Antenna Showers with 2nd-Order Kernels

Peter Skands (Monash University)

1. **Global Showers with NLO MECs**
    - Hartgring, Laenen, PS, **arXiv:1303.4974**
2. **Global Showers with NLO Kernels**
    - Li, PS, **arXiv:1611.00013**
    - Iterated 2→3 + direct 2→4 branchings
    - Second-order corrections to the 2→3 kernels
3. **New Developments: Sector Showers**
    - **ee:** Lopez-Villarejo, PS, **arXiv:1109.3608**
    - **pp:** Brooks, Preuss, PS, **arXiv:2003.00702**

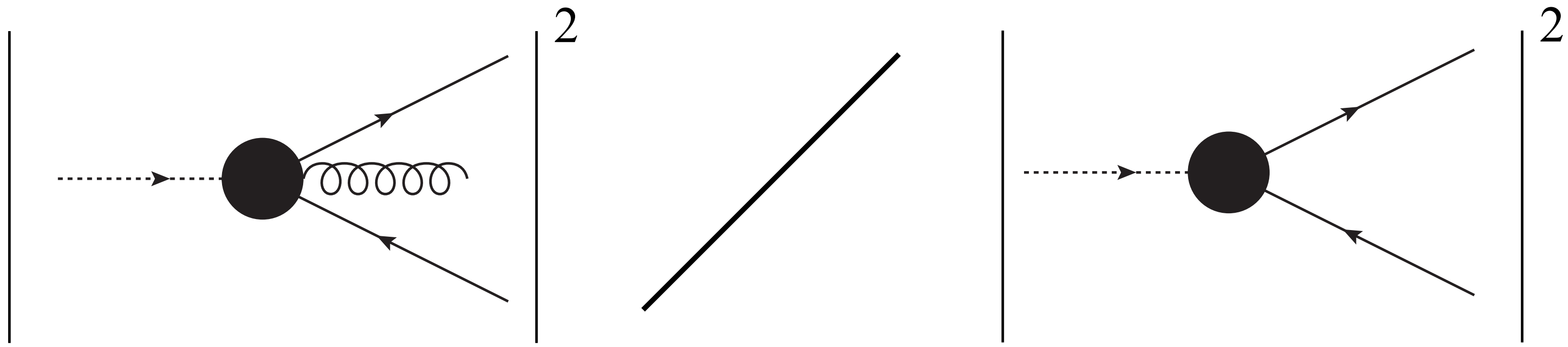
    } ➠ Pythia 8.304

    - Sector-Based Merging: *From Factorial* ➨ *Constant Time* **(in preparation)**
    - Outlook towards *2nd-order sector showers* **(ongoing work)**

June 2020
Taming the Accuracy of Event Generators

(Introduction): DGLAP, Antennae, and Dipoles

Factorisation of (squared) amplitudes in IRC singular limits



ij-collinear limit

jk-collinear limit

Full ME (modulo nonsingular terms)

DGLAP

$$\frac{P_{q\to qg}(z_i)}{s_{qg}} + \frac{P_{q\to qg}(z_k)}{s_{g\bar{q}}}$$

One term for each parton

Not a priori coherent.

Angular ordering restores azimuthally averaged eikonal

Antenna

$$\frac{2s_{q\bar{q}}}{s_{qg}s_{g\bar{q}}} + \frac{1}{s}\left(\frac{s_{g\bar{q}}}{s_{qg}} + \frac{s_{qg}}{s_{g\bar{q}}}\right)$$

eikonal term — collinear terms

partitioning of eikonal

One term for each colour connection

Coherent by construction

Dipole (CS/Partitioned)

$$\frac{\mathcal{K}_{qg,\bar{q}}(z_q)}{s_{qg}} + \frac{\mathcal{K}_{\bar{q}g,q}(z_{\bar{q}})}{s_{g\bar{q}}}$$

Two terms for each colour connection

Coherent by construction

Note: this is (intentionally) oversimplified. Many subtleties (recoil strategies, gluon parents, initial-state partons, and mass terms) not shown.

Why Antenna Showers?

Note: originally called "dipole showers" [Gustafson & Pettersson, 1988]; now confusing due to advent of new generation of (partitioned) dipole showers.

No need to partition the eikonal

→ easier to ensure **positive definite kernels**.

In dipole showers, two separate terms must be > 0, while in antenna showers only the equivalent of their sum needs to be > 0.

+ **Antenna-style recoils**: both parents absorb transverse recoil, rather than just one (though still not as general as PanGlobal)

Intrinsically coherent

Incorporates the **fully differential eikonal** (at Leading Colour)

➠ Coherent for any (sensible) choice of evolution variable

DGLAP + angular ordering only reproduces the eikonal in an integrated sense (averaged over azimuth).

Fewer terms:

Number of Histories for n Branchings

	$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$	$n=7$
CS Dipole	2	8	48	384	3840	46080	645120
Global Antenna	1	2	6	24	120	720	5040

(starting from a single colour-anticolour pair)

1. Early Proof of Concept

Hartgring, Laenen, PS, ***JHEP* 10 (2013) 127** (arXiv:1303.4974) ➠ Vincia 1.1 (Apr 2013)

Shower for $Z \to$ hadrons corrected through $\mathcal{O}(\alpha_s^2)$

- Double-real $(Z \to q\bar{q}gg \;\&\; Z \to q\bar{q}q'\bar{q}')$ based on iterated tree-level ME corrections [Giele, Kosower, Skands, 2011] through $Z \to 6$ from MG4, with "smooth ordering" (now abandoned)
- Hardcoded one-loop corrections to $Z \to q\bar{q}$ and $Z \to q\bar{q}g$ (massless quarks; LC)
- Double-virtual $Z \to q\bar{q}$ via unitarity (here just normalised total rate to unity).

Starting from $Z \to q\bar{q}$:

Compute NLO exclusive 3-jet cross section (with veto scale Q_4) at **fixed order** and in **shower**; define matching condition in limit $Q_4 \to 0$ (in dim.reg.)

(Could stop at hadronisation scale → power corrections in Q_{had})

$$\left|M_{Z\to q\bar{q}}\right|^2 A_3^0(Q^2)\left(1+V_3^{q\bar{q}}\right)\Delta_{2\to3}(Q_0^2,Q^2)\Delta_{3\to4}(Q^2,0) \stackrel{\mathcal{O}(\alpha_s^2)}{\rightarrow} |M_3^0|^2\left(1+\frac{2\mathrm{Re}\left[M_3^0 M_3^{1*}\right]}{|M_3^0|^2}\right)$$

Normalisation (best cross section available) (consistent with defining unit probability via unitarity at given order)

2→3 antenna function (tree-level, with MEC) with α_s evaluated at μ_{PS}

One-loop matching term, to be solved for

2→3 Sudakov factor : no branchings from starting scale Q_0 to resolution scale of 3-parton configuration, Q

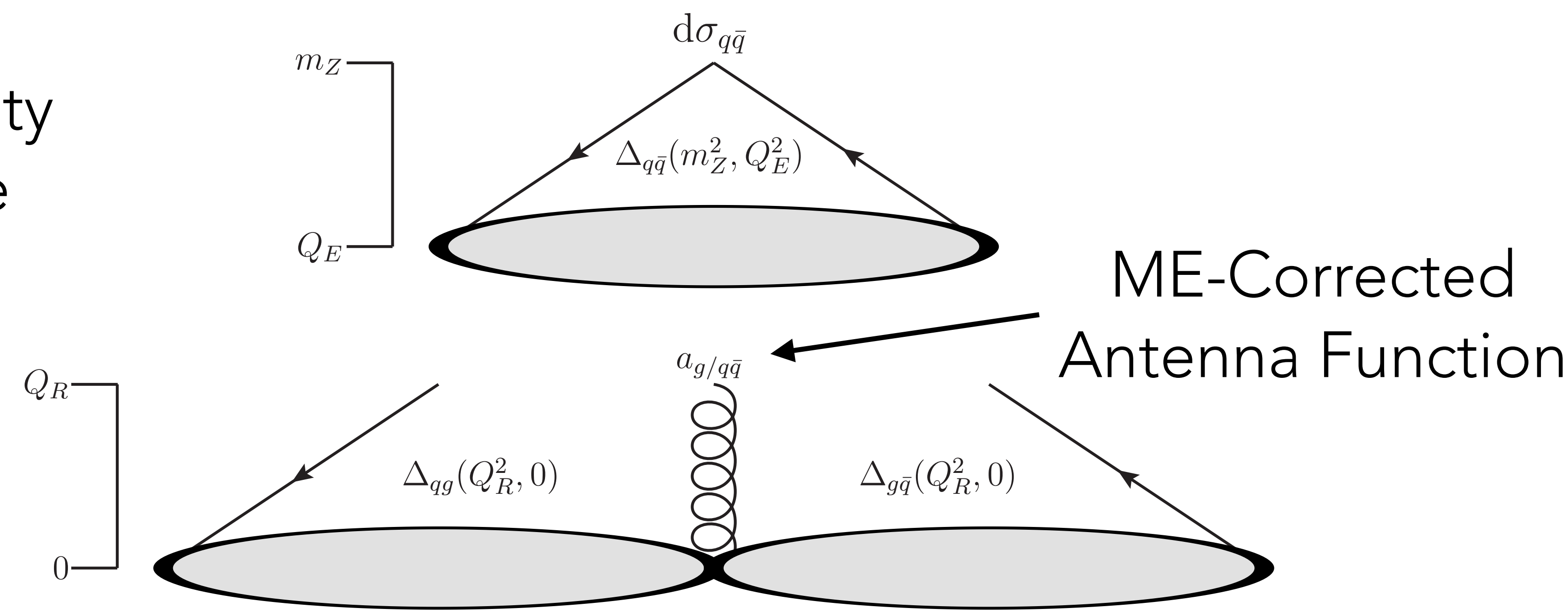
3→4 Sudakov factor : no branchings from scale of 3-parton configuration, Q to zero (in dim. reg.)

Fixed-Order $O(\alpha_s^2)$ (in dim. reg.) (renormalised at $\mu = \mu_{ME}$)

How it works

No-emission probability above 3-parton scale

No-emission probability below 3-parton scale



Solve for V3

$$
\begin{aligned}
V_{3Z}(q,g,\bar{q}) = {} & \left[\frac{2\,\mathrm{Re}[M_3^0 M_3^{1*}]}{|M_3^0|^2}\right]^{\mathrm{LC}} - \frac{\alpha_s}{\pi} - \frac{\alpha_s}{2\pi}\left(\frac{11N_C - 2n_F}{6}\right)\ln\left(\frac{\mu_{\mathrm{ME}}^2}{\mu_{\mathrm{PS}}^2}\right) \\
& + \frac{\alpha_s C_A}{2\pi}\left[-2I_{qg}^{(1)}(\epsilon,\mu^2/s_{qg}) - 2I_{qg}^{(1)}(\epsilon,\mu^2/s_{g\bar{q}}) + \frac{34}{3}\right] \\
& + \frac{\alpha_s n_F}{2\pi}\left[-2I_{qg,F}^{(1)}(\epsilon,\mu^2/s_{qg}) - 2I_{g\bar{q},F}^{(1)}(\epsilon,\mu^2/s_{g\bar{q}}) - 1\right] \\
& + \frac{\alpha_s C_A}{2\pi}\left[8\pi^2\int_{Q_3^2}^{m_Z^2} \mathrm{d}\Phi_{\mathrm{ant}}\; A_{g/q\bar{q}}^{\mathrm{std}} + 8\pi^2\int_{Q_3^2}^{m_Z^2} \mathrm{d}\Phi_{\mathrm{ant}}\; \delta A_{g/q\bar{q}}\right. \\
& \qquad \left. - \sum_{j=1}^{2} 8\pi^2\int_0^{s_j} \mathrm{d}\Phi_{\mathrm{ant}}\; (1 - O_{Ej})\; A_{g/qg}^{\mathrm{std}} + \sum_{j=1}^{2} 8\pi^2\int_0^{s_j} \mathrm{d}\Phi_{\mathrm{ant}}\; \delta A_{g/qg}\right] \\
& + \frac{\alpha_s n_F}{2\pi}\left[-\sum_{j=1}^{2} 8\pi^2\; P_{Aj}\int_0^{s_j} \mathrm{d}\Phi_{\mathrm{ant}}(1 - O_{Sj})\; A_{\bar{q}/qg}^{\mathrm{std}} + \sum_{j=1}^{2} 8\pi^2\int_0^{s_j} \mathrm{d}\Phi_{\mathrm{ant}}\; \delta A_{\bar{q}/qg}\right. \\
& \qquad \left. - \frac{1}{6}\,\frac{s_{qg} - s_{g\bar{q}}}{s_{qg} + s_{g\bar{q}}}\ln\left(\frac{s_{qg}}{s_{g\bar{q}}}\right)\right], \qquad (3.55)
\end{aligned}
$$

Here expressed in terms of the (N)LO $Z \to 3$ amplitudes, M_3^0 and M_3^1, and standard (GGG) antenna subtraction functions, A^{std}, with integrated poles $I^{(1)}$. The actual shower uses $A^{\mathrm{std}} + \delta A$, evolution scale Q, and ordering functions O_E and O_S for emissions and splittings respectively

→ Differential "K-factor" for 2→3 branchings

$$\left|M_{Z\to q\bar{q}}\right|^2 A_3^0(Q^2)\left(1+V_3^{q\bar{q}}\right)\Delta_{2\to3}(Q_0^2,Q^2)\Delta_{3\to4}(Q^2,0) \stackrel{\mathcal{O}(\alpha_s^2)}{\rightarrow} |M_3^0|^2\left(1+\frac{2\mathrm{Re}\left[M_3^0 M_3^{1*}\right]}{|M_3^0|^2}\right)$$

Non-divergent NLO correction
→ **positive-definite** NLO antenna

Poles ← Cancel if Q is IR safe → Poles

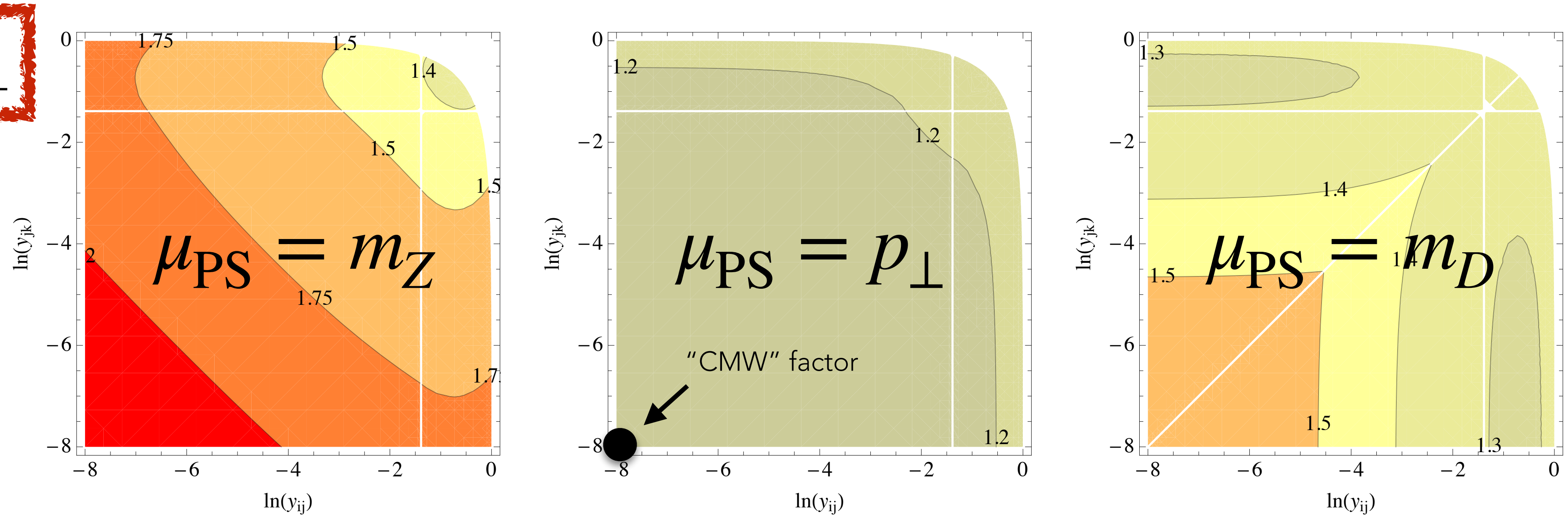
Partial cancellations

Use to define LL evolution so as to have no (resummable) logs left → Double Logs, Single Logs (incl β-dep)

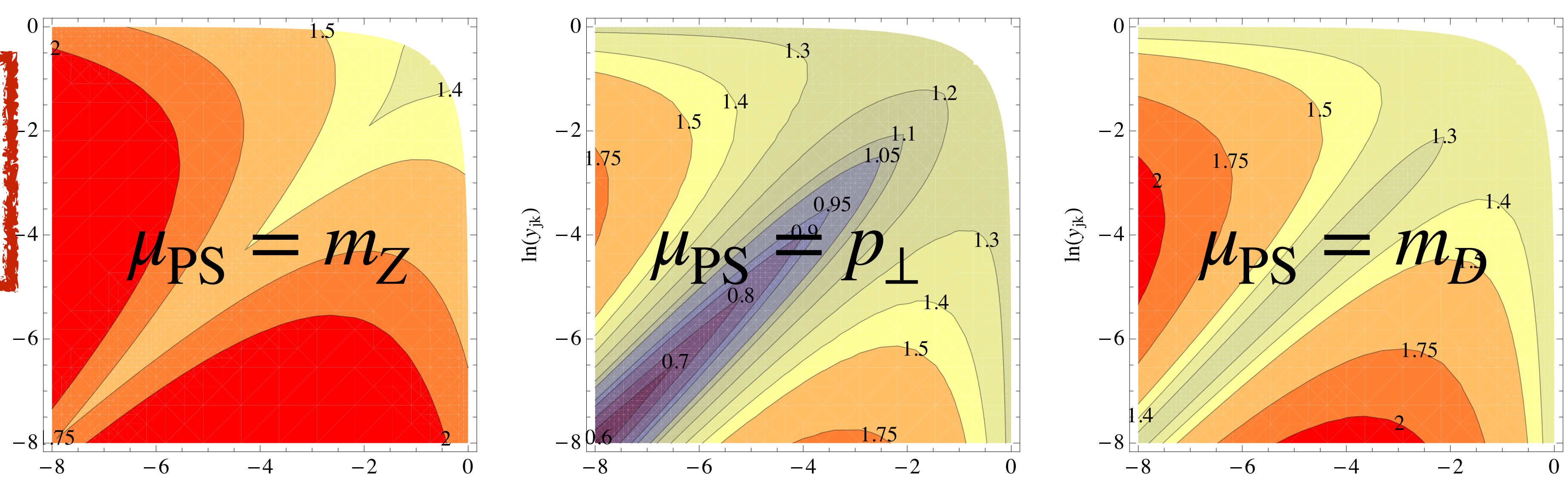
Nonsingular terms

Plots of V_3

$Q_E = p_\perp$



$Q_E = m_D$
~
virtuality



Note: no longer interested in smooth ordering; these plots are for strong ordering

2. From MECs → Shower kernels?

Li & PS, ***PLB* 771 (2017) 59** (arXiv:1611.00013)

Possible to base a shower framework on similarly derived "differential K-factors" for all antenna functions?

Elements

Iterated $2 \to 3$ and new "direct $2 \to 4$" branchings (in lieu of "smooth ordering") populate complementary phase-space regions.

- Ordered clusterings ➡ iterated $2 \to 3$
- Unordered clusterings ➡ direct $2 \to 4$ (+ higher, for sequential unordered steps)

Need appropriate scale definitions, $2 \to 4$ kernels, kinematics maps, and a $2 \to 4$ Sudakov sampler (with good efficiency in the relevant phase-space regions).

+ Virtual corrections to $2 \to 3$ kernels

Considerations of Shower Type: Global vs Sector Antennae

Conventional ("Global") antenna functions can be integrated over all of their phase spaces $\Longrightarrow$ **simple one-loop integrals**. (But **scale definitions are tricky**; see later.)

The $2 \to 4$ Branching Phase Space

Nesting of $2 \to 3$ Phase Spaces

For a given clustering: $d\Phi_{n+1} = d\Phi_n \times d\Phi_{\rm ant}$

↳ Generalisation to many possible clusterings: $d\Phi_{n+1} = \sum_{i=1}^{n_{\rm ant}} f_i \, d\Phi_{\rm ant,i} d\Phi_n^i$

Ordering/partitioning function (global or sector)

Global showers: f_i = 1 multiple cover (⊗ strong ordering)

~ conventional showers; antenna functions sum to total singularities

Sector showers: f_i = partition of unity (⊗ strong ordering)

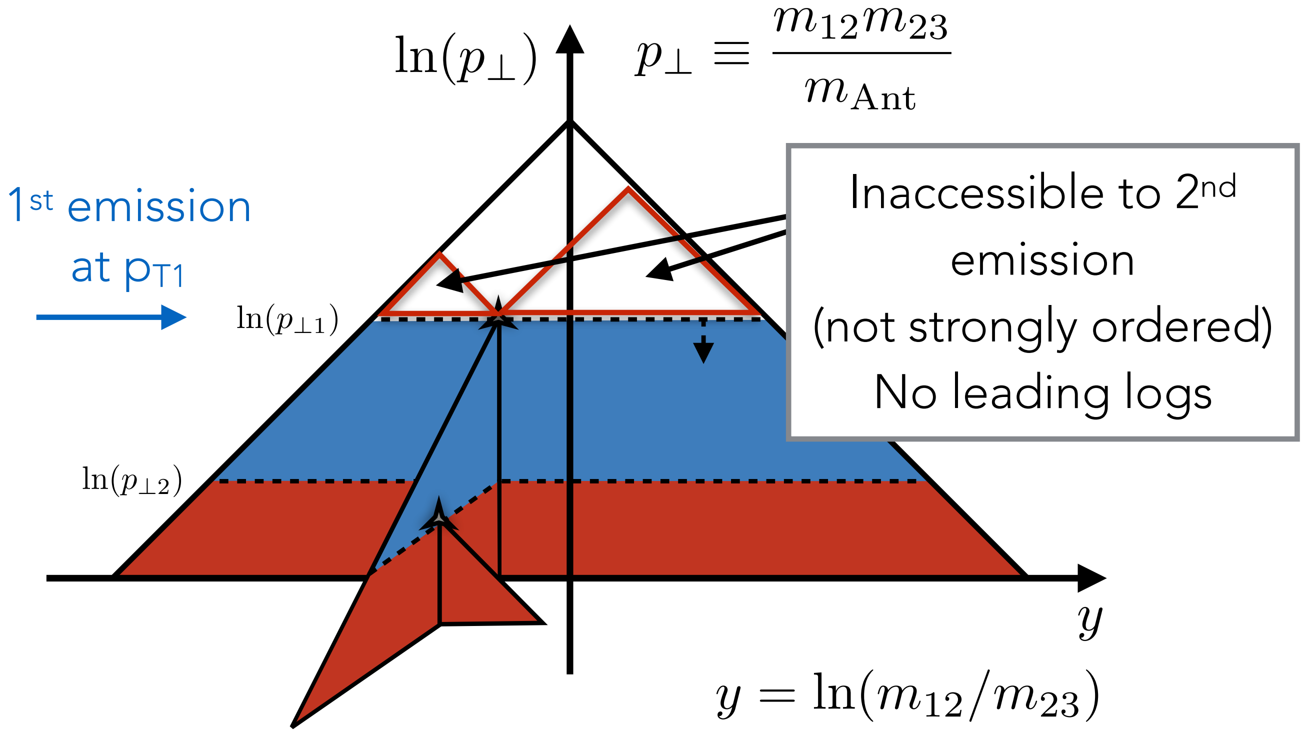
~ deterministic jet algorithms (e.g., Lopez-Villarejo & PS: JHEP 1111 (2011) 150)

Either can technically cover all of the multiple-emission PS;

but ⊗ strong ordering

$\Longrightarrow$ regions with all $f_i = 0$ (no ordered paths)

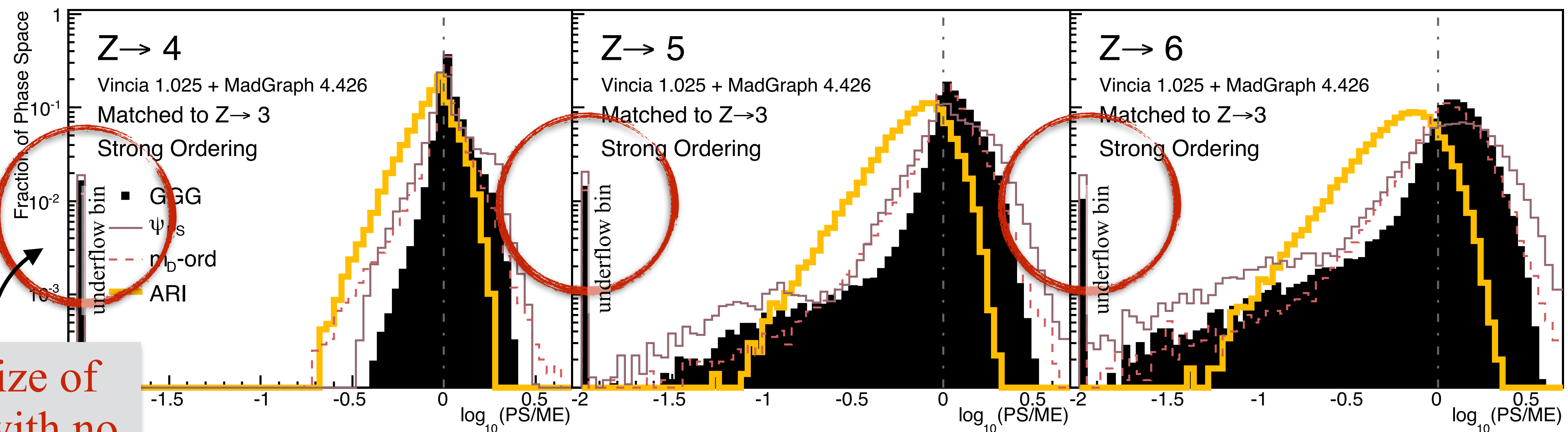
inaccessible to ordered shower based on iteration of n→n+1



How big are these regions? And what logs live there?

Giele, Kosower, PS: PRD84 (2011) 054003

Flat scans of N-parton phase space (RAMBO)



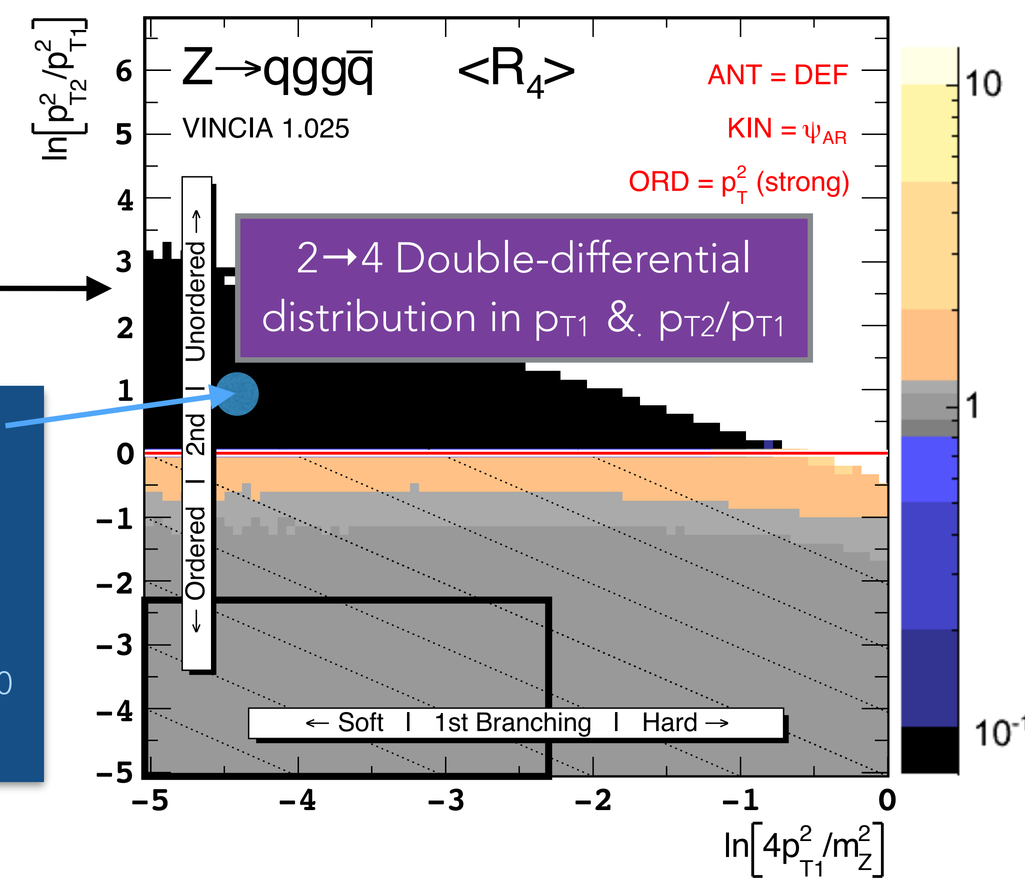
Total size of region with no ordered paths ~ 2% of PS

$$R_N = \log_{10}\left(\frac{\text{Sum(shower-paths)}}{|M_N^{(\text{LO,LC})}|^2}\right)$$

PS = shower expanded to tree level, summed over all ordered paths

ME = LO matrix element (MADGRAPH @ leading colour)

$Q_0 = 91\,\text{GeV}$
$p_{T1} = 5\,\text{GeV}$
$p_{T2} = 8\,\text{GeV}$
Unordered with $p_{T2} << Q_0$
"Double Unresolved"



Getting There: Direct 2→4 Branchings

Li & PS: PLB771 (2017) 59

Redefine the shower resolution scale

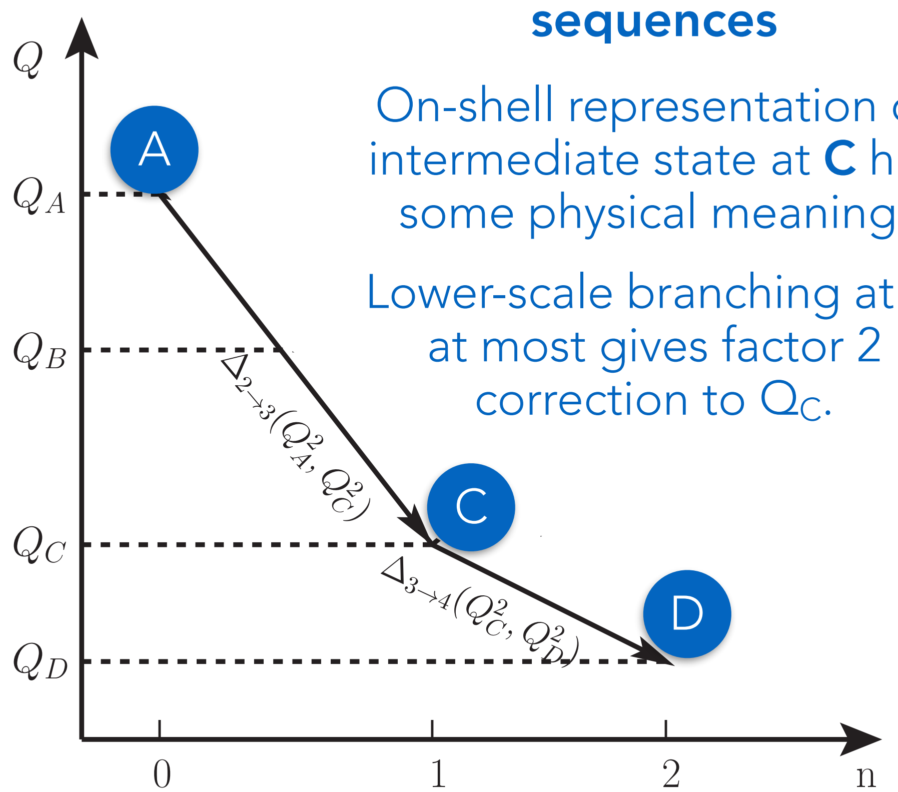
For **unordered** 2→4 paths: scale of **2nd** branching defines resolution

The intermediate on-shell 3-parton state is merely a convenient stepping stone in phase space ⇨ integrate out

Ordered sequences

On-shell representation of intermediate state at **C** has some physical meaning.

Lower-scale branching at **D** at most gives factor 2 correction to Q_C.

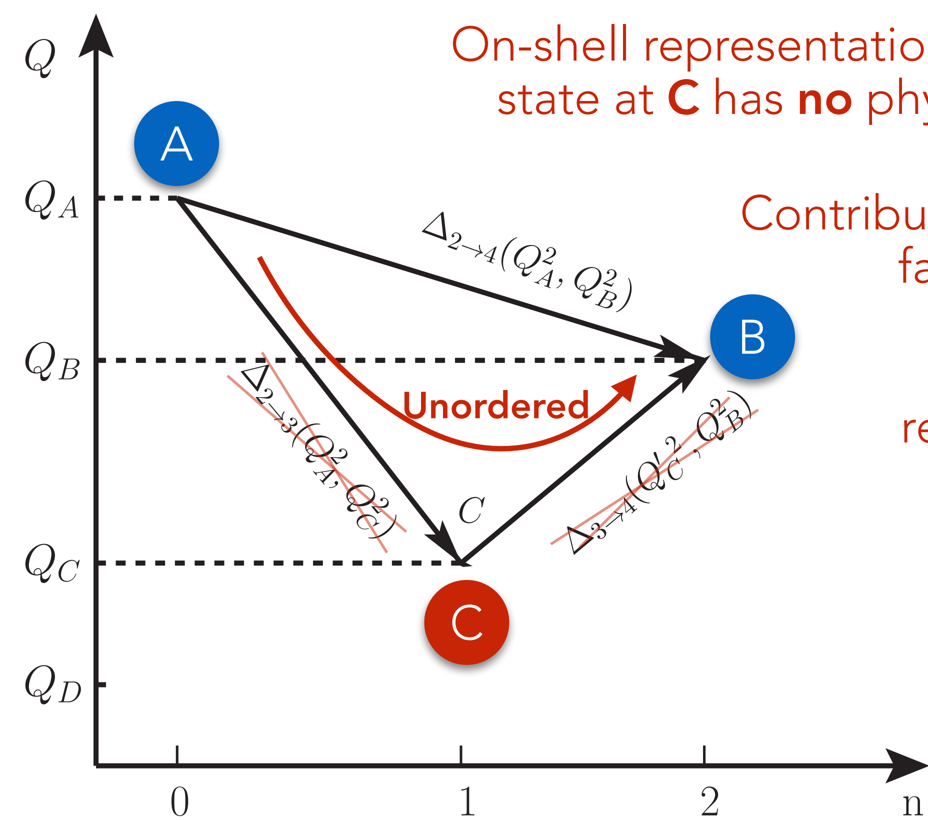


Unordered sequences

On-shell representation of intermediate state at **C** has **no** physical meaning.

Contributing diagrams are far off shell;

Q_B is the only relevant physical scale



Our approach: continue to exploit iterated on-shell $2 \to 3$ factorisations; but in unordered region let Q_B define evolution scale (integrate over Q_C)

Getting There: Direct 2→4 Branchings

Li & PS: PLB771 (2017) 59

Redefine the shower resolution scale

For **unordered** 2→4 paths: scale of **2nd** branching defines resolution

The intermediate on-shell 3-parton state is merely a convenient stepping stone in phase space ⇨ **integrate out**

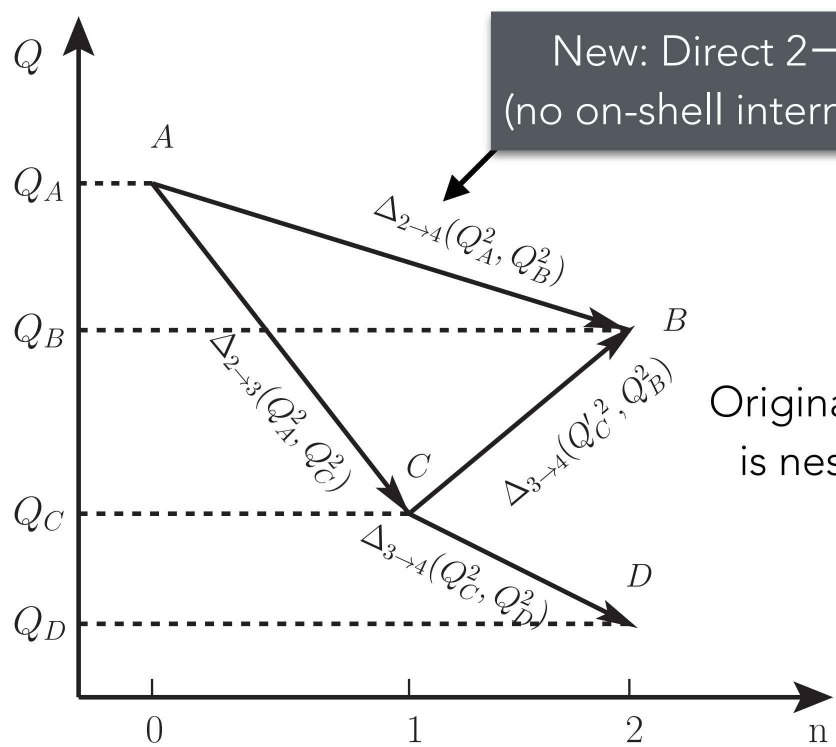


Figure 1: Illustration of scales and Sudakov factors in strongly ordered (ACD), smoothly (un)ordered (ACB), and direct 2 → 4 (AB) branching processes, as a function of the number of emitted partons, n.

Interchange order of integrations
$Q_{2\to3} \leftrightarrow Q_{3\to4}$

Unordered phase space: $Q_4 > Q_3$

$$\int_0^{Q_0^2} dQ_3^2 \int_{Q^2}^{Q_0^2} dQ_4^2\, \Theta(Q_4^2 - Q_3^2)\, f(Q_3^2, Q_4^2) = \int_{Q^2}^{Q_0^2} dQ_4^2 \int_0^{Q_4^2} dQ_3^2\, f(Q_3^2, Q_4^2)\,,$$

Originally, the 3→4 phase space is nested inside the 2→3 one

Now the intermediate (unordered) scale is integrated over for each value of Q_4

for a generic integrand, f, with the result:

$$\Delta_{2\to4}(Q_0^2, Q^2) = \exp\left[-\sum_{s\in a,b} \int_{Q^2}^{Q_0^2} dQ_4^2 \int_0^{Q_4^2} dQ_3^2 \int_{\zeta_{4-}}^{\zeta_{4+}} d\zeta_4 \int_{\zeta_{3-}}^{\zeta_{3+}} d\zeta_3 \frac{|J_3 J_4|}{(16\pi^2)^2 m^2 m_s^2} \int_0^{2\pi} \frac{d\phi_4}{2\pi} R_{2\to4}\, s_3\, s_3' \right], \quad (11)$$

Product of 2→3 functions

Jacobian for dLIPS → $dQ_3 dQ_4 d\zeta_3 d\zeta_4$

2→4 MEC

Note: this is not a very pedagogical exposition; will try to come up with a better one

Combining 2→4 with Iterated 2→3

Split the 2→4 phase space into non-overlapping sectors

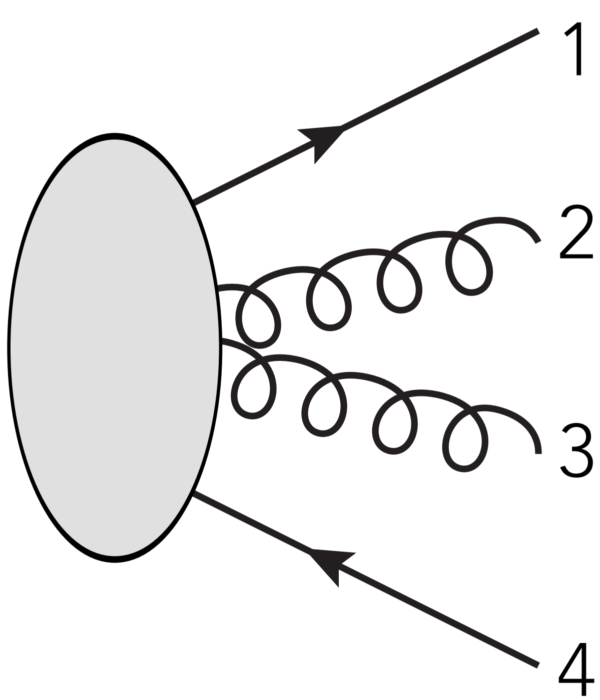
Pure 2→4 sector: inaccessible to iterated 2→3 (no ordered paths)

⇨ add new "direct" 2→4 branchings without risk of double-counting

Rest of phase space (accessible to at least one ordered 2→3 path)

Unitarity (Sudakov exponentials and virtual corrections): want to sum inclusively over the "least resolved" degree of freedom

Classify according to what a jet algorithm (with shower evolution parameter as clustering measure) would do. E.g., for a (colour-connected) double-emission:



$$p_{\perp 2} \equiv \frac{m_{12}m_{23}}{m_{123}}$$

$$p_{\perp 3} \equiv \frac{m_{23}m_{34}}{m_{234}}$$

A jet clustering algorithm (ARCLUS) would grab the smallest of these p_T values, and cluster

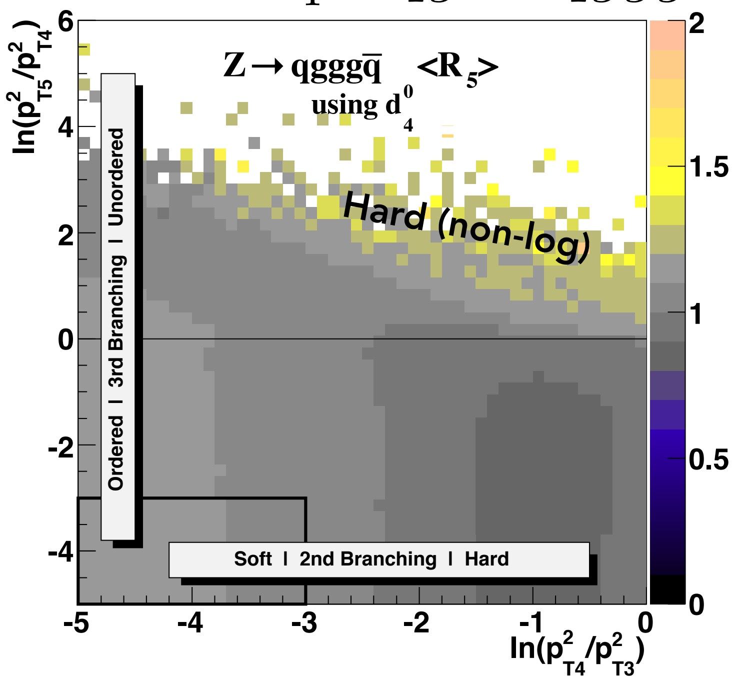
If the resulting path is **ordered**: populate by iterated 2→3 (with 2→4 MEC factors)
If **unordered**, keep clustering; direct 2→n

Clustering terminates when we reach a $Q_n > \min(p_{T2}, p_{T3}, \ldots)$
⇨ defines point as 2→2+m (so far we only do 2→3 and 2→4!)

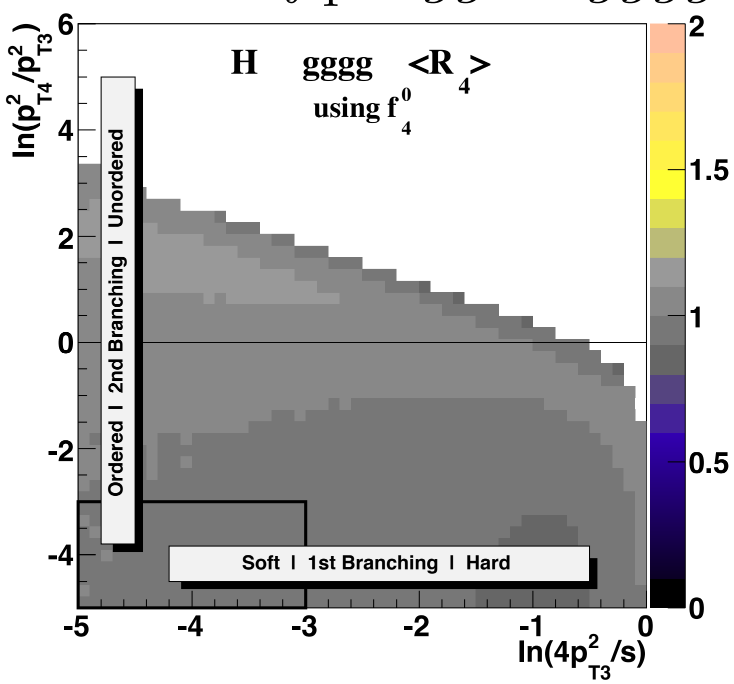
Phase-Space Distributions

Li & PS: PLB771 (2017) 59

Test of d_4^0 : $qg \to qggg$



Test of f_4^0 : $gg \to gggg$



Actual shower runs:

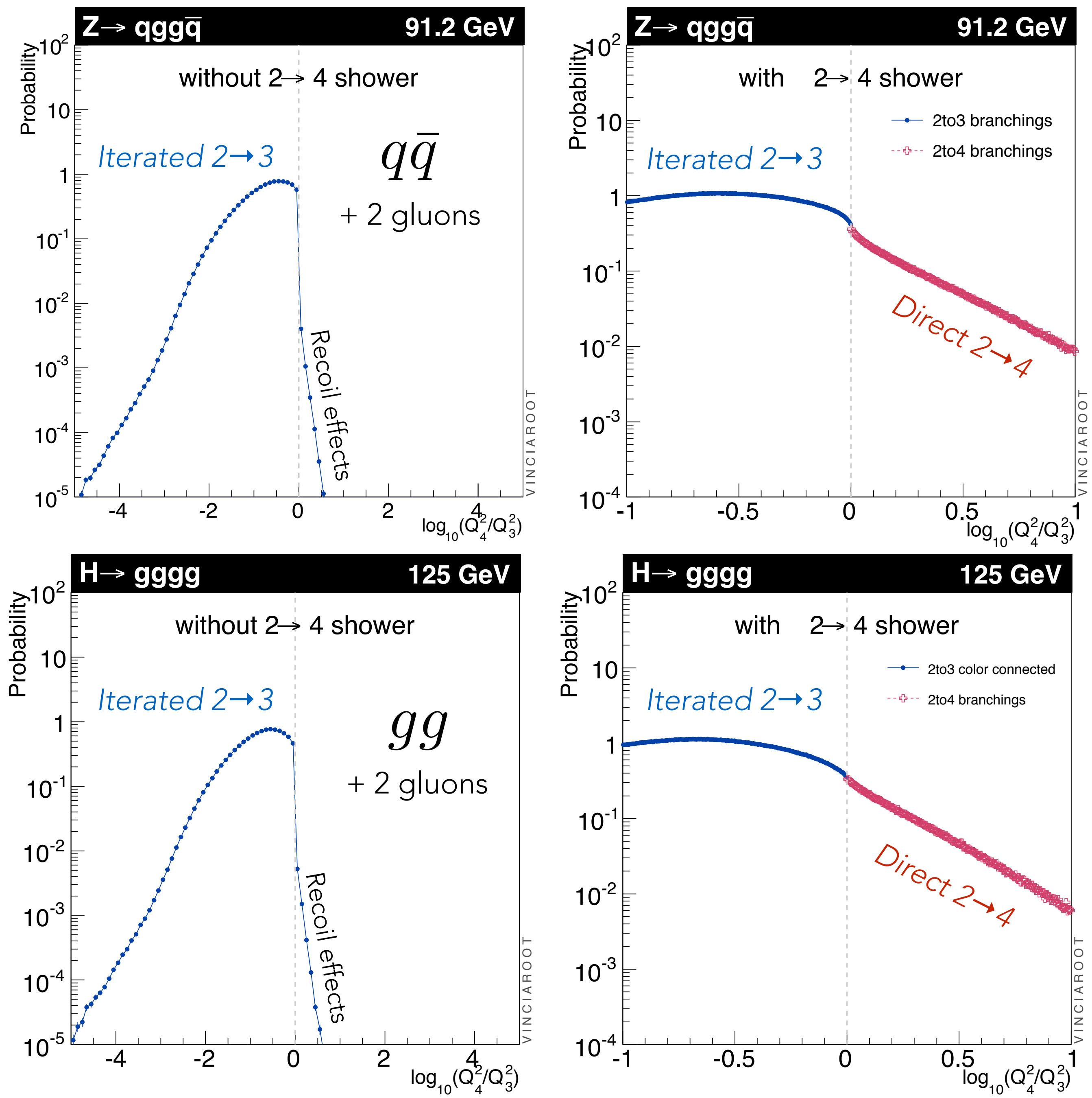


Figure 3: Top left: the ratio of sequential clustering scales Q_4/Q_3 for a strongly ordered $2 \to 3$ shower, for $Z \to qgg\bar{q}$ (on log-log axes). Top right: closeup of the region around $Q_4/Q_3 \sim 1$, with 2→4 branchings included. Bottom row: the same for $H \to gggg$. .

Details of trial functions etc, see Li & PZS: PLB771 (2017) 59

Second-Order Evolution Equation (Global Shower)

Li & PS: PLB771 (2017) 59

Putting 2→3 and 2→4 together ⇨ evolution equation for dipole-antenna at O(α_s^2):

~ POWHEG inside exponent

(Hoeche, Krauss, Prestel ~ MC@NLO inside exponent)

$$\frac{d\Delta(Q_0^2,Q^2)}{dQ^2} = \int d\Phi_{\rm ant} \Big[\delta(Q^2 - Q^2(\Phi_3))\, a_3^0$$

- → 0

Iterated 2→3 with (finite) one-loop correction →

$$\times \left(1 + \frac{a_3^1}{a_3^0} + \sum_{s\in a,b} \int_{\rm ord} d\Phi^s_{\rm ant}\, R_{2\to4}\, s_3' \right) \Delta(Q_0^2,Q^2)$$

(2→)3→4 antenna function

(2→)3→4 MEC

Direct 2→4 (as sum over "a" and "b" subpaths) →

$$+ \sum_{s\in a,b} \int_{\rm unord} d\Phi^s_{\rm ant} \delta(Q^2 - Q^2(\Phi_4)) R_{2\to4}\, s_3\, s_3' \Delta(Q_0^2,Q^2) \Big]$$

2→4 as explicit product x MEC

Only generates double-unresolved singularities, not single-unresolved

Note: the equation is formally identical to:

$$\frac{d}{dQ^2}\Delta(Q_0^2,Q^2) =$$

$$\int \frac{d\Phi_3}{d\Phi_2}\, \delta(Q^2 - Q^2(\Phi_3)) \left(a_3^0 + a_3^1\right) \Delta(Q_0^2,Q^2)$$

$$+ \int \frac{d\Phi_4}{d\Phi_2}\, \delta(Q^2 - Q^2(\Phi_4))\, a_4^0\, \Delta(Q_0^2,Q^2)\,, \quad (3)$$

But on this form, the pole cancellation happens *between* the two integrals

Differential 2nd-Order Corrections (Global Shower)

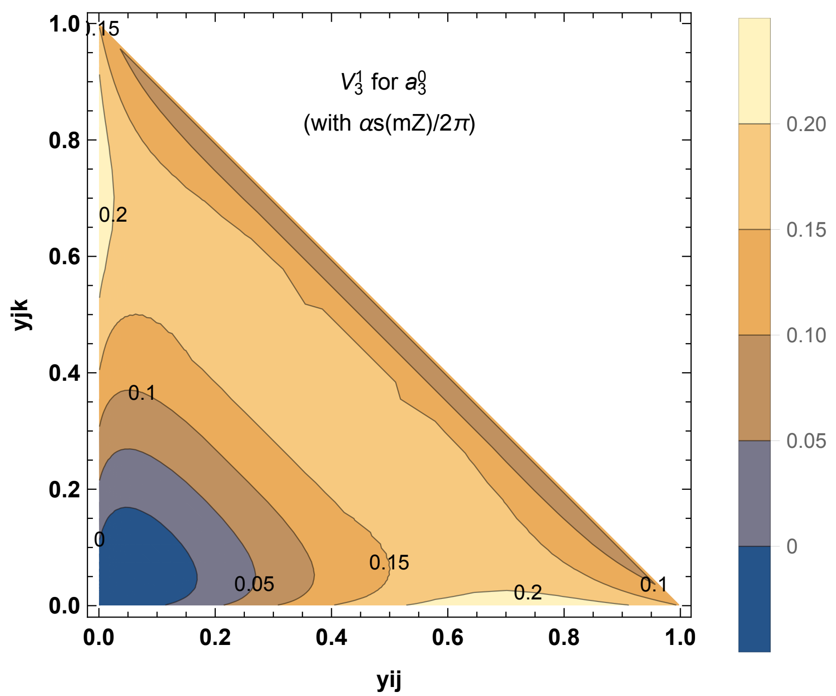
(with direct 2→4 instead of smooth ordering)

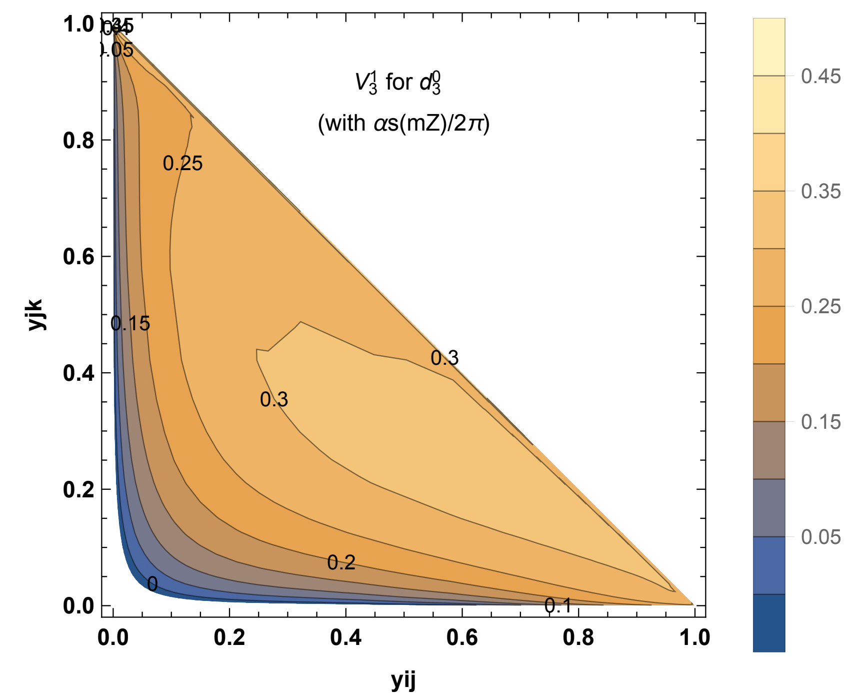
Work in progress…
Plots by Hai Tao Li

Differential 2nd-Order Correction for 2→3 kernels

$Q\bar{Q} \to QG\bar{Q}$
(new treatment)

From Z decay



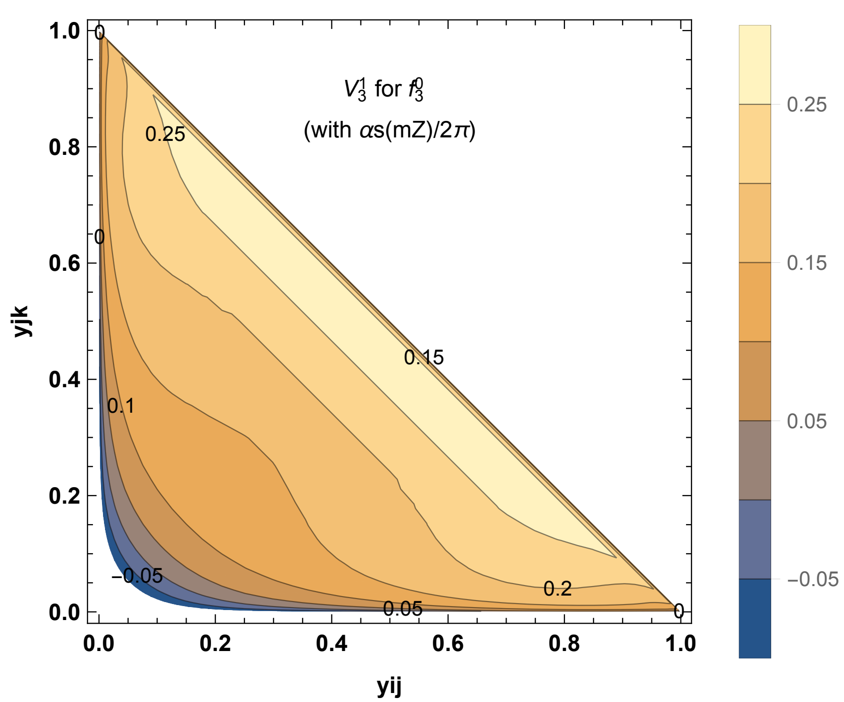


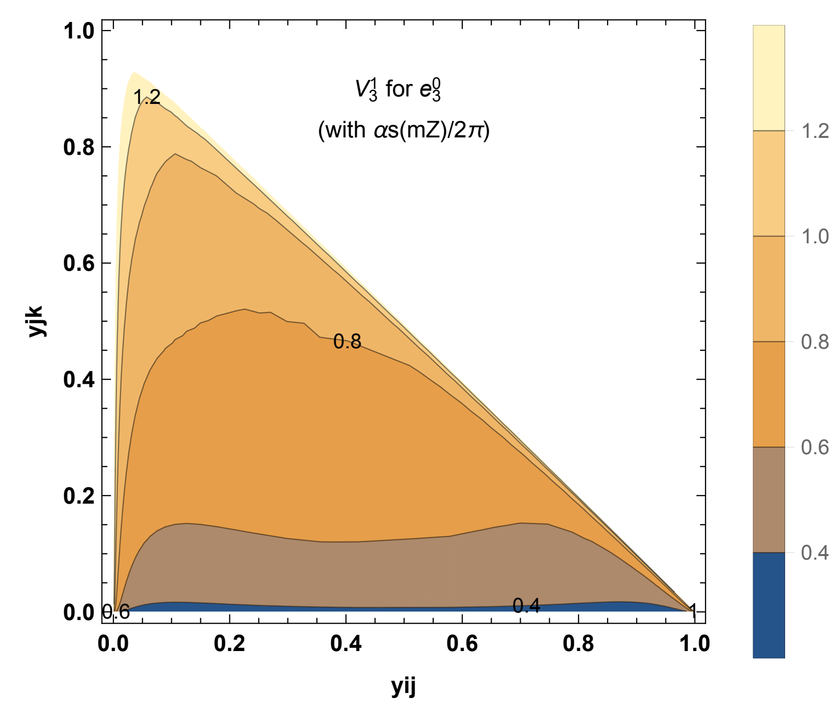
$QG \to QGG$

From χ decay

$GG \to GGG$

From H decay





$QG \to Q\bar{Q}'Q'$

Note: large corrections for g→qq

(leading pole only $1/y_{jk}$)

Yesterday, Christian Gutschow asked for a magic wand that could speed up MC calculations by a factor 2.

I **don't** have a factor-2 wand :(

But I **do** have a factorial-complexity to constant-complexity wand, for merging!

This requires a different shower paradigm

Which is anyway the one we are now pursuing for our final go at constructing the 2nd-order shower

➠ Pythia 8.304

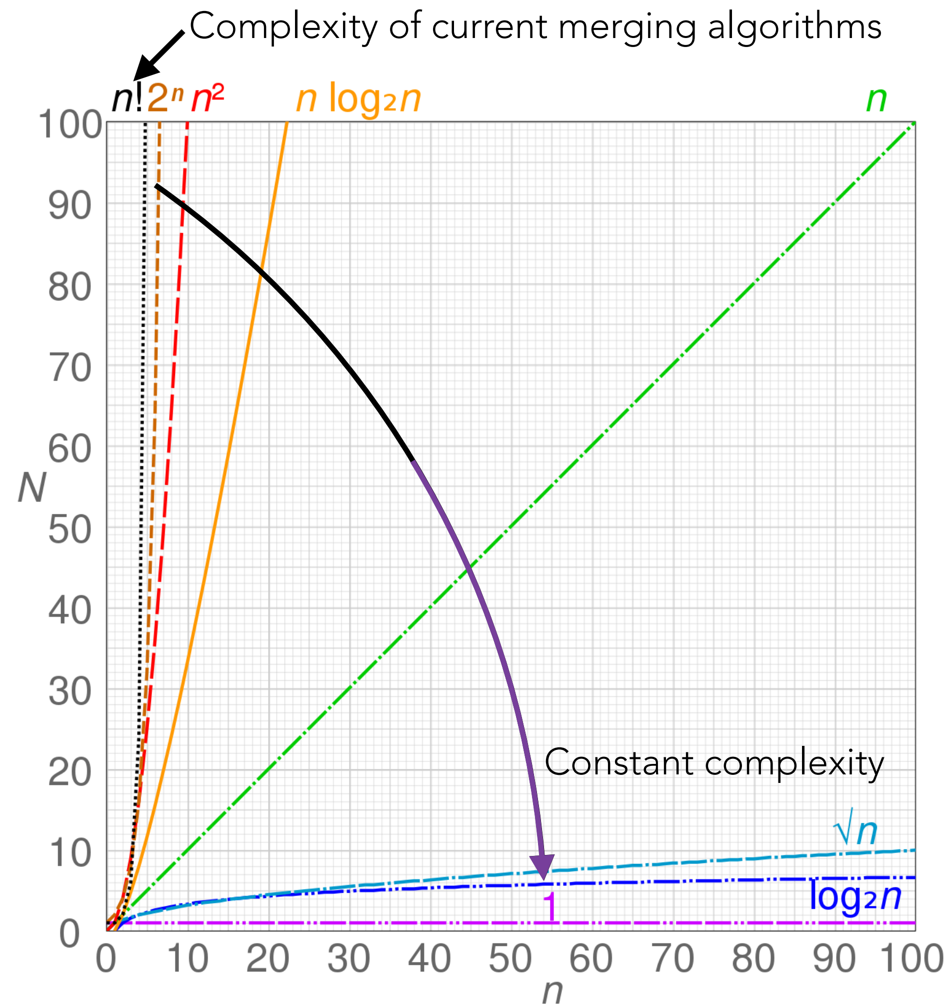


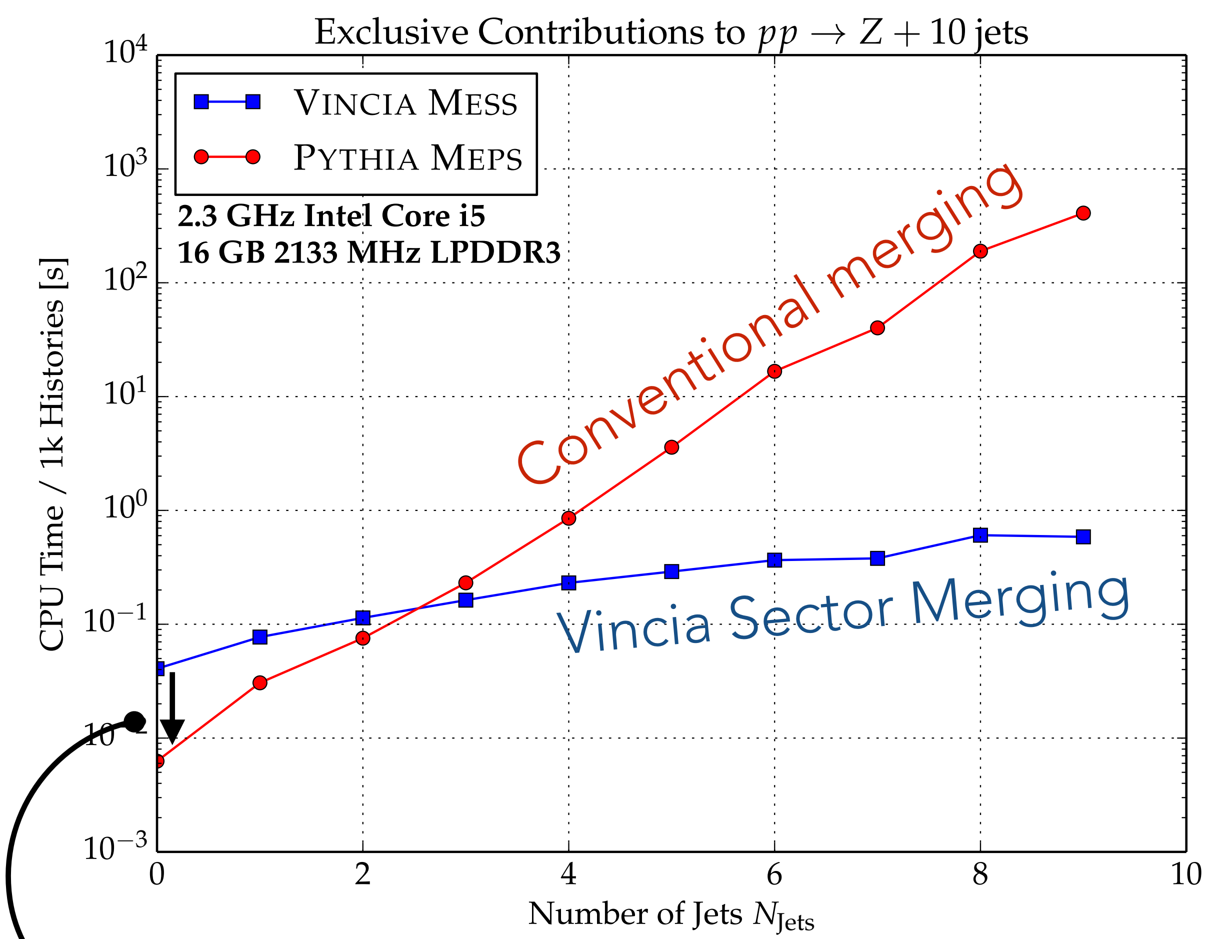
Figure 1: *Number of operations, N vs number of input items, n for algorithms of common complexities, assuming a constant of 1. Polynomial (n^2) or better scaling is usually considered efficient for complex problems, while exponential (2^n) or factorial ($n!$) scaling are infamous for being highly resource demanding. Plot from Ref. [2].*

Vincia with Sector Merging: CPU and Memory Usage

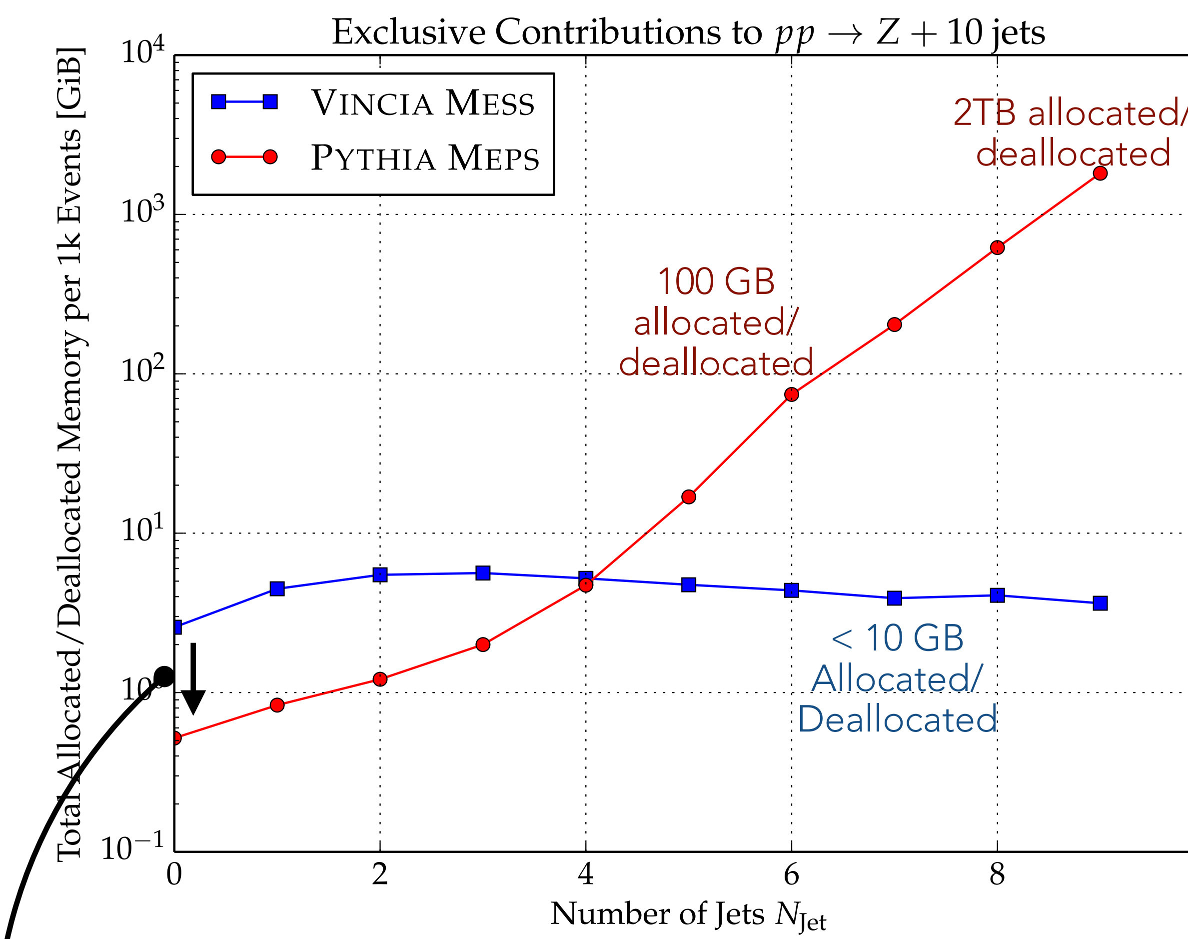
Test case: $pp \to Z$ + merging with up to 9 jets + valgrind

Based on HDF5 ME samples [Höche, Prestel, Schulz: *PRD* 100 (2019) 1, 014024] with 20 GeV merging scale

CPU Time / 1k Events



GB of Allocated/Deallocated Memory



(Some work to do to optimise the basic shower; so far we focused on the scaling to high n)

What are Sector Showers?

Idea first suggested to me by D. Kosower

- Kosower, *PRD* 57 (1998) 5410; *PRD* 71 (2005) 045016
- But also, e.g., Larkoski & Peskin, *PRD* 81 (2010) 054010; *PRD* 84 (2011) 034034

In conventional ("global") showers, each branching kernel can populate the full $\mathrm{d}\Phi_{n+1}/\mathrm{d}\Phi_n$, subject only to the condition of ordering in the evolution variable.

- As highlighted earlier, this generates a multiple covering of phase space.
- The overlapping PS regions are not a problem if the shower kernels are defined such that their **sum** reproduces the full singularity structure of the (squared) matrix elements.
- This is how all modern dipole and (global) antenna showers work (to my knowledge).
- This is also what produces the proliferation of histories.

In a **sector shower**, only **one kernel** is allowed to populate each $\mathrm{d}\Phi_{n+1}$ point.

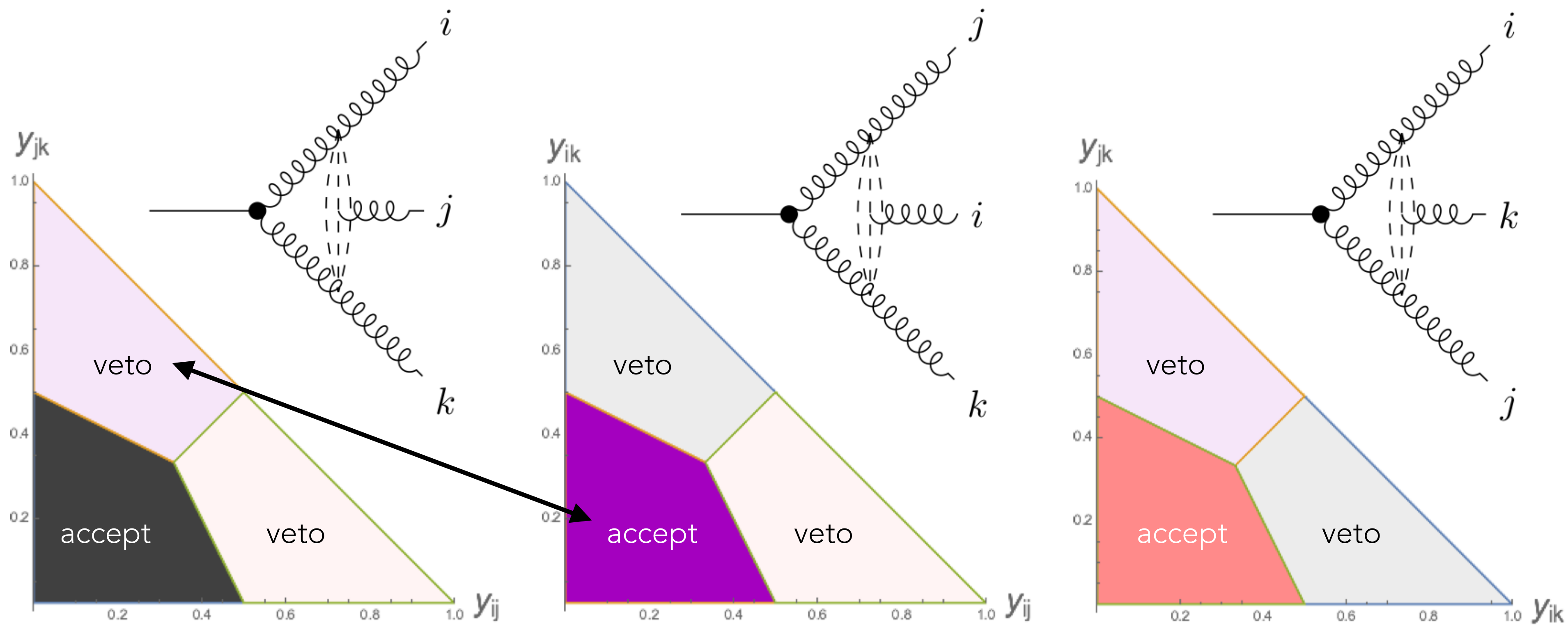
- Each kernel must therefore contain the **full** singularity structure of its sector (generally corresponding to a sum over global functions that, at least, includes any singular ones).
- First implementation, arXiv:1109.3608, later abandoned (for NLO corrections and the move to pp), now resurrected for pp, **arXiv:2003.00702**, with full mass and helicity dependence.

Sector Showers

Brooks, Preuss, Skands, **arXiv:2003.00702**

Consider $H \to gg$ + shower

At $g_i g_j g_k$ level, there are three possible clusterings



Sector shower trial emission (of gluon j) is **vetoed** if $p_{\perp j}$ is **not** the smallest scale in the event **after** the branching. (Recoils not allowed to make any other $p_\perp$ smaller than $p_{\perp j}$.)

➨ Scale of $g_i g_j g_k$ is **uniquely defined** (history independent) $\equiv \min(p_{\perp j}, p_{\perp i}, p_{\perp k})$

Creates a unique (bijective) shower history that corresponds exactly to a jet algorithm (anyone remember ARCLUS?) $\Longrightarrow$ **one term per PS point at any *n*** (constant complexity)

Outlook Towards 2nd-Order Sector Showers

Full-fledged sector shower (including II, IF, RF, and FF antennae with mass effects)

Ready for upcoming Pythia 8.303 or 8.304.

Will **replace** the existing Vincia global antenna-shower model in Pythia 8.

Brooks, Preuss, Skands, **arXiv:2003.00702**

Full-fledged implementation of sector merging algorithm in final validation stages.

Expect public release soon after shower itself (before end of 2020).

2nd order corrections; focus so far on *what we can do:*

Baseline check: all (LC) single- and double-unresolved limits explicitly reproduced, apart from some confusion remaining for the global case in the triple-collinear limit. (Should be solved by the move to sector showers.)

No work has so far gone into further measuring or testing its log accuracy.

Adapting direct $2 \to 4$ branchings to sector context relatively straightforward (?)

Interested in the PanScales work on recoils and ordering variables.

Current work focuses on the sector integrals for the 2nd-order virtual corrections

A rollercoaster of eureka moments and dead ends.

Extra Slides

The Solution that worked at LO: Smooth Ordering

Wanted starting point for (LO) matrix-element corrections over **all** of phase space (good approx → small corrections)

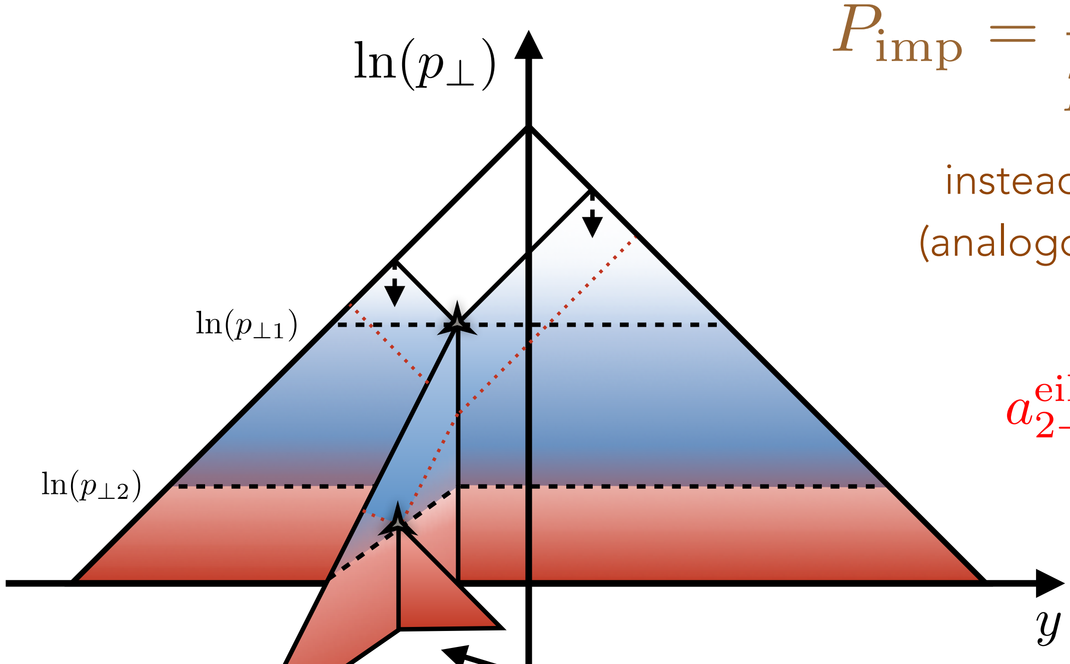
Allow newly created antennae to evolve over their full phase spaces, with suppressed (beyond-LL) probability: **smooth ordering**

Giele, Kosower, PZS: PRD84 (2011) 054003

$$P_{\mathrm{imp}} = \frac{p_{\perp n-1}^2}{p_{\perp n-1}^2 + p_{\perp n}^2}$$

$\to 1$ for $p_{\perp n} \ll p_{\perp,n-1}$
$\to 1/2$ for $p_{\perp n} \sim p_{\perp,n-1}$
$\to 0$ for $p_{\perp n} \gg p_{\perp,n-1}$

instead of strong ordering
(analogous to POWHEG hfact)



Figures from Fischer, Prestel, Ritzmann, PZS: EPJC76 (2016) 11, 589

(b) Smooth Ordering

$$a_{2\to 4}^{\mathrm{eik}} \sim \frac{1}{p_{\perp n-1}^2} P_{\mathrm{imp}} \frac{1}{p_{\perp n}^2} \propto \begin{cases} 1/p_{\perp n}^2 & \text{ordered} \\ 1/p_{\perp n}^4 & \text{unordered} \end{cases}$$

Leading Logs unchanged

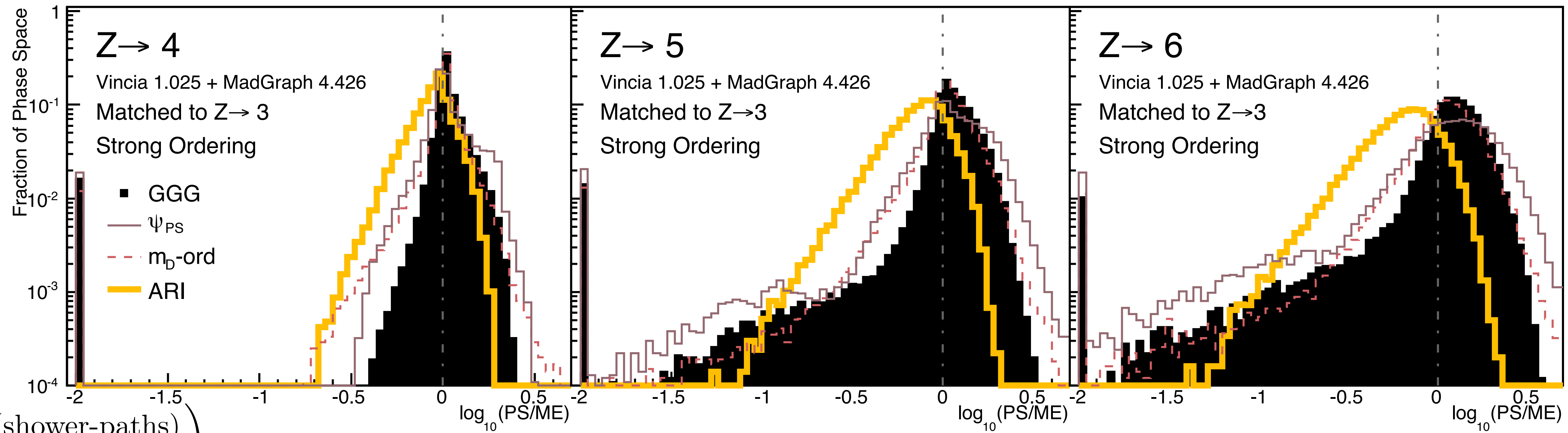
Fischer, Prestel, Ritzmann, PZS: EPJC76 (2016) 11, 589

$$-\ln\Delta \propto \int_{p_\perp^2}^{m^2} \frac{1}{1+\frac{q_\perp^2}{Q_\perp^2}} \frac{dq_\perp^2}{q_\perp^2} \ln\left[\frac{m^2}{q_\perp^2}\right] \sim \left(\frac{1}{2}\ln^2\left[\frac{Q_\perp^2}{p_\perp^2}\right] + \ln\left[\frac{Q_\perp^2}{p_\perp^2}\right]\ln\left[\frac{m^2}{Q_\perp^2}\right]\right)$$

Note: this conclusion appears to differ from that of Bellm et al., Eur.Phys.J. C76 (2016) no.1
My interpretation is that, in the context of a partonic angular ordering, they neglect the additional rapidity range from the extra origami folds

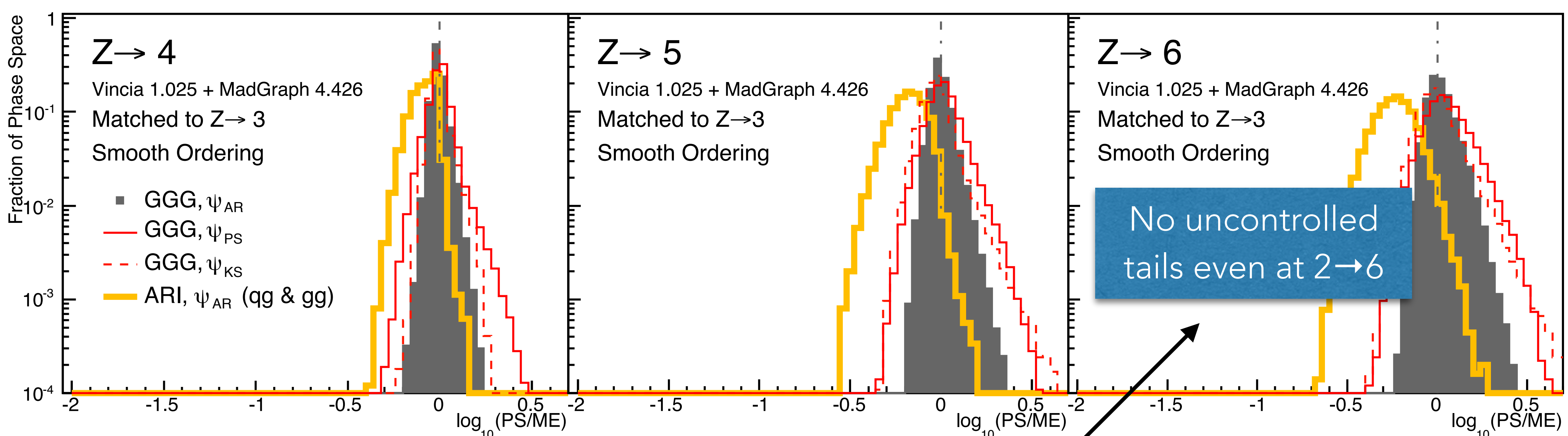
Smooth ordering: An excellent approximation (at tree level)

Strong



$$R_N = \log_{10}\left(\frac{\text{Sum(shower-paths)}}{|M_N^{(\text{LO,LC})}|^2}\right)$$

Smooth



No uncontrolled tails even at 2→6

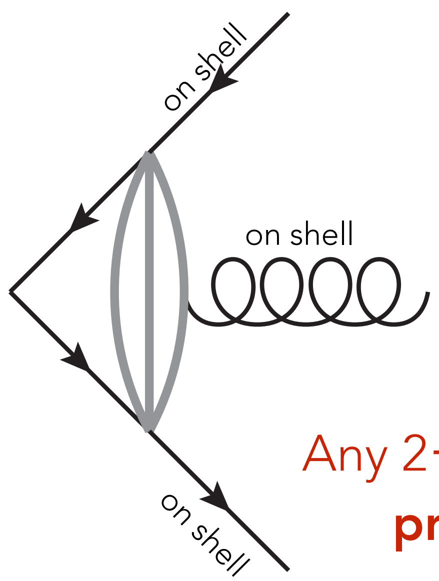
Even after three sequential shower emissions, the smooth shower approximation is still a very close approximation to the matrix element **over all of phase space**

(Why it works?)

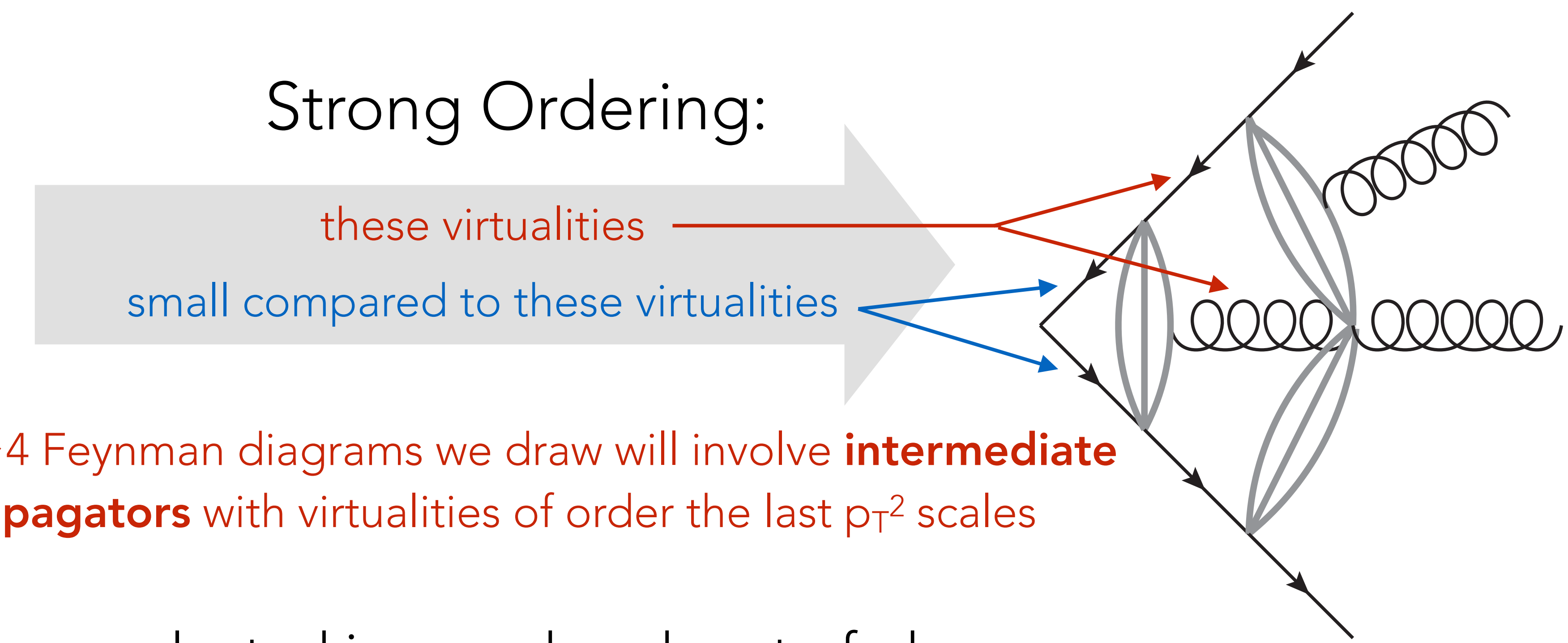
The antenna factorisations are **on shell**

n on-shell partons → **n+1** on-shell partons

In the first 2→3 branching, final-leg virtualities assumed ~ 0



Strong Ordering:



Any 2→4 Feynman diagrams we draw will involve **intermediate propagators** with virtualities of order the last p_T^2 scales

Cannot be neglected in unordered part of phase space

Interpretation: off-shell effect

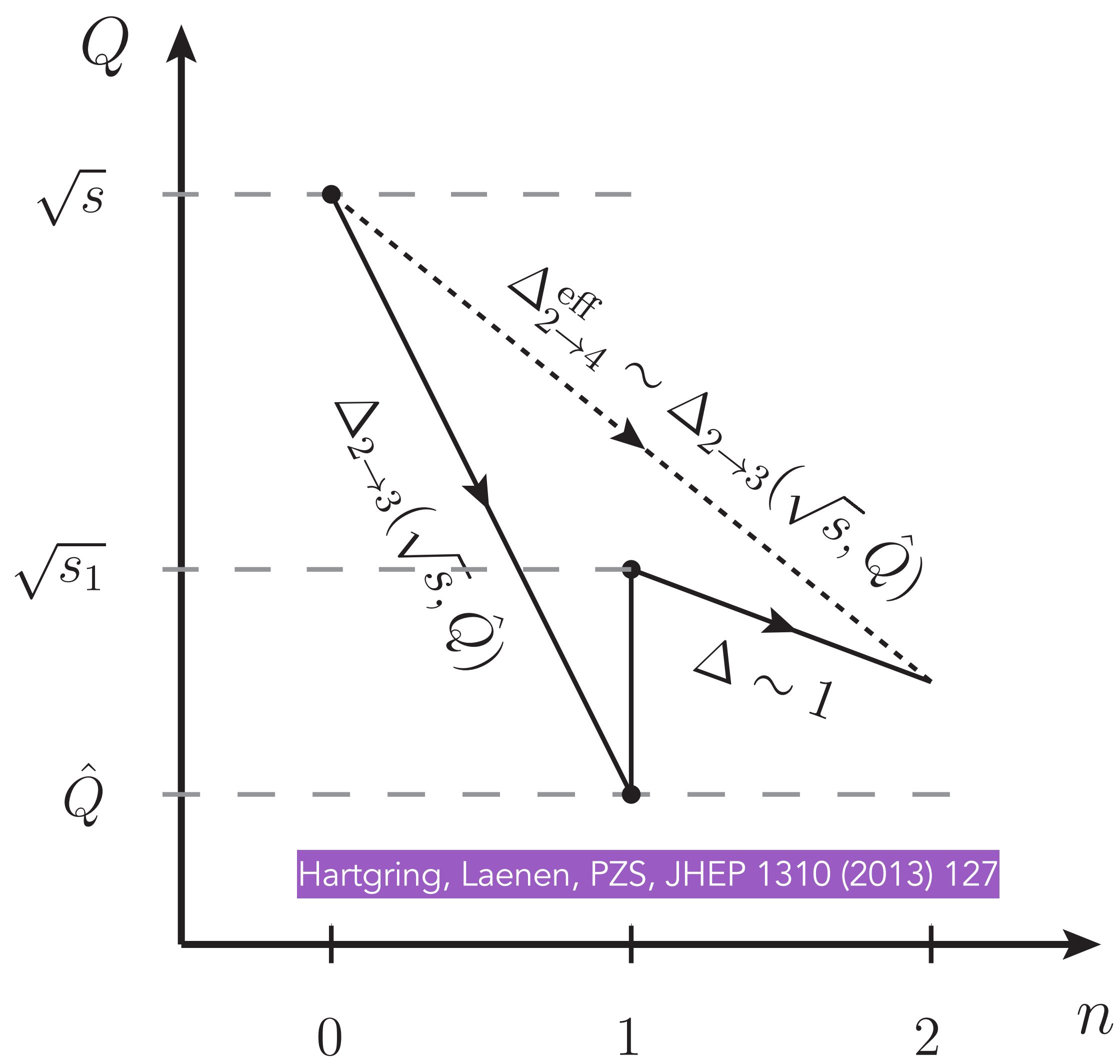
$$\frac{1}{2p_i \cdot p_j} \to \frac{P_{\mathrm{imp}}(n \to n+1)}{2p_i \cdot p_j} = \frac{1}{2p_i \cdot p_j + \mathcal{O}(p^2_{\perp n+1})}$$

Good agreement with ME → good starting point for 2→4

The problem with Smooth Ordering

Smooth ordering: nice tree-level expansions (small ME corrections) ⇨ good 2→4 starting point

But we worried the Sudakov factors were "wrong" ⇨ not good starting point for 2→3 virtual corrections? Not good exponentiation?



For unordered branchings (e.g., double-unresolved)

effective 2→4 Sudakov factor effectively → LL Sudakov for intermediate (unphysical) 3-parton point

2→4 Trial Generation

$$\frac{1}{(16\pi^2)^2} a_{\rm trial}^{2\to 4} = \frac{2}{(16\pi^2)^2} a_{\rm trial}^{2\to 3}(Q_3^2) P_{\rm imp} a_{\rm trial}^{2\to 3}(Q_4^2)$$
$$= C \left(\frac{\alpha_s}{4\pi}\right)^2 \frac{128}{(Q_3^2 + Q_4^2) Q_4^2} \,. \qquad (15)$$

In particular, the trial function for sector A (B) is independent of momentum p_6 (p_3) which makes it easy to translate the $2 \to 4$ phase spaces defined in eq. (6) to shower variables. Technically, we generate these phase spaces by oversampling, vetoing configurations which do not fall in the appropriate sector.

Accept ratio: $P_{\rm trial}^{2\to 4} = \frac{\alpha_s^2}{\hat{\alpha}_s^2} \frac{a_4}{a_{\rm trial}^{2\to 4}}$

Solution for constant trial α_s

$$\mathcal{A}_{2\to 4}^{\rm trial}(Q_0^2, Q^2) = C\, I_\zeta \frac{\ln(2)\hat{\alpha}_s^2}{8\pi^2} \ln\frac{Q_0^2}{Q^2} \ln\frac{m^4}{Q_0^2 Q^2}$$

$$\Rightarrow \quad Q^2 = m^2 \exp\left(-\sqrt{\ln^2(Q_0^2/m^2) + 2 f_R/\hat{\alpha}_s^2}\right)$$

where $f_R = -4\pi^2 \ln R / (\ln(2) C I_\zeta)$. (Same $I_{\rm zeta}$ as in GKS)

Solution for first-order running α_s (also used as overestimate for 2-loop running):

$$Q^2 = \frac{4\Lambda^2}{k_\mu^2} \left(\frac{k_\mu^2 m^2}{4\Lambda^2}\right)^{-1/W_{-1}(-y)} . \qquad (20)$$

Lambert W

where

$$y = \frac{\ln k_\mu^2 m^2/4\Lambda^2}{\ln k_\mu^2 Q_0^2/4\Lambda^2} \exp\left[-f_R b_0^2 - \frac{\ln k_\mu^2 m^2/4\Lambda^2}{\ln k_\mu^2 Q_0^2/4\Lambda^2}\right] ,$$

→ Differential "K-factor" for 2→3 branchings

Hartgring, Laenen, PZS, JHEP 1310 (2013) 127

Solve for V_3

$$\left|M_{Z\to q\bar{q}}\right|^2 A_3^0(Q^2)\left(1+V_3^{q\bar{q}}\right)\Delta_{2\to3}(Q_0^2,Q^2)\Delta_{3\to4}(Q^2,0) \overset{\mathcal{O}(\alpha_s^2)}{\rightarrow} |M_3^0|^2\left(1+\frac{2\mathrm{Re}\left[M_3^0M_3^{1*}\right]}{|M_3^0|^2}\right)$$

Poles ← Cancel if Q is IR safe → Poles

Non-divergent NLO correction
→ **positive-definite** NLO antenna

Partial cancellations
Use to define LL evolution so as to have no (resummable) logs left

Double Logs
Single Logs
(β-dependent logs)

+ transcendentality-0

Can do some Sudakov integrals analytically

But not all → split into analytic and numerical parts

Use that smooth-ordering already gave a good approximation, which can be integrated fairly easily

E.g.: $$\Delta_{3\to4} = 1 - \sum_{a\in1,2}\int_{\mathrm{ord}} \mathrm{d}\Phi_{\mathrm{ant}}\, a_{3\to4}\frac{a_{2\to4}}{a_{2\to3}a_{3\to4}+a'_{2\to3}a'_{3\to4}} + \mathcal{O}(\alpha_s^2)$$

ordering boundary

complicated 2→4 ME-correction factor

$$\pm\sum_{a\in1,2}\int \mathrm{d}\Phi_{\mathrm{ant}}\, a_{3\to4}P_{\mathrm{imp}}$$

Doable analytically;
contains all single-unresolved poles

Difference done numerically;
(slow but can be parametrised in terms of two invariants)

Sector Showers

Scale definition

Global showers are not truly Markovian (history independent), in the sense that a generic *n*-parton configuration could have been produced by many different histories (all contributing to one and the same configuration).

Not a problem from the pure (LL) shower point of view. But each history has its own (set of) intermediate (and final) scales. This makes the analytical calculation of, and matching to, deterministic NLO jet rates delicate and difficult on the shower side, and casts doubt on the iteration.

Sector showers, on the other hand, have a single unique history, with a single clearly defined set of scales. Simplifies matching conditions (at the price of harder integrals).

Natural sectorisation in $2 \to 4$

When separating the $2 \to 3$ and $2 \to 4$ phase spaces, we split the $2 \to 4$ phase space into two sectors. Part of the iterated $2 \to 3$ phase-space was included in the $2 \to 4$ sectors.

Awkward to keep global structure for the remaining iterated $2 \to 3$ part.

Scaling of Histories with Multiplicity: Magic Wand for Merging

For merging applications, the factorial growth in the number of histories can be a computational bottleneck. This would be obviated in a sector shower approach.