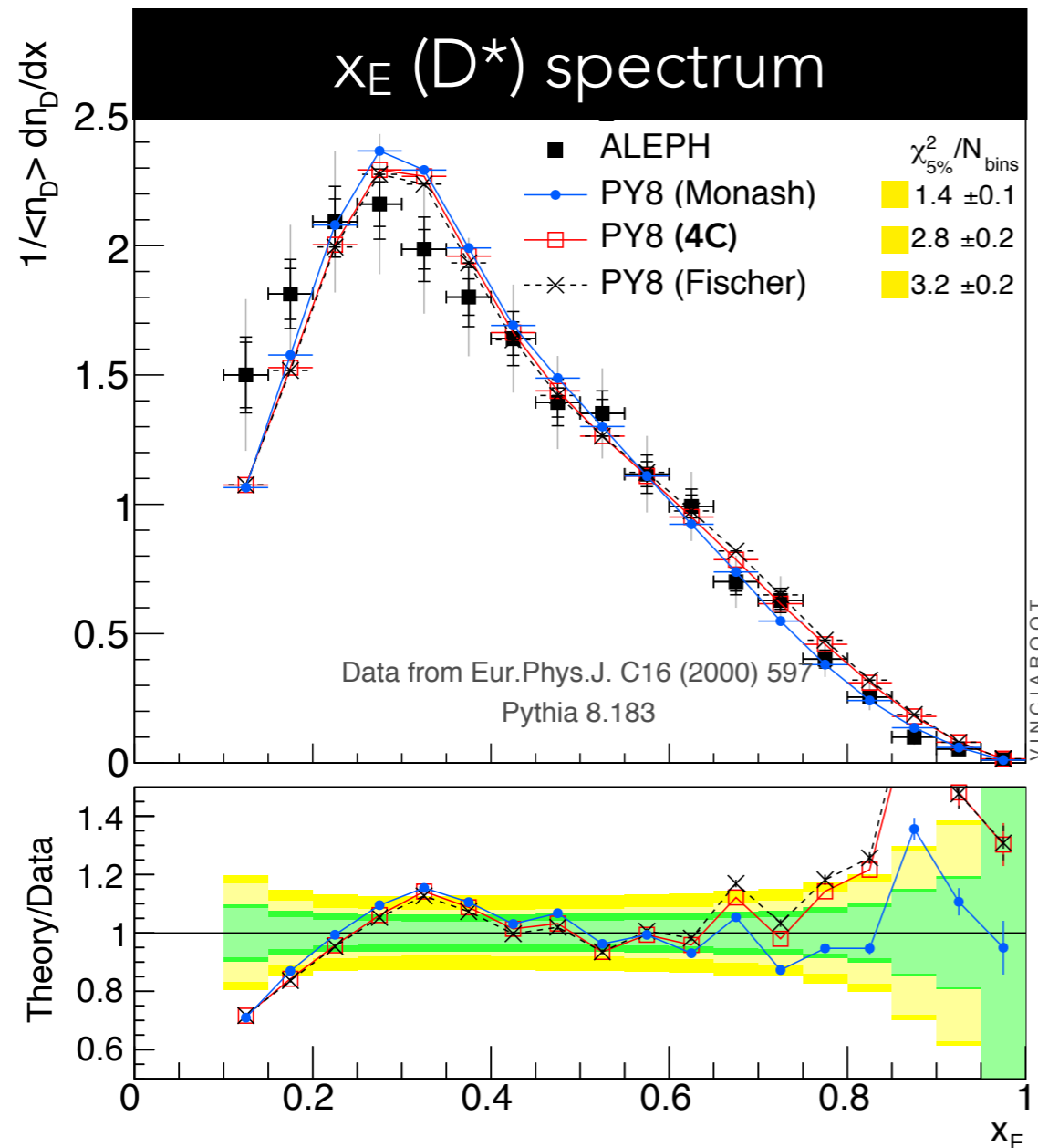
Problem: most LEP measurements are inclusive (including 3-jet events)

\rightarrow Would need 3-jet NLO merging to ensure correct 3-jet admixture.

Constraints : Charm

No "C-tagged" data from LEP (that I am aware of)

Monash tune only used a single D^* spectrum (ALEPH) $\rightarrow r_c$

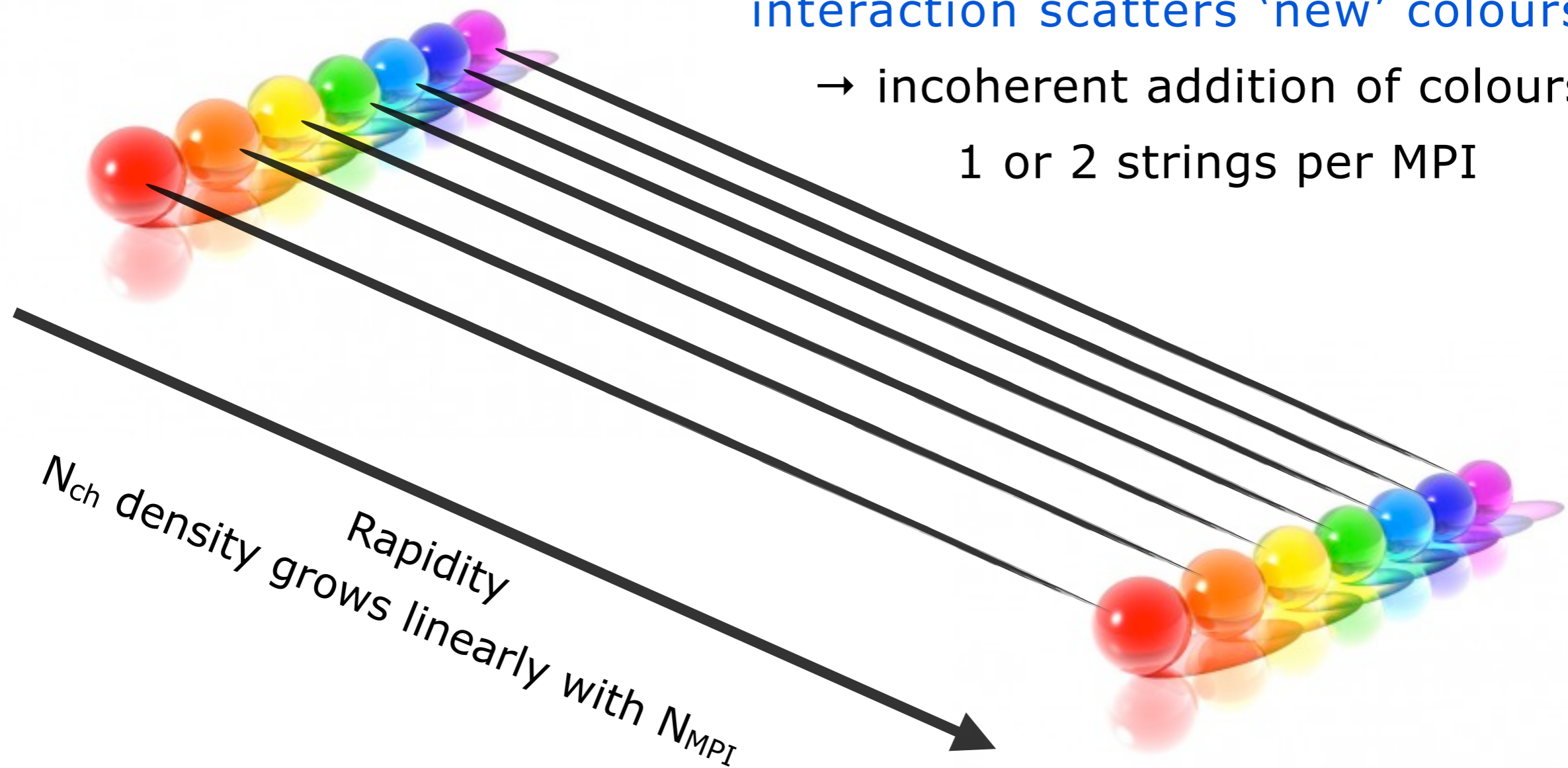


Actual charm fragmentation not seen very clearly. For $x < 0.5$, the inclusive D^* spectrum is dominated by B decays

From ee to pp: multiple parton interactions (MPI)

Leading N_c : each parton-parton interaction scatters 'new' colours

→ incoherent addition of colours
1 or 2 strings per MPI

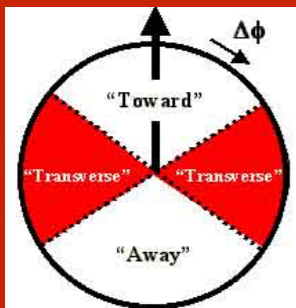


Simple, clean, factorized picture ...

WRONG!

Anticipated already in first Pythia MPI model (*Sjöstrand & van Zijl, 1987*)

"CR" parameter = probability for MPI to just generate "kinks" on hard-process colour structure, rather than new strings of their own



Tevatron $\langle N_{ch} \rangle$ and $\langle \Sigma p_T \rangle$ in "Transverse" UE region
Required $\sim 100\%$ CR (*Rick Field, "Tune A", 2002*)



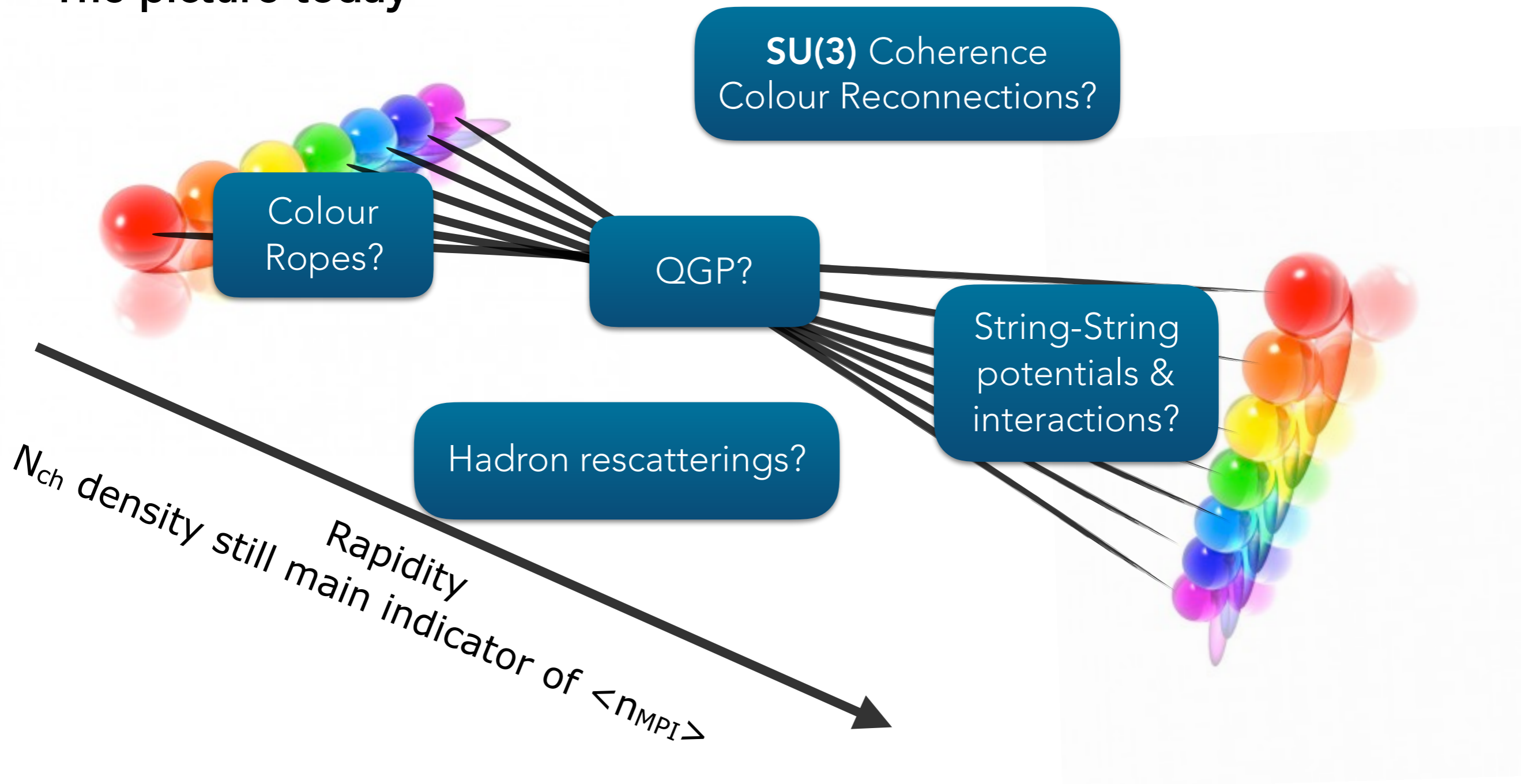
➤ Not a small effect, then ...

+ Many new measurements and discoveries from LHC (& RHIC)

(e.g., CMS ridge, ALICE strangeness vs N_{ch} , ...)

The MPI are all within a proton radius of each other (in pp)

The picture today



Fertile ground for model building

Brief Summary of “New” Theory Models in Pythia

QCD-inspired CR `ColourReconnection:mode = 1` Christiansen, Skands, JHEP 1508 (2015) 003

Stochastically sample subleading- N_C connections according to **SU(3) weights** and choose among possibilities (incl colour- ϵ ones) based on **string-length minimisation**.

➤ **some flow effects & additional baryons** (incl multiply-heavy); no extra strangeness

Ropes & Shoving Bierlich, Gustafson, Lönnblad, Tarasov, JHEP 1503 (2015) 148
Bierlich, Gustafson, Lönnblad, PLB 779 (2018) 58

Ropes: allow QCD charges to combine into higher representations: 6, 10, 15, 21, 28, ... with **higher string tensions** (Casimir scaling) ➤ **more strangeness & more baryons**

Shoving: explicit dynamical model of repulsion between different strings/ropes ➤ **flow**

Thermodynamical String fragmentation Fischer, Sjöstrand, JHEP 1701 (2017) 140

+ Much ongoing work ...

Hadronic Rescattering (Sjöstrand+Utheim)

HI extensions (Angantyr, PISTA) & extensions with **UrQMD** (Bierlich et al.)

Interacting Strings: momentum-space alternative to ropes+shoving (Duncan+Skands)

Back to basics: fragmentation of a single string: early / out-of-equilibrium, and thermal effects. **Time-varying string tension** out soon. + other variants? E.g., **UCLA model?**

Some Suggestions for New Measurements

Want to disentangle $\langle p_T \rangle$, $\langle \text{strangeness} \rangle$, $\langle \text{baryons} \rangle$, $\langle N_{\text{ch}} \rangle$
 $\langle \zeta \rangle$ ($\backslash \text{varsigma}$) $\langle \mathcal{B} \rangle$

E.g., CR and “flow” increase $\langle p_T \rangle$ without (directly) affecting $\langle \zeta \rangle$

“Baryonic” CR can increase $\langle \mathcal{B} \rangle$

Higher tensions/temperatures: correlated $\langle p_T \rangle$, $\langle \zeta \rangle$, and $\langle \mathcal{B} \rangle$

Some Simple Questions:

How **local** are the $\langle \zeta \rangle$ and $\langle \mathcal{B} \rangle$ enhancement mechanisms?

How far in phase space **is nearest anti-strange / anti-baryon?**

For different values of N_{ch} density, p_{TB} or $p_{\text{T}}(\text{b-jet})$, and ζ density

E.g., heavy-flavour tag, say B_s \blacktriangleright know the endpoint flavour \blacktriangleright look for nearest anti-strange quark.

What is the distance in p_T ? in rapidity (along z / along b-jet)? in ΔR ?

How do the HF fractions depend on **event multiplicity?**

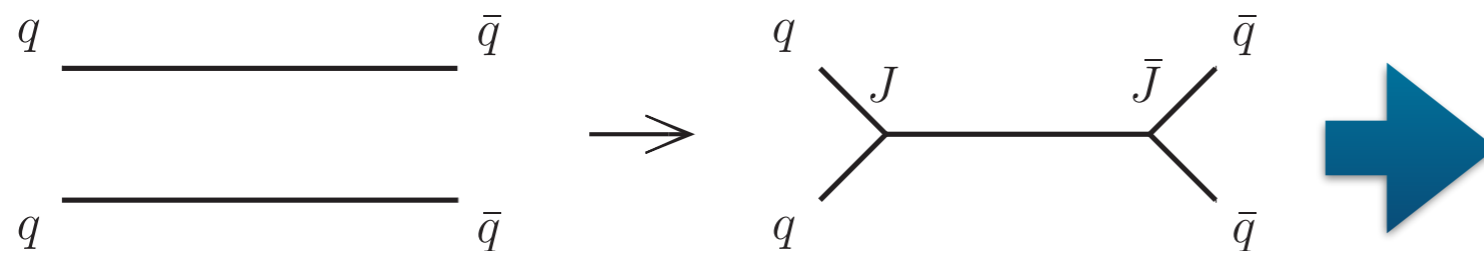
Heavy-Flavour Baryons

Christiansen, Skands, JHEP 1508 (2015) 003

Example: QCD-inspired CR

ColourReconnection:mode = 1 = 0

Allows "junction reconnections", e.g.:



(b) Type II: junction-style reconnection

For the parameters used in that study,

Λ_c/D^+ increased by factor 2

Λ_b/B^+ by factor 3

+ potentially larger changes for $\Sigma_{c,b}^{(*)}$

| Particle | $N_{\text{par}}/N_{\text{events}}$ | |
|------------------|------------------------------------|---------------------|
| D^+ | $5.3 \cdot 10^{-2}$ | $6.5 \cdot 10^{-2}$ |
| Λ_c^+ | $1.2 \cdot 10^{-2}$ | $6.6 \cdot 10^{-3}$ |
| Σ_c^{++} | $1.3 \cdot 10^{-2}$ | $5.4 \cdot 10^{-4}$ |
| Σ_c^+ | $1.5 \cdot 10^{-2}$ | $5.2 \cdot 10^{-4}$ |
| Σ_c^0 | $1.3 \cdot 10^{-2}$ | $5.1 \cdot 10^{-4}$ |
| Σ_c^{*++} | $2.2 \cdot 10^{-3}$ | $9.5 \cdot 10^{-4}$ |
| Σ_c^{*+} | $2.4 \cdot 10^{-3}$ | $9.4 \cdot 10^{-4}$ |
| Σ_c^{*0} | $2.2 \cdot 10^{-3}$ | $9.1 \cdot 10^{-4}$ |
| ccq^7 | $2.1 \cdot 10^{-4}$ | $1.0 \cdot 10^{-7}$ |
| B^+ | $1.6 \cdot 10^{-3}$ | $2.3 \cdot 10^{-3}$ |
| Λ_b^0 | $8.2 \cdot 10^{-4}$ | $3.9 \cdot 10^{-4}$ |
| Σ_b^+ | $9.5 \cdot 10^{-4}$ | $3.1 \cdot 10^{-5}$ |
| Σ_b^0 | $1.0 \cdot 10^{-3}$ | $3.7 \cdot 10^{-5}$ |
| Σ_b^- | $9.4 \cdot 10^{-4}$ | $3.2 \cdot 10^{-5}$ |
| Σ_b^{*+} | $9.5 \cdot 10^{-4}$ | $3.1 \cdot 10^{-5}$ |
| Σ_b^{*0} | $1.0 \cdot 10^{-3}$ | $3.7 \cdot 10^{-5}$ |
| Σ_b^{*-} | $9.4 \cdot 10^{-4}$ | $3.2 \cdot 10^{-5}$ |
| bcq^7 | $1.8 \cdot 10^{-5}$ | 0 |
| bbq^7 | $1.1 \cdot 10^{-6}$ | 0 |

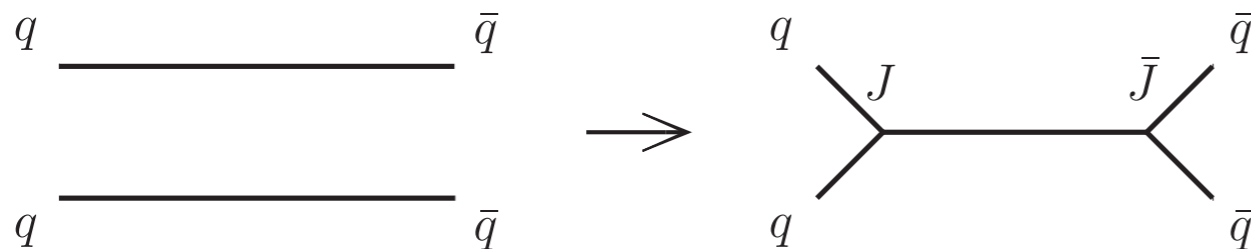
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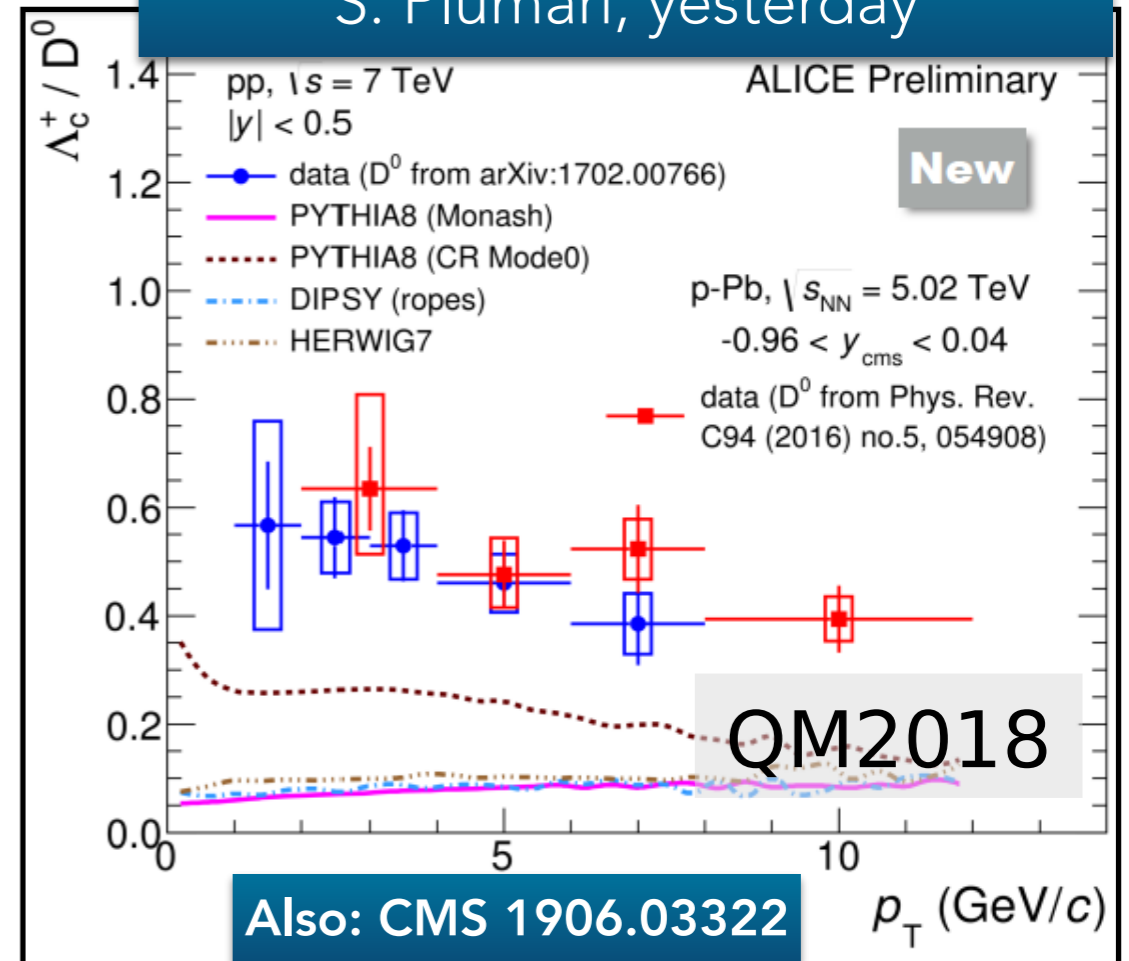
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| Σ^0 | $1.3 \cdot 10^{-2}$ | $5.1 \cdot 10^{-4}$ |

S. Plumari, yesterday

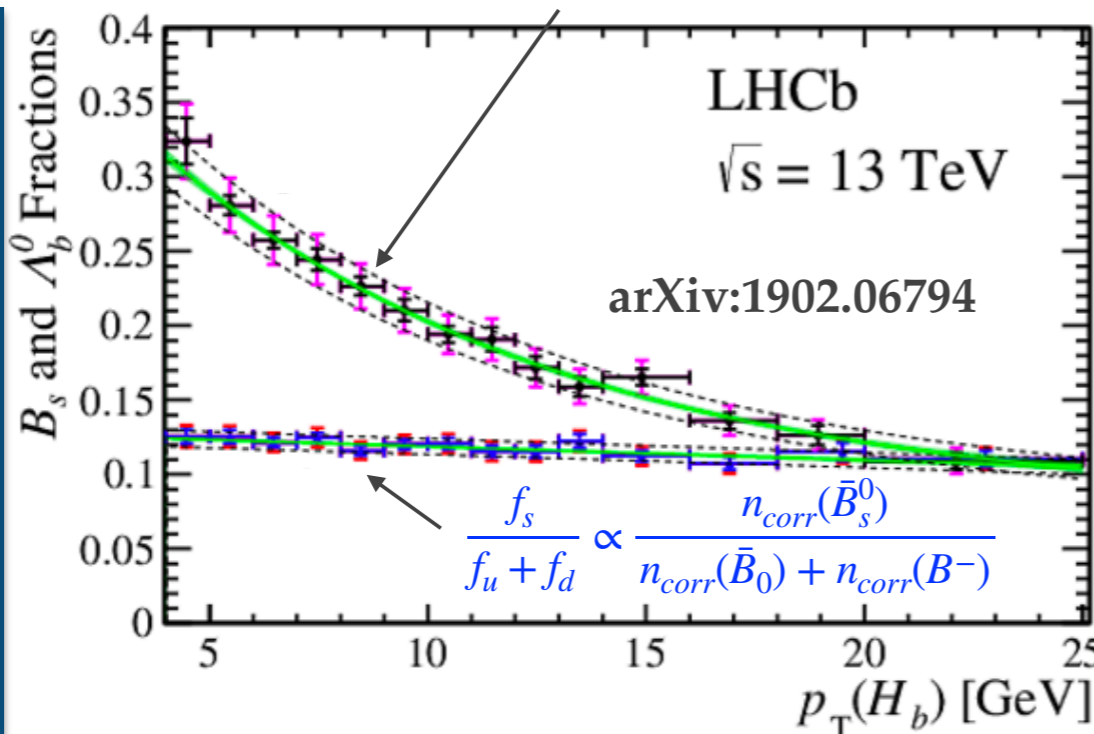
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B. Audurier, yesterday



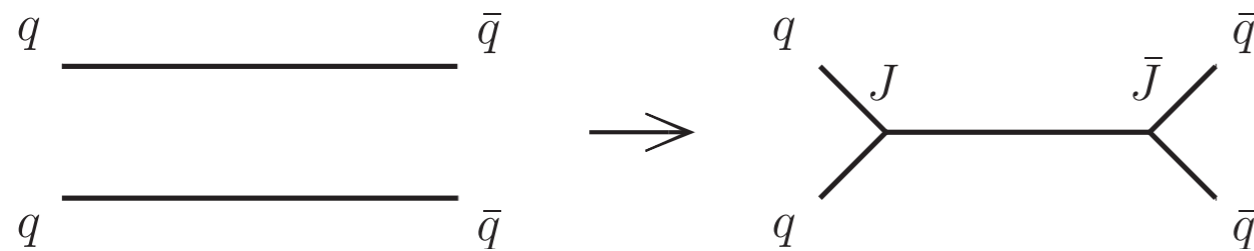
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Christiansen, Skands, JHEP 1508 (2015) 003

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From S. Plumari, yesterday

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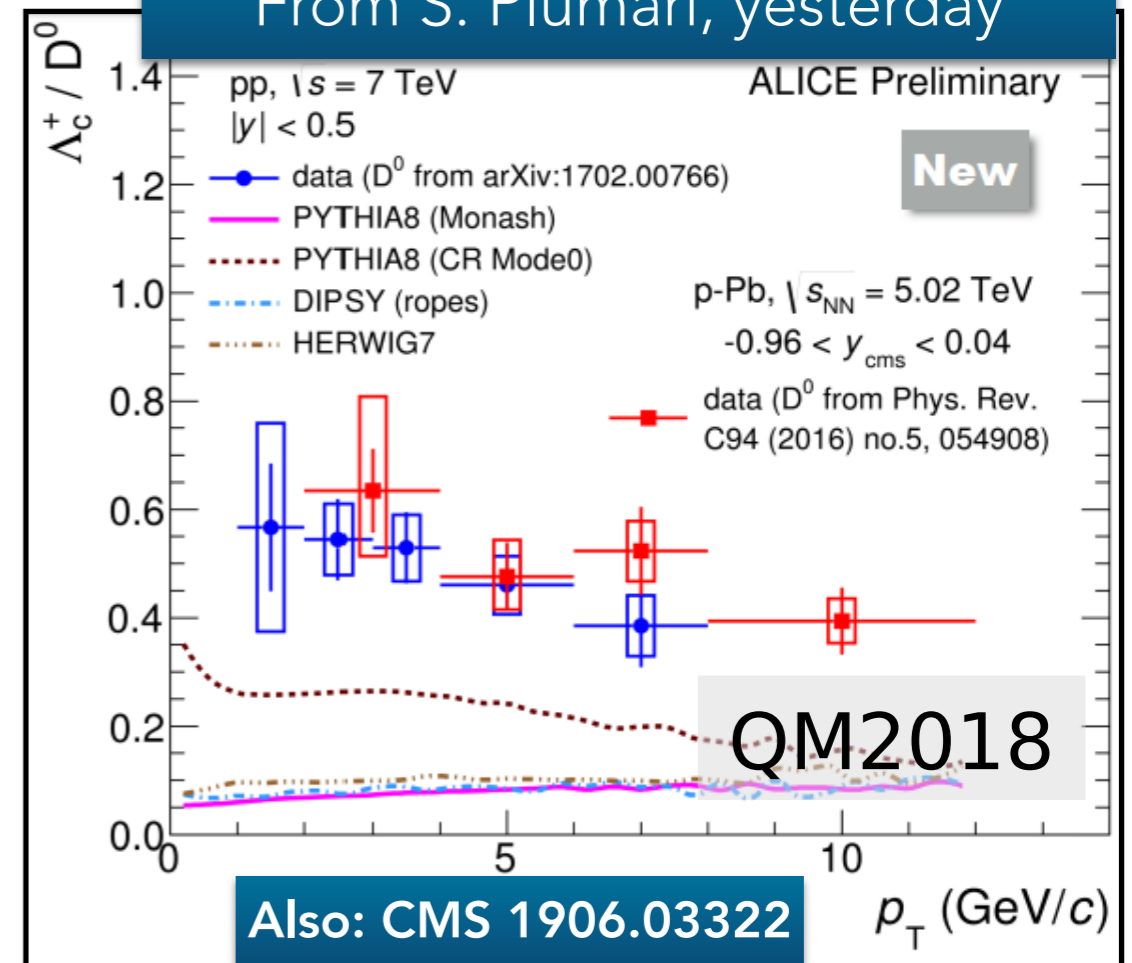
Λ_b/B^+ by factor 3

+ potentially larger changes for $\Sigma_{c,b}^{(*)}$

Generically expect dependence on multiplicity \blacktriangleright Measure $\langle B/M \rangle (N_{\text{ch}})$?

(Should be true for ropes, hydro, ... too)

+ **baryon-antibaryon** rapidity dependence?



First step, e.g., ALICE D meson associated track multiplicities in arXiv:1910.14403

(Some) LHCb measurements

B_s/B^+ vs event kinematics

From $B_s \rightarrow J/\psi \varphi$ & $B^+ \rightarrow J/\psi K^+$

No dependence on p_{LB}, η_B

Decreasing trend with p_{TB} 

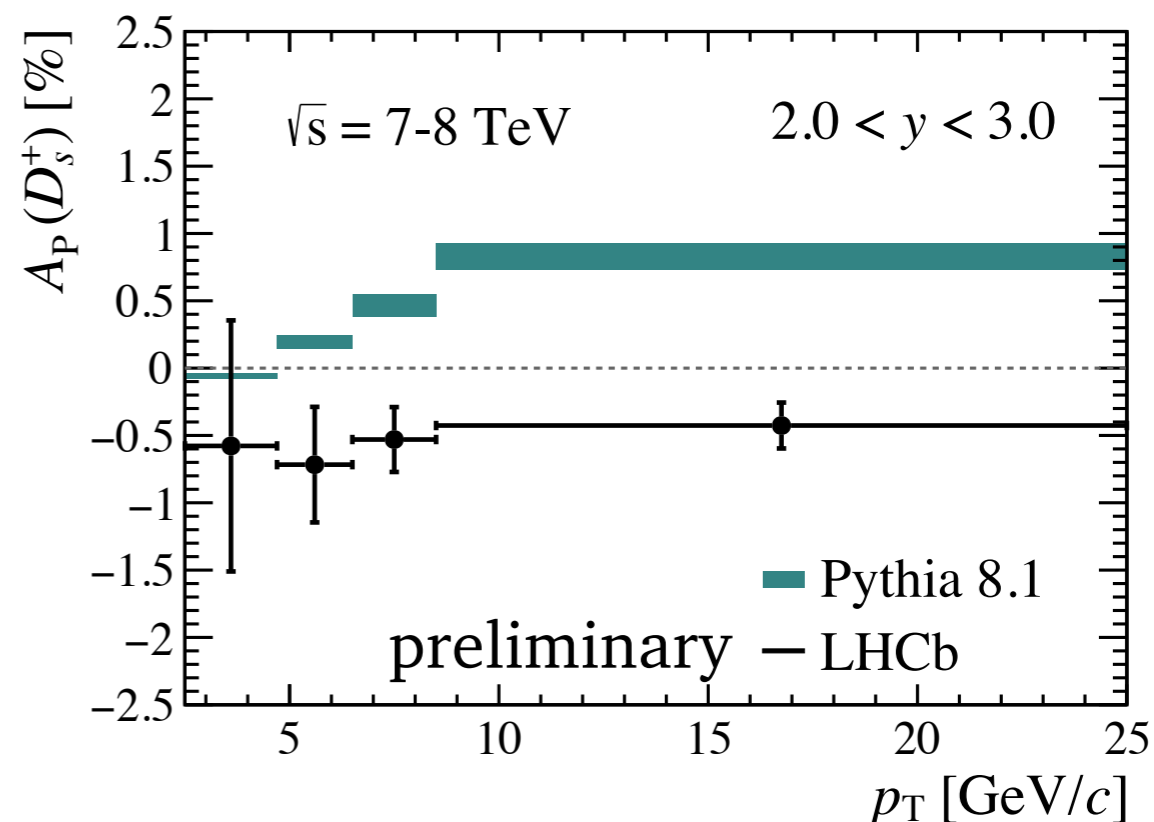
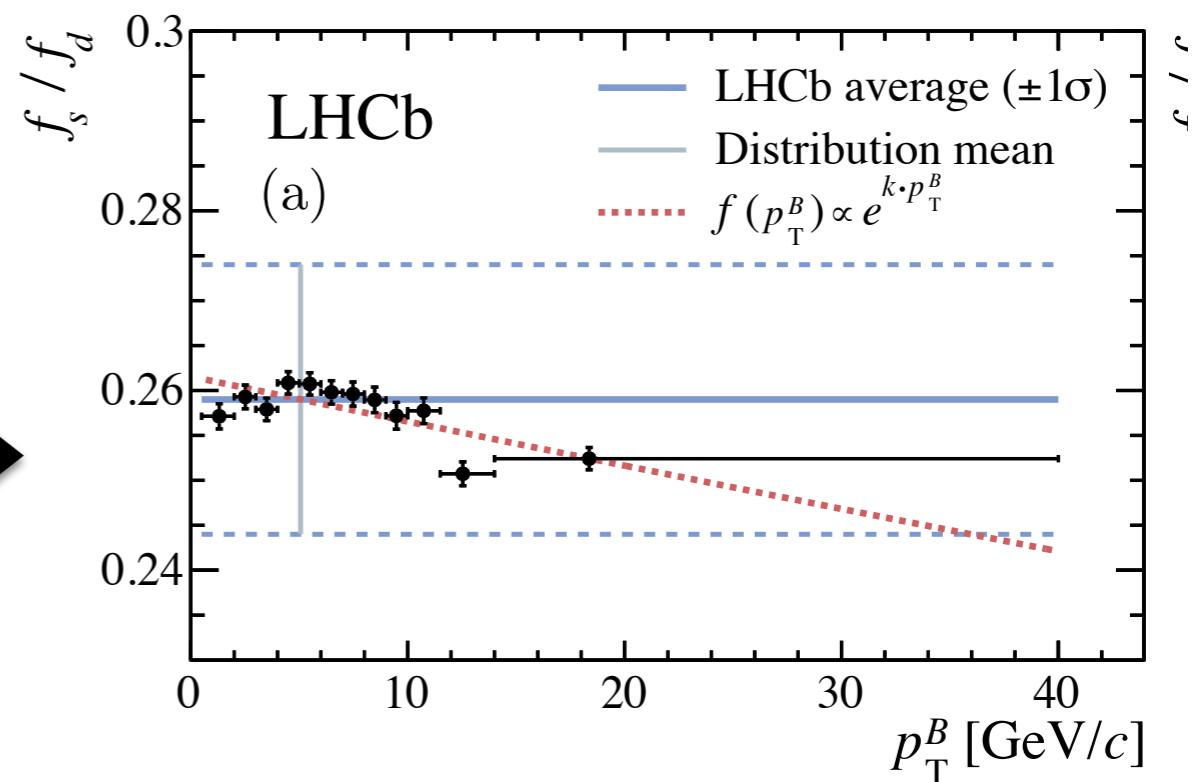
Would be highly interesting to see vs event multiplicity / associated track multiplicity

D_s asymmetry (S. Klaver, Moriond 2018)

$$\frac{\sigma(pp \rightarrow D_s^+) - \sigma(pp \rightarrow D_s^-)}{\sigma(pp \rightarrow D_s^+) + \sigma(pp \rightarrow D_s^-)}$$

Strong p_T dependence in Pythia, not seen in data

High p_T \blacktriangleright Coherence effect?



Multiply Heavy Hadrons?

Christiansen, Skands, JHEP 1508 (2015) 003

Heavy flavours produced perturbatively, not in string/cluster breakups;

So why would multiply heavy hadrons be interesting as soft probes?

Because they also **probe the confinement field** in unique ways (colour- ϵ_{ijk})

E.g., the Ξ_{cc} has been measured [LHCb-PAPER-2019-037](#)

Does its rate vary with associated track density?

| Particle | ColourReconnection:mode = 1 | | | = 0 |
|------------------|---|---------------------|---------------------|--|
| | New CR model ($N_{\text{par}}/N_{\text{events}}$) string | junction | all | Old CR model $N_{\text{par}}/N_{\text{events}}$ (all) |
| D^+ | $5.3 \cdot 10^{-2}$ | 0 | $5.3 \cdot 10^{-2}$ | $6.5 \cdot 10^{-2}$ |
| Λ_c^+ | $4.0 \cdot 10^{-3}$ | $7.9 \cdot 10^{-3}$ | $1.2 \cdot 10^{-2}$ | $6.6 \cdot 10^{-3}$ |
| Σ_c^{++} | $2.7 \cdot 10^{-4}$ | $1.3 \cdot 10^{-2}$ | $1.3 \cdot 10^{-2}$ | $5.4 \cdot 10^{-4}$ |
| Σ_c^+ | $2.5 \cdot 10^{-4}$ | $1.5 \cdot 10^{-2}$ | $1.5 \cdot 10^{-2}$ | $5.2 \cdot 10^{-4}$ |
| Σ_c^0 | $2.5 \cdot 10^{-4}$ | $1.3 \cdot 10^{-2}$ | $1.3 \cdot 10^{-2}$ | $5.1 \cdot 10^{-4}$ |
| Σ_c^{*++} | $5.1 \cdot 10^{-4}$ | $1.7 \cdot 10^{-3}$ | $2.2 \cdot 10^{-3}$ | $9.5 \cdot 10^{-4}$ |
| Σ_c^{*+} | $4.9 \cdot 10^{-4}$ | $1.9 \cdot 10^{-3}$ | $2.4 \cdot 10^{-3}$ | $9.4 \cdot 10^{-4}$ |
| Σ_c^{*0} | $4.8 \cdot 10^{-4}$ | $1.7 \cdot 10^{-3}$ | $2.2 \cdot 10^{-3}$ | $9.1 \cdot 10^{-4}$ |
| ccq^7 | 0 | $2.1 \cdot 10^{-4}$ | $2.1 \cdot 10^{-4}$ | $1.0 \cdot 10^{-7}$ |
| B^+ | $1.6 \cdot 10^{-3}$ | 0 | $1.6 \cdot 10^{-3}$ | $2.3 \cdot 10^{-3}$ |
| Λ_b^0 | $1.9 \cdot 10^{-4}$ | $6.3 \cdot 10^{-4}$ | $8.2 \cdot 10^{-4}$ | $3.9 \cdot 10^{-4}$ |
| Σ_b^+ | $1.1 \cdot 10^{-5}$ | $9.3 \cdot 10^{-4}$ | $9.5 \cdot 10^{-4}$ | $3.1 \cdot 10^{-5}$ |
| Σ_b^0 | $1.2 \cdot 10^{-5}$ | $1.0 \cdot 10^{-3}$ | $1.0 \cdot 10^{-3}$ | $3.7 \cdot 10^{-5}$ |
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| bcq^7 | 0 | $1.8 \cdot 10^{-5}$ | $1.8 \cdot 10^{-5}$ | 0 |
| bbq^7 | 0 | $1.1 \cdot 10^{-6}$ | $1.1 \cdot 10^{-6}$ | 0 |

Note: the baryon "predictions" depend on poorly constrained model parameters; highlight measurement sensitivity

To discuss: **observables to tell apart ...**

CR: longitudinal (1D) strings + transverse boosts: flow-like effects

No $\langle \zeta \rangle$ enhancement; low velocity dispersions relative to common boosts

Additional tracers: multiply heavy baryons (will at least \blacktriangleright constraints!)

Ropes etc: longitudinal (1D) strings with higher effective tensions

Strangeness enhancement + higher $\langle p_T \rangle$, but still "1D"

\blacktriangleright rank ordering, const dN/dy ?

Shoving etc.: Longitudinal strings with transverse repulsions

1D "rank" still relevant for $\langle \mathcal{B} \rangle$, $\langle \zeta \rangle$, and (local) p_T conservation \blacktriangleright correlations?

+ higher tensions? Is $\langle p_T \rangle$ correlated or anti-correlated with $\langle \zeta \rangle$, $\langle \mathcal{B} \rangle$?

Thermal/Statistical systems: 3D systems with higher effective T

Very high dispersions, 3D.

Quantum number and p_T conservation not ordered in "rank" at all?

Extra Slides

What a strange world we live in, said Alice [to the queen of hearts]

We wanted to know if “violent” collision events produced higher-strength fields.

Smoking gun would be a higher fraction of strange particles being produced

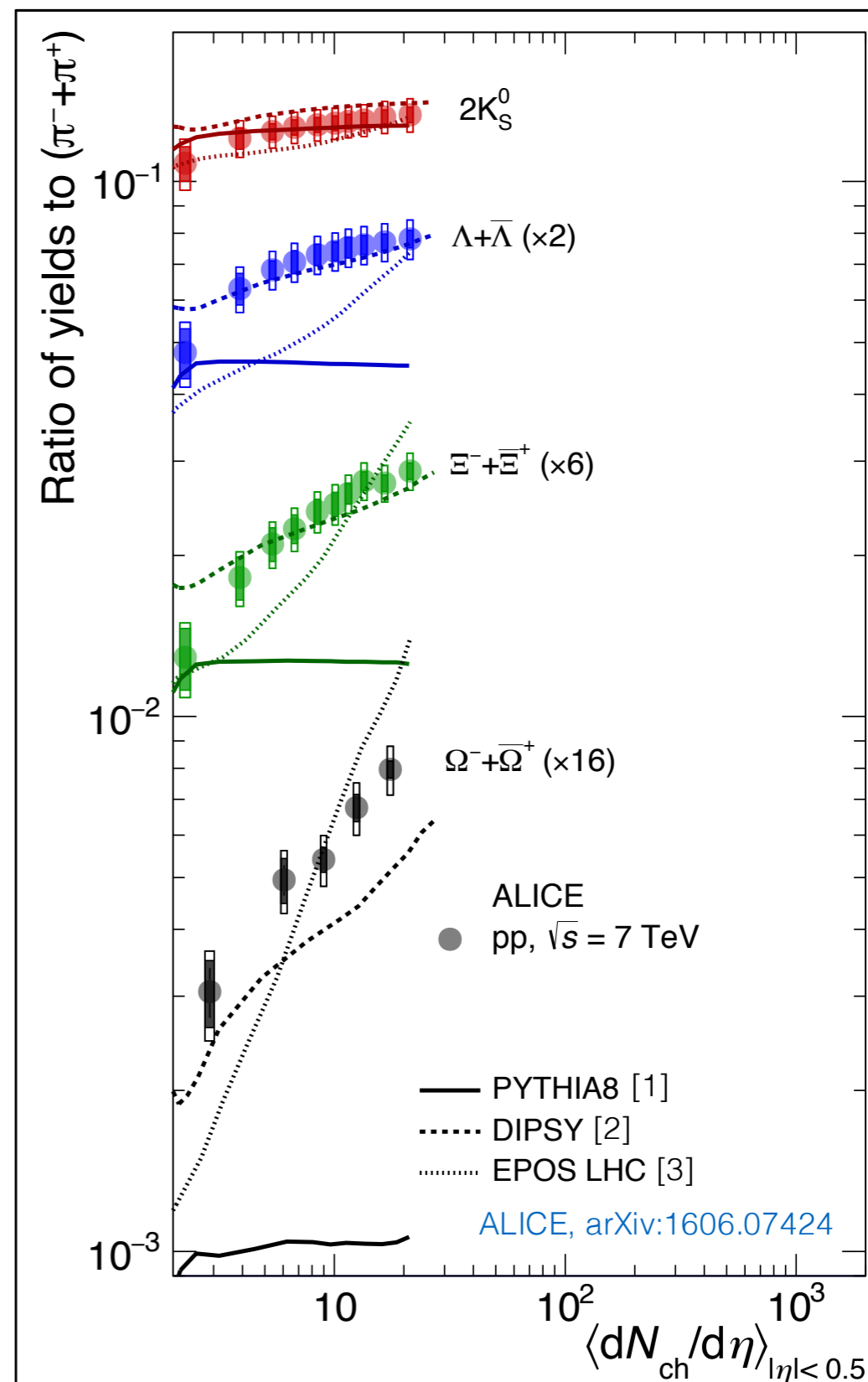
(higher-strength fields \implies more energy per “space-time volume” \implies easier to produce higher-mass quark-antiquark pairs)

Jackpot!

Now working on models in which nearby fragmenting fields interact with each other.

Interactions between QCD strings!

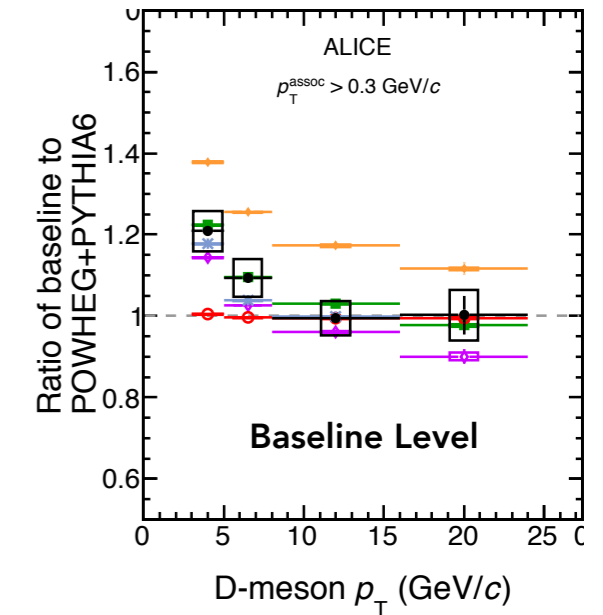
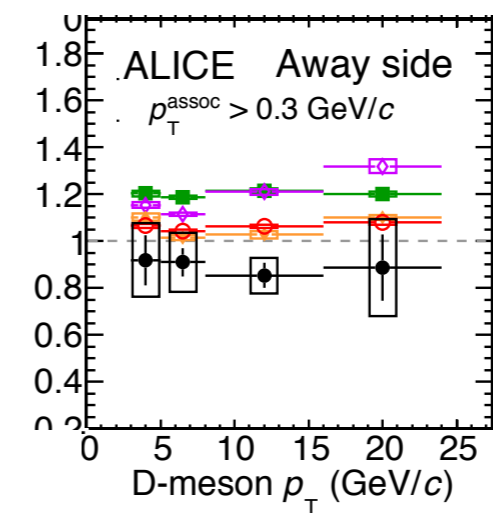
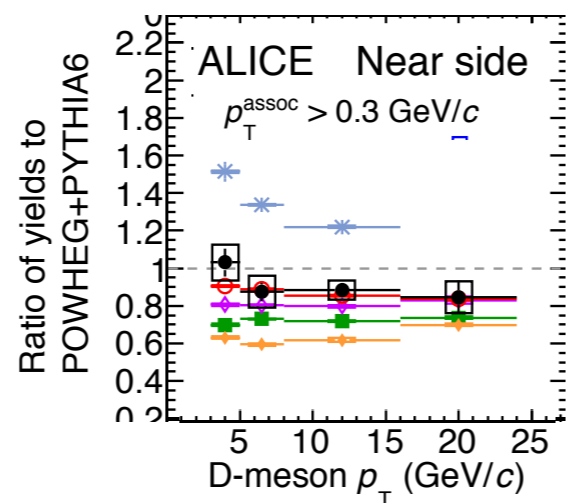
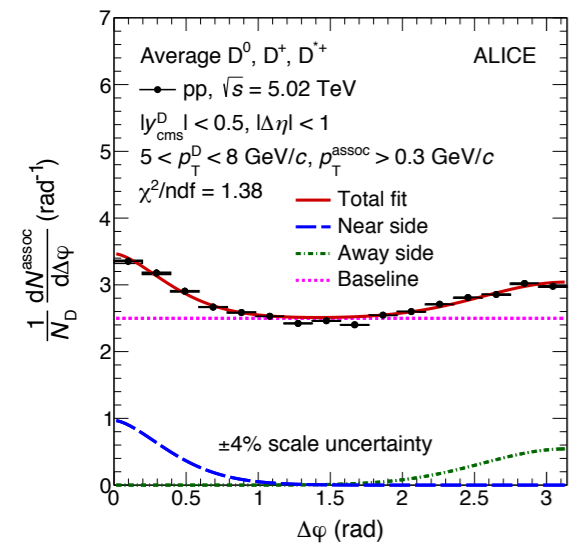
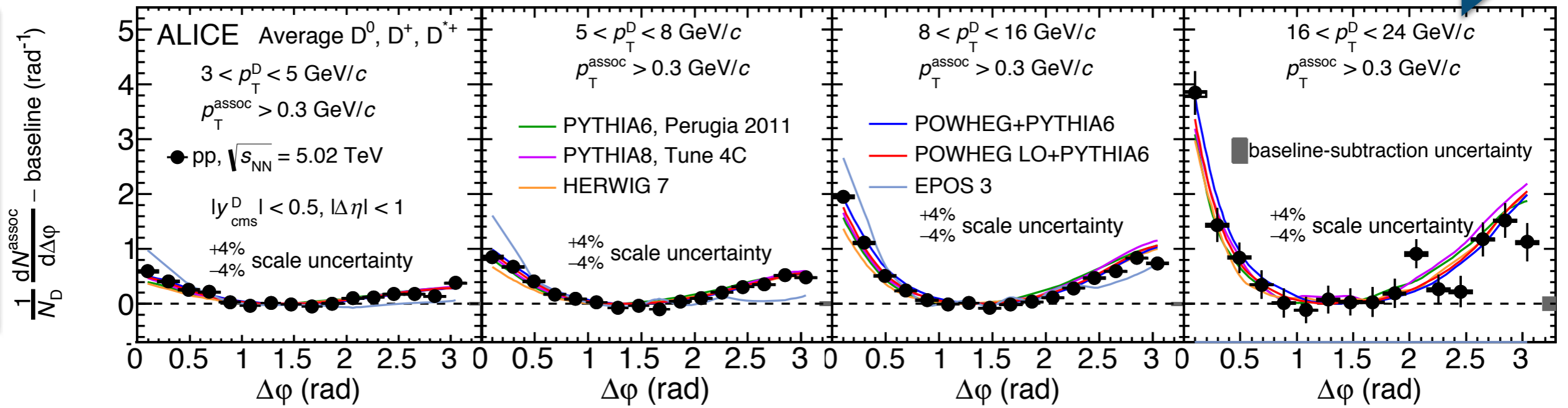
Higher tensions + repulsion effects \blacktriangleright modifications in high-density environments
(Competing idea: the whole thing turns into a near-perfect liquid which gets heated up.)



D meson associated tracks (ALICE)

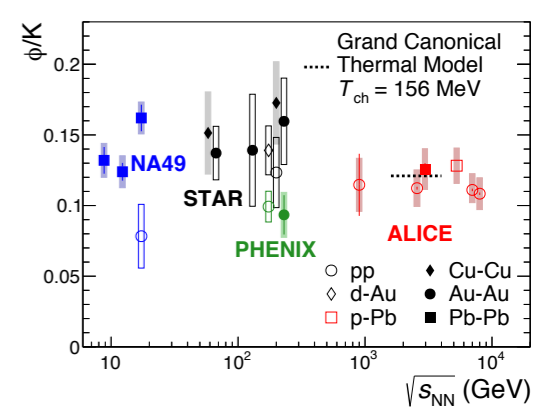
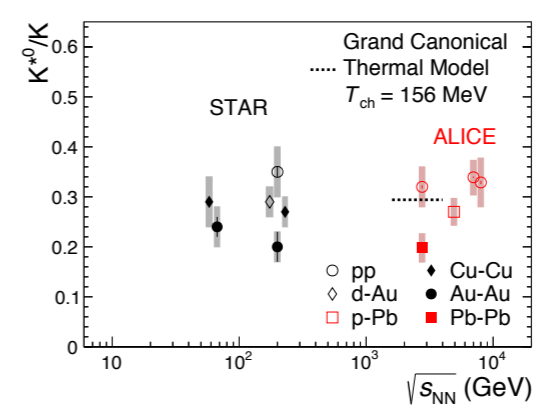
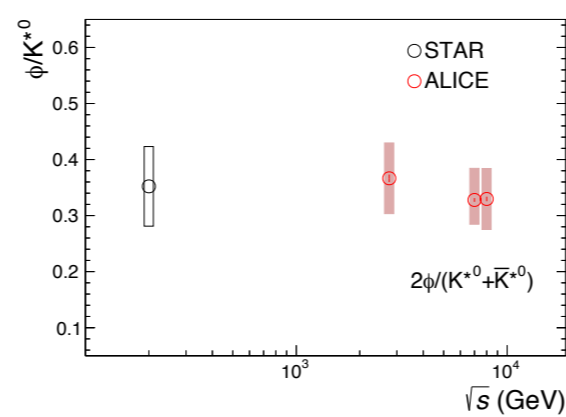
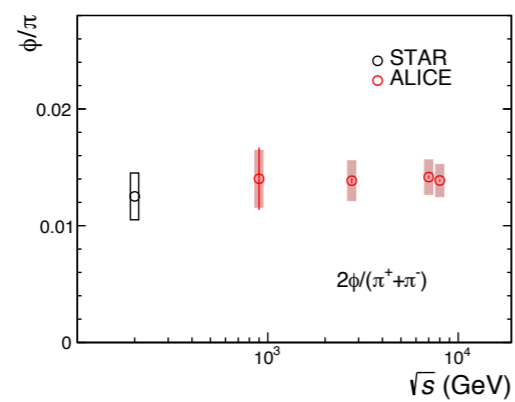
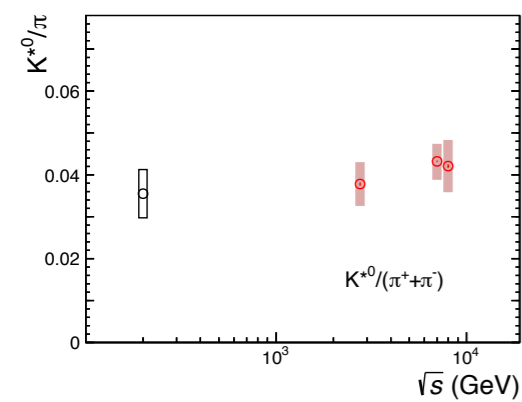
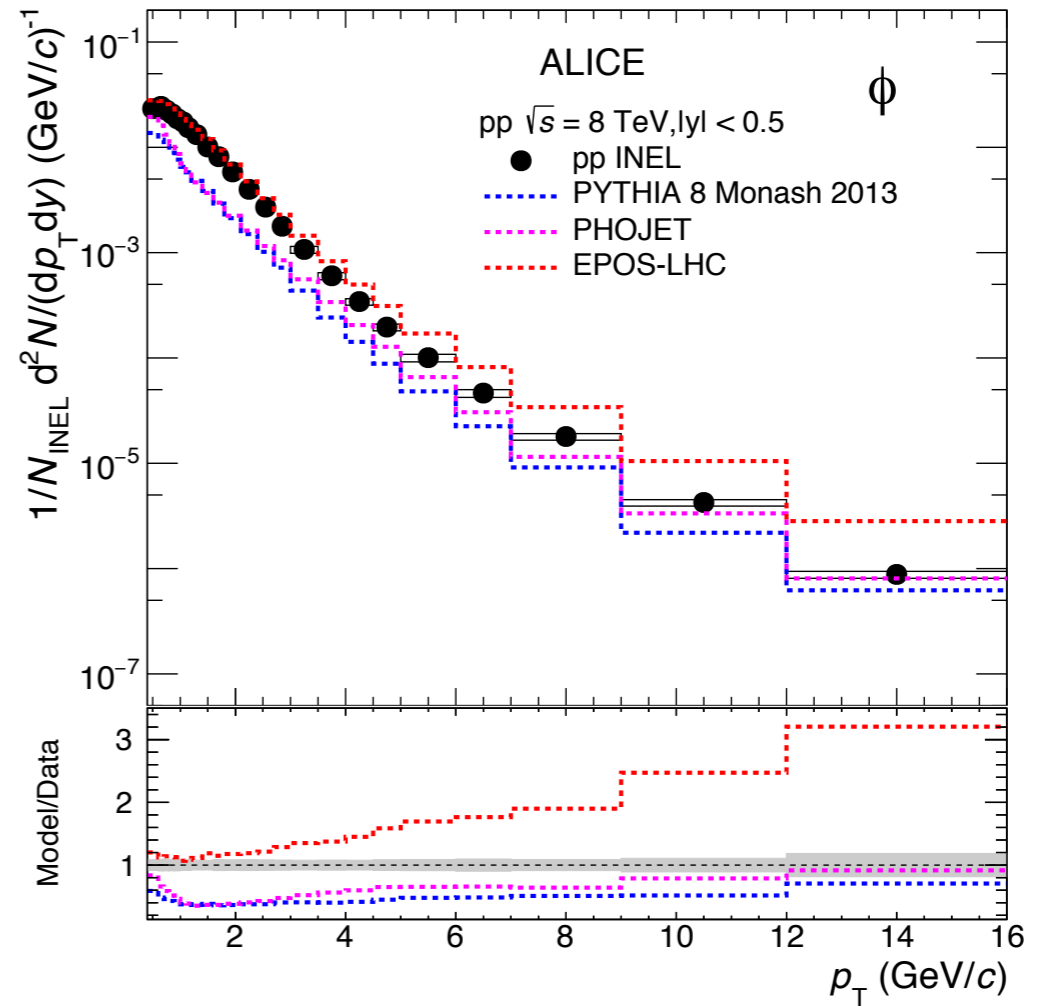
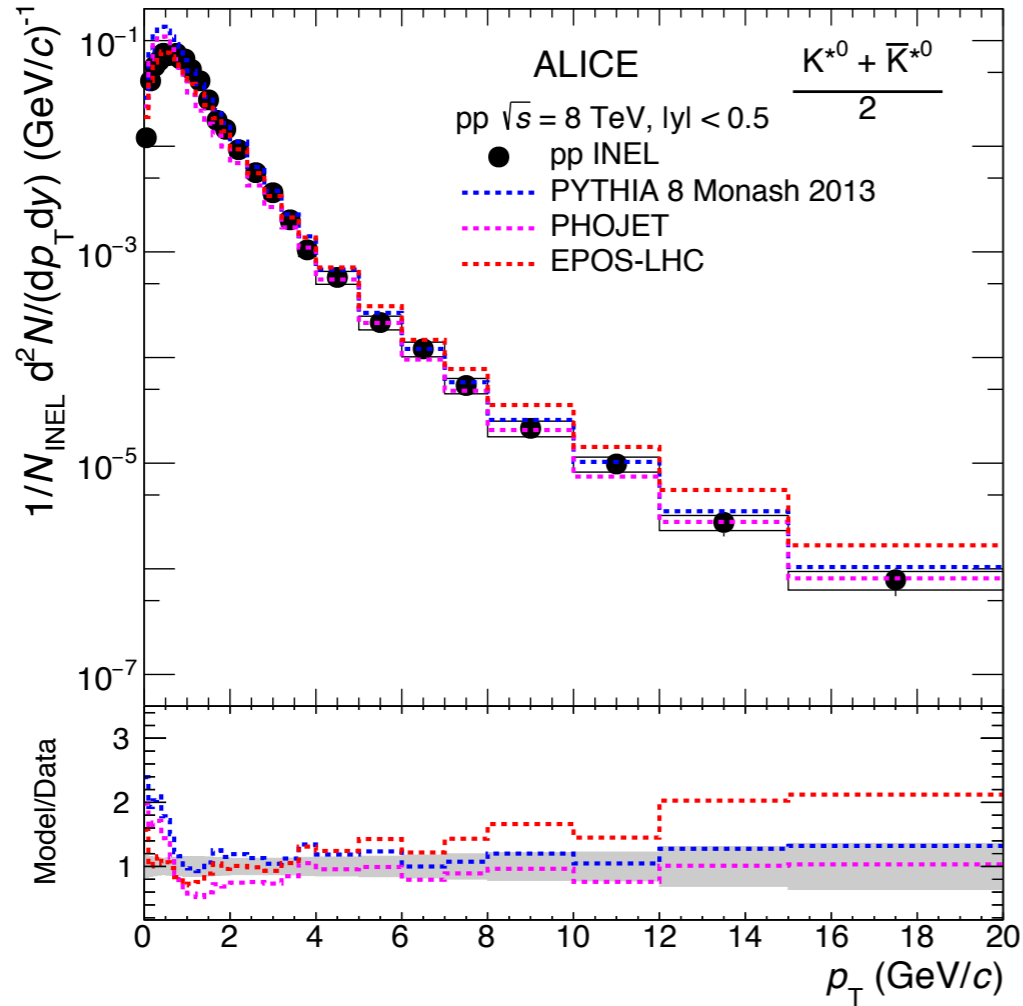


arXiv:1910.14403v1



K* and ϕ (ALICE)

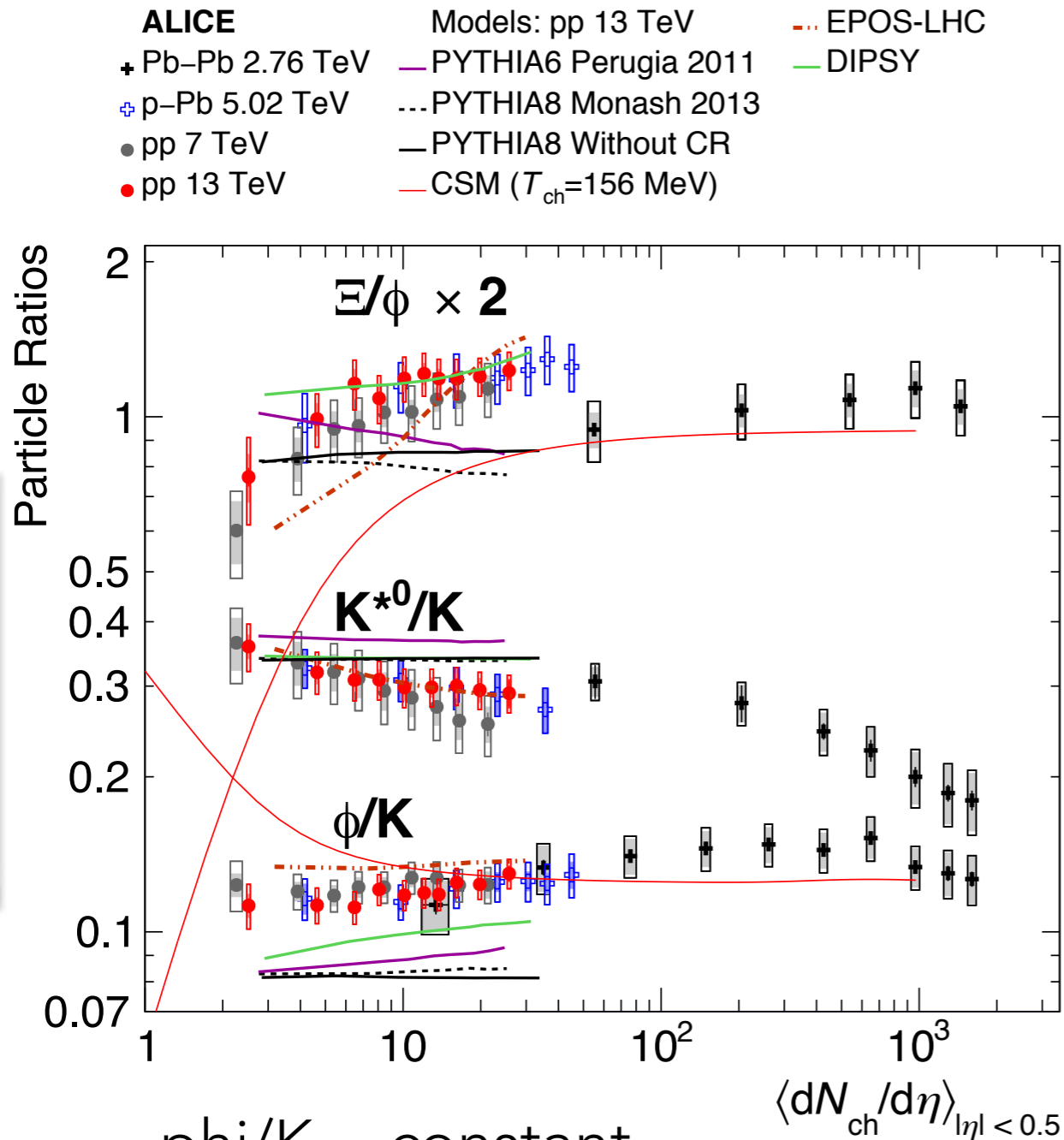
arXiv:1910.14410v1



NB: n_{ch} dependence measured separately, in arXiv:1910.14397

K* and ϕ multiplicity dependence (ALICE)

arXiv:1910.14397v1



$\phi/K \sim \text{constant}$
 K^*/K decreasing
 Ξ/ϕ increasing

