

Particle Physics (Phenomenology)

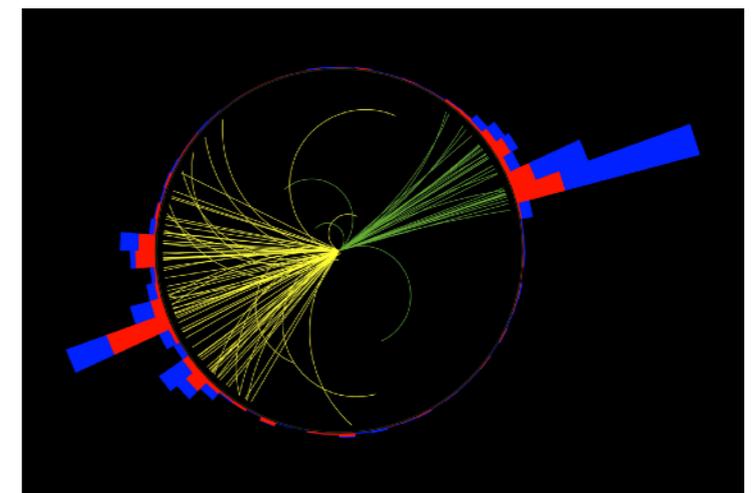
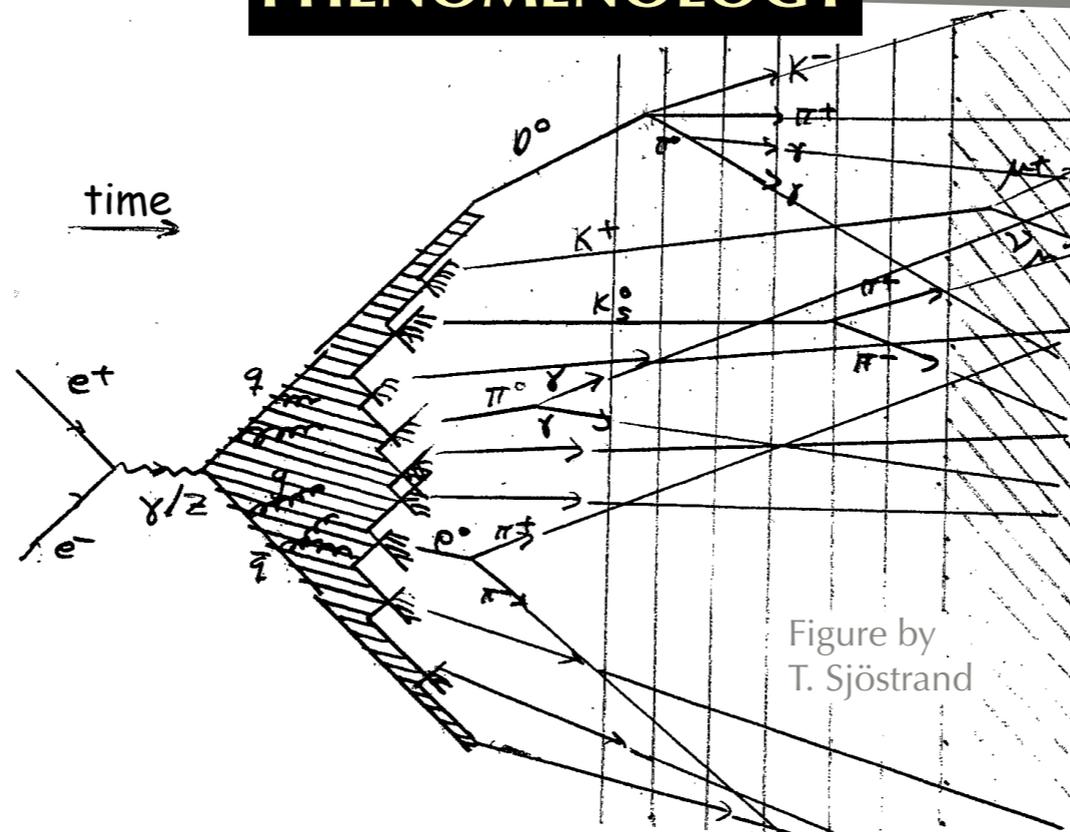
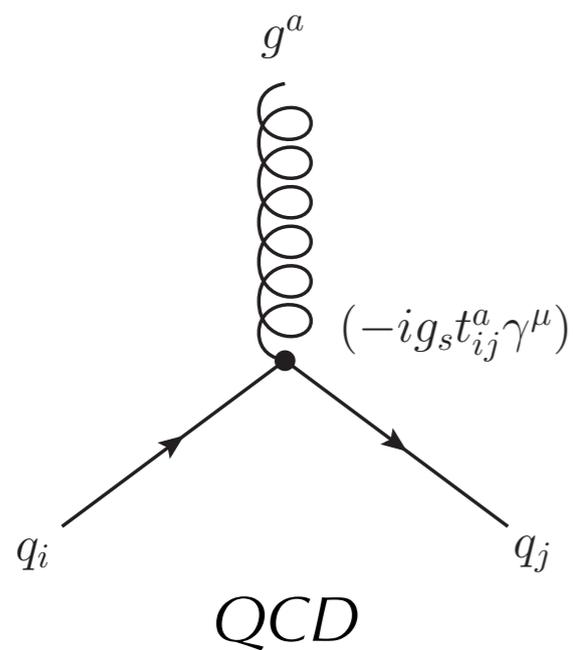
Lecture
1/2

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THEORY

PHENOMENOLOGY

EXPERIMENT



"Jets"

INTERPRETATION

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Spring School on Particle Physics & Cosmology



1) Units in Particle Physics

The main particle-physics units of energy is eV (& MeV, GeV)

1 electron-Volt = kinetic energy obtained by a unit-charge particle (eg an electron or proton) accelerated by potential difference of 1 Volt

$$1 \text{ eV} = Q_e \cdot 1 \text{ V} = 1.602176565(35) \times 10^{-19} \text{ C} \cdot 1 \text{ J/C} = 1.6 \times 10^{-19} \text{ J}$$

(So for accelerators, the beam energy in eV is a measure of the corresponding electrostatic potential difference)

Planned linear accelerators (ILC, CLIC) could reach $E_{\text{CM}} \sim 1,000 \text{ GeV} = 1 \text{ TeV}$

The highest-energy (circular) accelerator LHC $\sim 6500 \text{ GeV/beam}$.

Using $E=mc^2$ we typically express all masses in units of eV/c²

An example

$$m_e = 9.1 \times 10^{-31} \text{ kg} \quad \begin{array}{l} \text{J} = \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2} \\ \text{=} \end{array} \quad 9.1 \times 10^{-31} \frac{\text{s}^2 \text{ J}}{\text{m}^2}$$
$$\text{=} \frac{\text{eV}}{1.6 \times 10^{-19}} \quad 5.7 \times 10^{-12} \text{ eV} \frac{\text{s}^2}{\text{m}^2}$$
$$\text{=} \frac{c = 3 \times 10^8 \text{ m/s}}{\text{=} \quad 511 \times 10^3 \text{ eV/c}^2$$

$$m_e = 0.511 \text{ MeV/c}^2$$
$$m_\mu = 106 \text{ MeV/c}^2$$
$$m_\tau = 1780 \text{ MeV/c}^2$$

(sometimes we forget to say the 1/c²; it is implied by the quantity being mass)

In fact, we use MeV and GeV for *everything!*

Define a set of units in which $\hbar = c = 1$

Action [Energy*Time] : dimensionless ($\hbar = 1$)

All actions are measured in units of \hbar

Velocity [Length/Time] : dimensionless ($c = 1$)

All velocities are measured in units of c (i.e., $\beta = v/c$)

Energy : dimension 1 [E] = eV, MeV, GeV, ...

Mass : dimension 1 ($E=mc^2$)

E.g., $m_e = 0.511 \text{ MeV}/c^2$

Time : dimension -1 ($[\hbar]=[E*t]=1$)

Convert: use $\hbar = 6.58 \times 10^{-22} \text{ MeV s}$ to convert 1s

E.g., $\tau_\mu = 2.2 \mu\text{s} = 2.2 \times 10^{-6} \text{ s} / (6.58 \times 10^{-16} \text{ eV s}) = 3.3 \times 10^{-9} / \text{eV}$

Length : dimension -1 (velocity is dimensionless)

Momentum : dimension 1 (same as energy and mass; $E^2-p^2=m^2$)

Example:
lengths \rightarrow energies

	λ \rightarrow	E
HEP	< 1 fm	> 1 GeV
gamma	1 pm	1 MeV
X-rays	0.1 nm	10 keV
UV	100 nm	10 eV

2) Energy and Momentum in Particle Physics

We define “4-momentum” as:

$$p^0 = \gamma mc$$

$$\vec{p} = \gamma m \vec{v} = \frac{m \vec{v}}{\sqrt{1 - \beta^2}}$$

Q: what does it mean that this is a “4-vector”?

$$x^\mu \rightarrow x'^\mu = \Lambda^\mu{}_\nu x^\nu$$

$$\implies p^\mu \rightarrow p'^\mu = \Lambda^\mu{}_\nu p^\nu$$

Zero-component = Relativistic Energy ($/c$), defined as :

$$E = \gamma mc^2 \rightarrow p^\mu = (E/c, \vec{p}) = (E/c, p_x, p_y, p_z)$$

$p^2 = 0$: lightlike; $p^2 > 0$: timelike; $p^2 < 0$: spacelike

Expand γ around small β :

$$E = mc^2 \left(1 + \frac{1}{2} \frac{v^2}{c^2} + \frac{3}{8} \frac{v^4}{c^4} + \dots \right) = mc^2 + \frac{1}{2} mv^2 + \frac{3}{8} m \frac{v^4}{c^2} + \dots$$

Reminder: Relativistic Centre-of-Mass

The frame in which the total 3-momentum, $\mathbf{p} = 0$ defines the **rest frame** of a particle, or the **CM frame** for a **system of particles**
In that frame, the total energy is equal to the invariant mass = E_{CM}

For a single particle, at rest

$$p^\mu = (m, 0, 0, 0) \quad p^\mu p_\mu = m^2 \quad = E^2 - \mathbf{p}^2 = m^2$$

Lorentz invariant Squared rest mass In any frame

For a system of particles:

$$p^\mu = \sum_i p_i^\mu \quad p^\mu p_\mu = E_{\text{CM}}^2$$

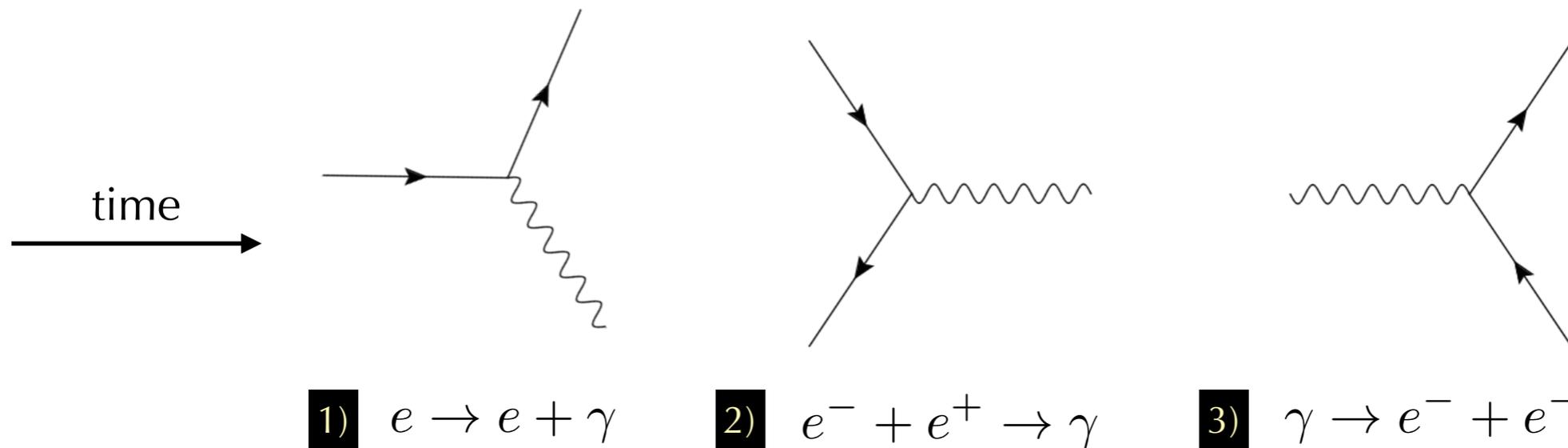
Lorentz invariant squared invariant mass of the system

How to find the CM frame of (a system of) particles?

1. Sum up their 3-momenta \rightarrow total \mathbf{p} . If it is zero: done
2. If non-zero, find their overall velocity $\boldsymbol{\beta} = \mathbf{p} / E$
3. Construct and do the relevant (inverse) Lorentz boost. (& Check 1).

Feynman diagrams and 4-momentum conservation

Consider the QED vertex:



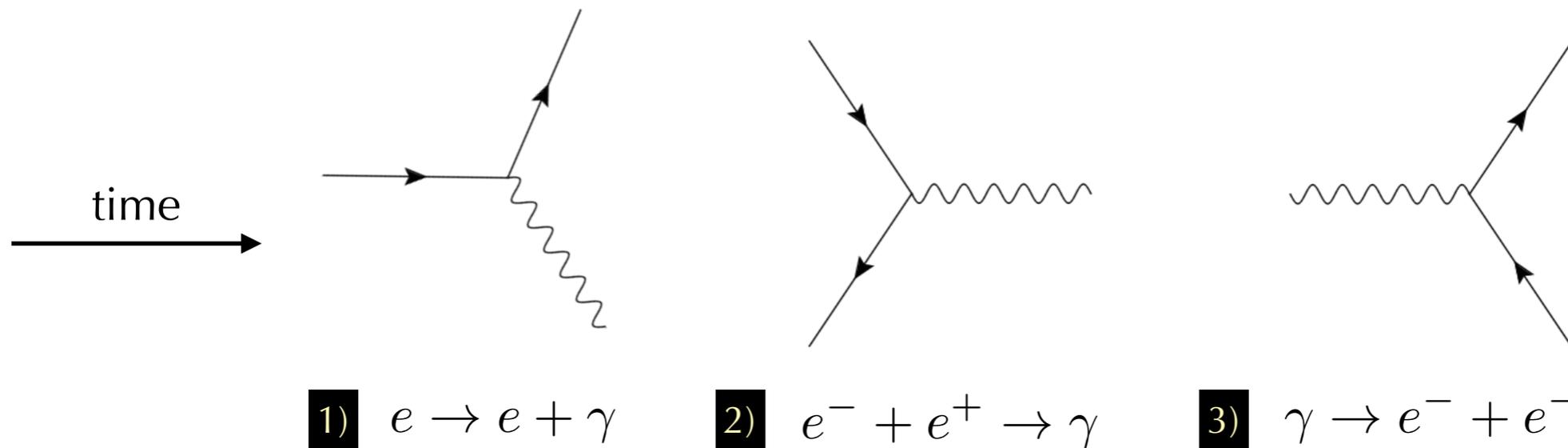
What about 4-momentum conservation?

- 1) Electron at rest decaying to a recoiling electron + a photon?
- 2) Two massive particles reacting to produce a massless photon?
- 3) Massless photon decaying to two massive electrons?

This all sounds very strange (even for relativity)

Feynman diagrams and 4-momentum conservation

Consider the QED vertex:



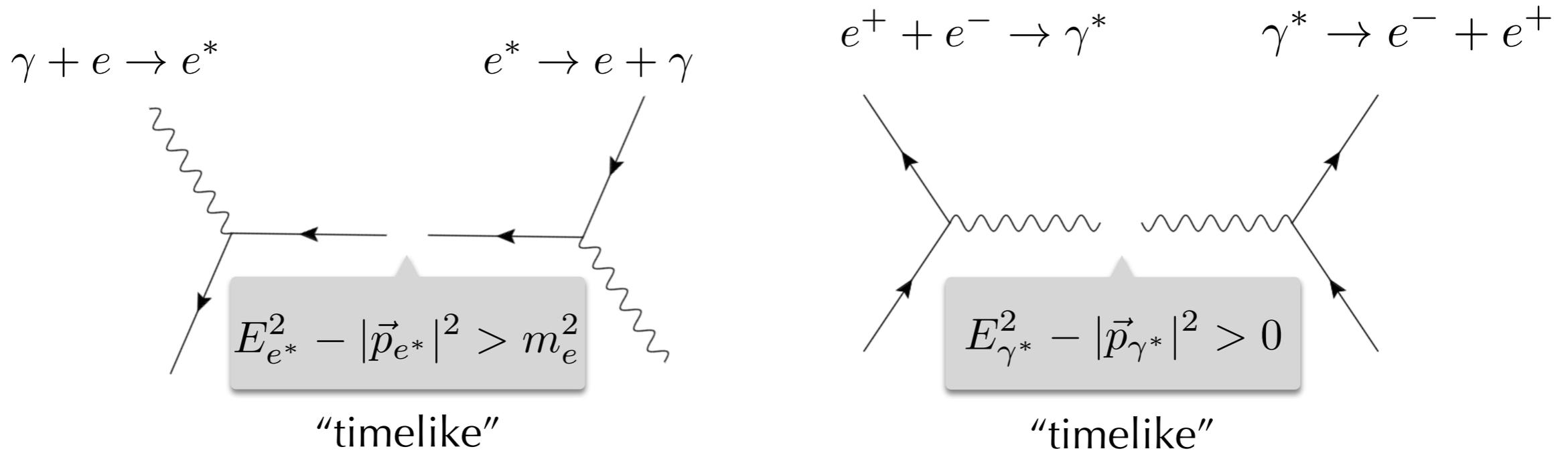
What about 4-momentum conservation?

At least one of the involved particles must have $E^2 - p^2 \neq m^2$

Equivalent to Heisenberg ΔE but here expressed in **L.I.** form

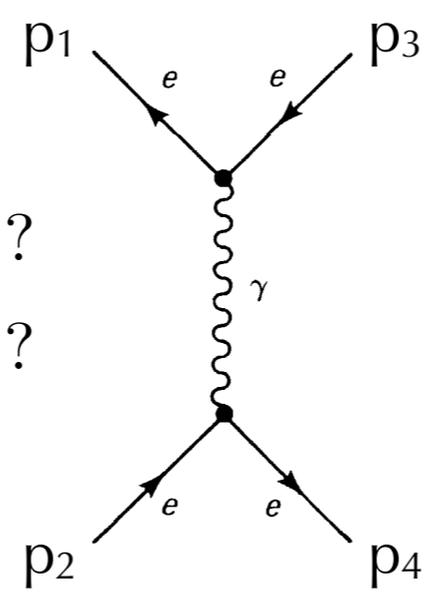
We call such particles **virtual**; and say they are **off mass shell**

Virtual Particles: Examples



Q: exchanged virtual photon is:

- A) “timelike” $E_{\gamma^*}^2 - |\vec{p}_{\gamma^*}|^2 > 0 ?$
- B) “spacelike” $E_{\gamma^*}^2 - |\vec{p}_{\gamma^*}|^2 < 0 ?$



(for $m_e = 0$)
 $p_\gamma^2 = (p_1 - p_3)^2$

4) Perturbation Theory: Fermi's Golden Rule

Two basic ingredients to **calculate** decay rates and cross sections

1) The **amplitude** for the process: \mathcal{M}

Contains all the *dynamical* information; couplings, propagators, ...

Calculated by evaluating the relevant Feynman Diagrams, using the "**Feynman Rules**" for the interaction(s) in question

2) The **phase space** available for the process

Contains only *kinematical* information;

Depends only on external masses, momenta, energies;

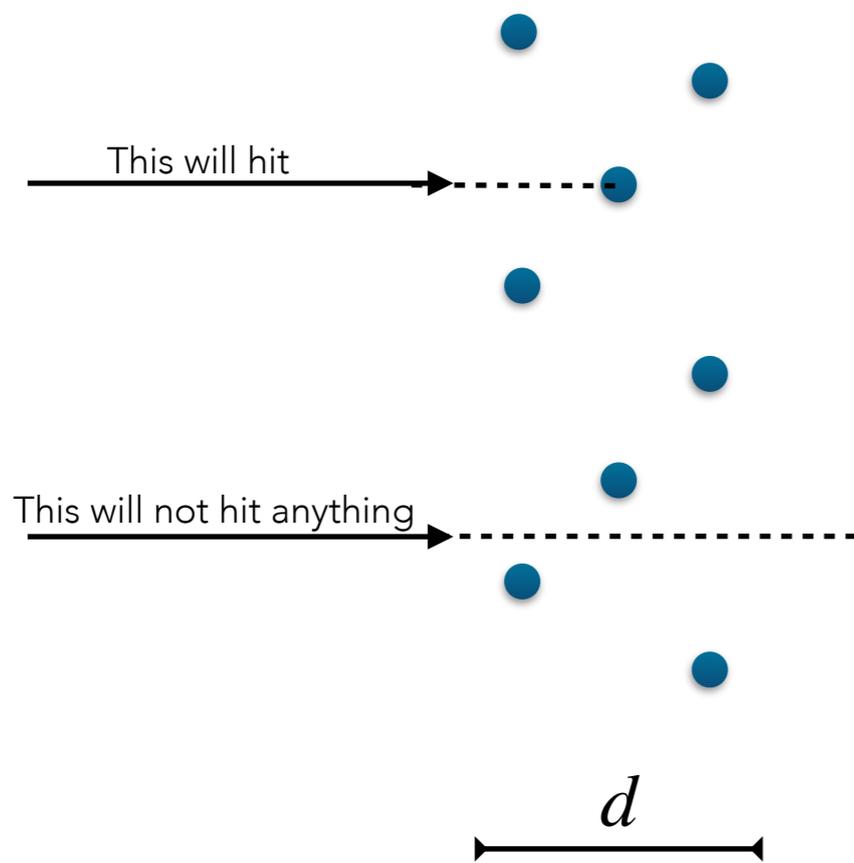
"Counts" the number/density of available final states

The Golden Rule is*: $\text{transition rate} = \frac{2\pi}{\hbar} |\mathcal{M}|^2 \times (\text{phase space})$

*For a derivation, see QM (nonrelativistic) or QFT (relativistic)

5) Cross Sections and Decay Rates in Particle Physics

Consider a beam of point-like particles traversing a depth d of a “target region”. The target region consists of tiny solid spheres, each having radius r . The number density of the tiny spheres is ρ .



What is the probability that a single incoming particle (uniformly chosen) will hit one of the little spheres ? (assuming they do not “shadow” each other)

ρ : Total number of scattering centres per unit volume

(ρd) = Total number of scattering centres presented to the beam per unit area

(πr^2) = Cross sectional area of each scattering centre as presented to the beam

$$P_{\text{scatter}} = \rho d (\pi r^2)$$

(Could eg use this to define scattering length = $1/(\rho\sigma)$... but that is not our goal today)

Generalising the notion of cross section

1) The beam is usually made of little spheres too

Should really talk about mutual cross-sectional area that the beam and target particles present **to each other**

+ Lorentz Invariance: Whether the beam hits the target, or vice versa, should be equivalent. They hit each other (as they indeed do in colliders).

2) Our spheres are not solid (nor do we think of them as spheres)

We're talking about (classical) potentials or (quantum) fields

Transition amplitudes, computed according to Fermi's Golden rule

~ wave function overlaps between in- and out-states

Classical hit or miss translates to **scattering or transmission**

Transmission: in = out

Scattering: in \neq out

Related by Probability Conservation: $P_{\text{transmission}} = 1 - P_{\text{scattering}}$

(Unitarity).

Cross Section in Particle Physics

Classical fixed-target solid spheres:

$$P_{\text{scatter}} = \rho d (\pi r^2)$$

Particle Physics:

$$\text{Event Rate} = \text{Luminosity} \times \sigma$$

L: determined by beam: densities, etc.

Has units of flux: per unit area per unit time

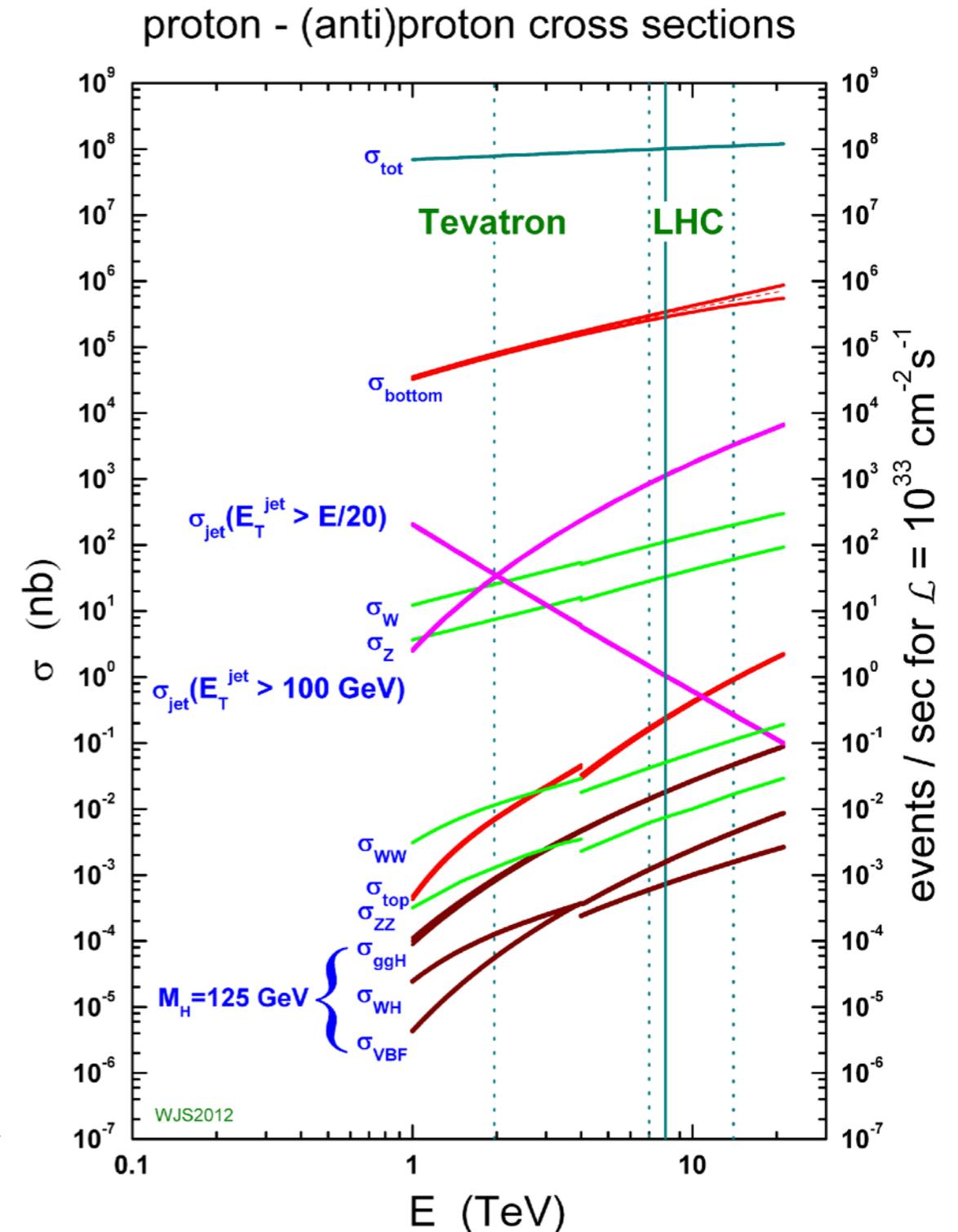
E.g., LHC Run 1 had $L_{pp} \sim 10^{33} \text{ cm}^{-2} \text{ s}^{-1}$

σ : What we compute

Has units of area.

$$1 \text{ barn} = 10^{-24} \text{ cm}^2 \quad (\leftrightarrow r_{\text{sphere}} \sim 6 \times 10^{-15} \text{ m})$$

\sim Area two colliding particles present to each other; **total or for specific interaction**

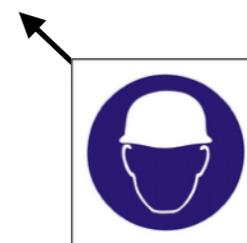


Event Rates with Decays

What we actually measure is typically a **cross section times a branching fraction**

E.g., the event rate for $h^0 \rightarrow \gamma\gamma$ observed at LHC is compared to a theoretical calculation of

$$N(h^0 \rightarrow \gamma\gamma)_{\text{LHC}} = \sigma(pp \rightarrow h^0)_{\text{LHC}} * \text{BR}(h^0 \rightarrow \gamma\gamma) * L_{pp} * \langle \text{efficiency} \rangle$$



Branching Ratio : $\text{BR}(i) = \frac{\Gamma_i}{\sum_j \Gamma_j}$
 or “branching fraction”

“Total Decay Width”:

$$\Gamma = \sum_i \Gamma_i$$

Γ_i : “Partial Width”

π^+ DECAY MODES K.A. Olive et al. (Particle Data Group), Chin. Phys. C38, 090001 (2014) (URL: <http://pdg.lbl.gov>)

	Mode	Fraction (Γ_i/Γ)	Confidence level
Γ_1	$\mu^+ \nu_\mu$	[a] (99.98770 ± 0.00004) %	
Γ_2	$\mu^+ \nu_\mu \gamma$	[b] (2.00 ± 0.25) × 10 ⁻⁴	
Γ_3	$e^+ \nu_e$	[a] (1.230 ± 0.004) × 10 ⁻⁴	
Γ_4	$e^+ \nu_e \gamma$	[b] (7.39 ± 0.05) × 10 ⁻⁷	
Γ_5	$e^+ \nu_e \pi^0$	(1.036 ± 0.006) × 10 ⁻⁸	
Γ_6	$e^+ \nu_e e^+ e^-$	(3.2 ± 0.5) × 10 ⁻⁹	
Γ_7	$e^+ \nu_e \nu \bar{\nu}$	< 5 × 10 ⁻⁶	90%

example from the “PDG book” pdg.lbl.gov

How does a particle decay?

(heuristic derivation)

It sits in its rest frame and gets time evolved, by e^{iHt}

Unstable \rightarrow H contains operators that want to annihilate it (+ create decay products)

Decay probability per unit time:

$$dN = -\Gamma N dt \quad \rightarrow \quad N(t) = N(0)e^{-\Gamma t} \quad \rightarrow \quad \tau = \frac{1}{\Gamma}$$

$N(t) \sim$ squared amplitude. Amplitude itself must then be proportional to:

$$A(t) \propto e^{-imt - \frac{\Gamma t}{2}}$$

Ordinary time evolution (of stable particle at rest with mass m)

Decay term

Exponential decay in time \rightarrow "Breit-Wigner" in momentum-space

$$|\phi(E)|^2 = \frac{1}{(E - m)^2 + \Gamma^2/4}$$

Fourier transform \rightarrow an infinite set of plane waves of different energies that add up to a decaying wave

$$\begin{aligned} \phi(E) &= \int_0^\infty e^{-t[i(m-E) + \Gamma/2]} dt \\ &= \frac{1}{i(m-E) + \Gamma/2} \end{aligned}$$

Note: I used only time component of plane wave in A(t) here \rightarrow **non-relativistic BW**
 \sim Heisenberg : $\Delta E = (E-m) \sim \Gamma \sim 1/\Delta t$

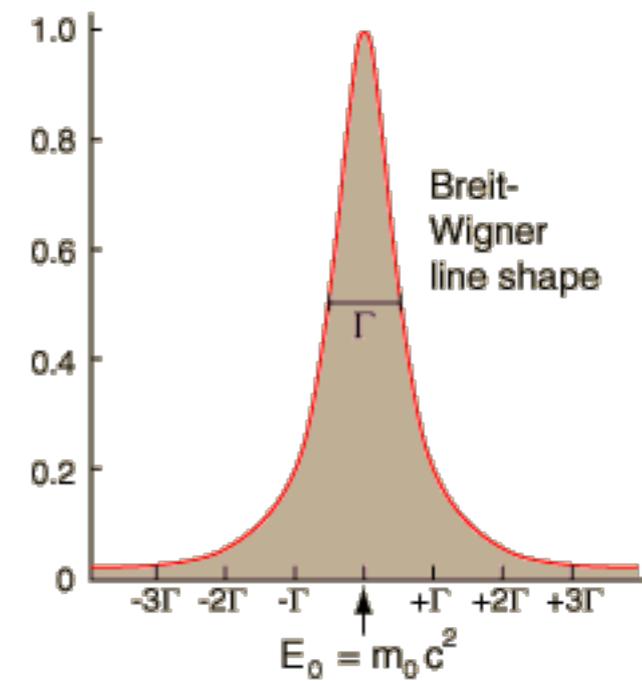
Resonance Shapes & Kinematic Thresholds

Resonance Shapes ...

Heisenberg: the energy is uncertain...

If a particle is unstable (has a non-zero decay rate), then we at most have the duration of its life to measure its energy.

Analogous to line-broadening of lines in spectra of excited atoms



Relativistic BW

~ distribution of **off-shellness** of **virtual particle** with $m^2 = E^2 - p^2 \neq m_0^2$

$$\mathcal{P}(m^2) dm^2 \propto \frac{1}{(m^2 - m_0^2)^2 + m_0^2 \Gamma^2} dm^2$$

(+ further subtleties, not covered here: *normalisation, running widths, multiple resonances, radiation off unstable particles, large widths, ...*)

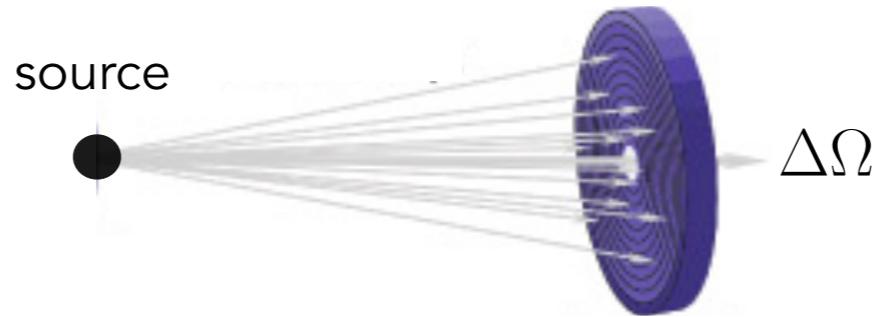
Kinematic Thresholds

A particle cannot be produced (on shell) unless the colliding particles have enough CM energy to create its rest mass

... & cannot decay to any (combination of) on-shell particles heavier than itself

Making Predictions

Scattering Experiments:



LHC detector
Cosmic-Ray detector
Neutrino detector
X-ray telescope
LIGO
...

Test model predictions by comparing to measurements

In particle physics → Integrate differential **cross sections** over specific **phase-space** regions

$$N_{\text{count}}(\Delta\Omega) \propto \int_{\Delta\Omega} d\Omega \frac{d\sigma}{d\Omega}$$

$$d\Omega = d\cos\theta dp$$

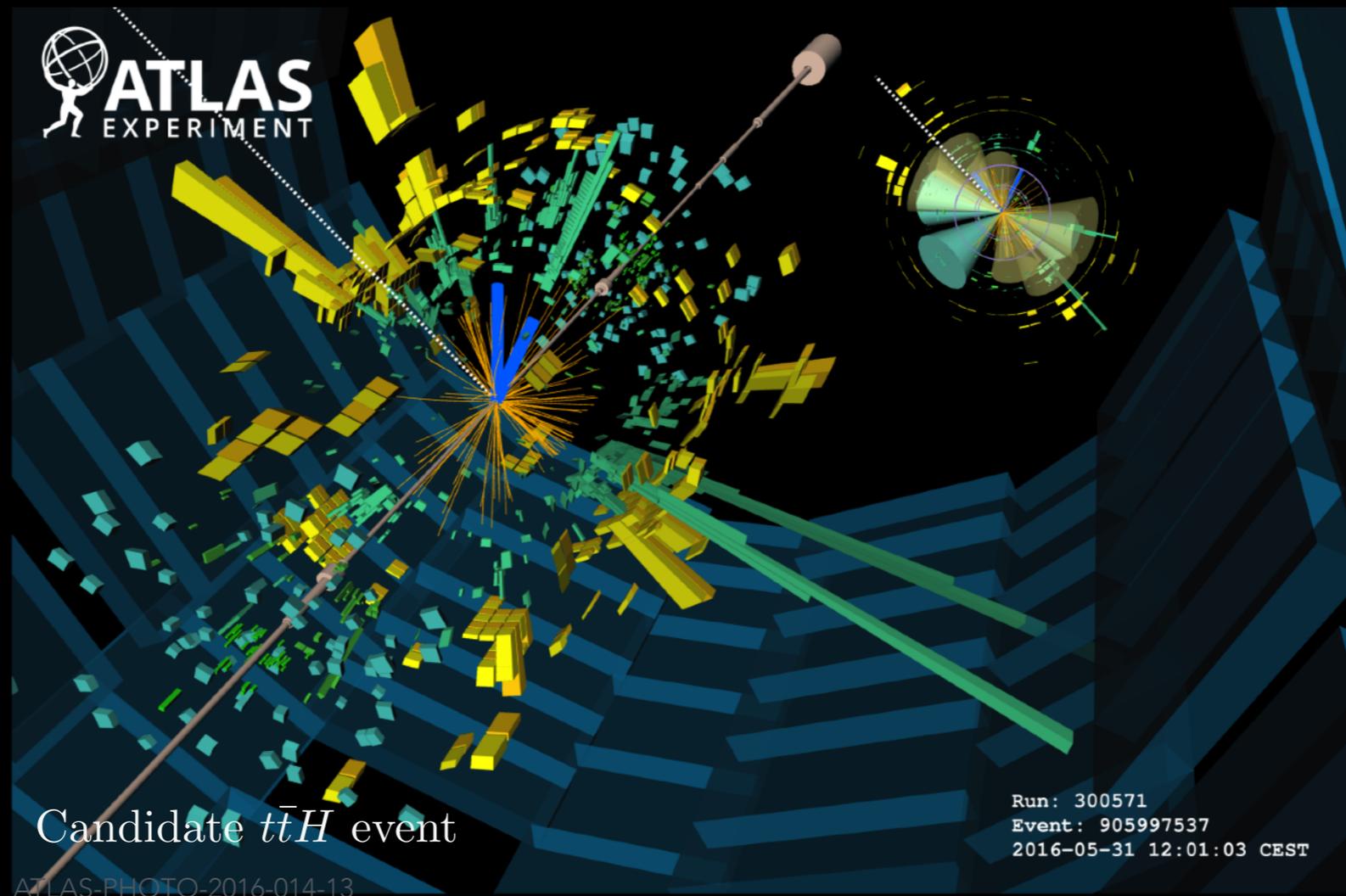
- What can our **incoming and outgoing states** be?
- How do we calculate **transition rates** between them?
- What kinds of **observables** can/should we measure?
- How do we **accurately relate** measured observables to calculated/predicted transition rates?

$d\sigma/d\Omega$; how hard can it be?

Approximate all contributing amplitudes for this ...

To all orders...then square including interference effects, ...

+ non-perturbative effects

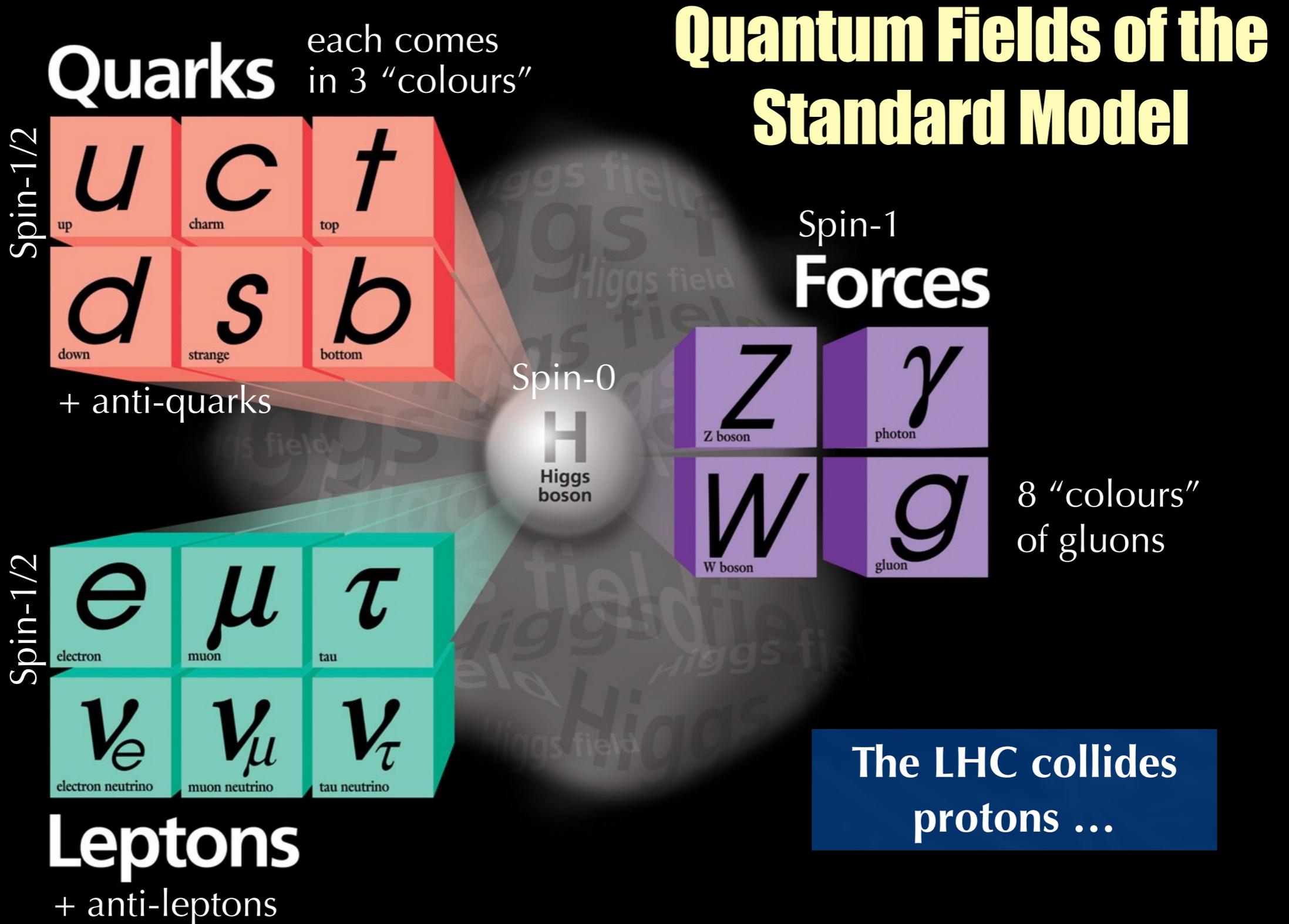


... integrate it over a ~ 300 -dimensional phase space

(+ recall that collider delivers 40 million of these per second)

Too much for us (today).

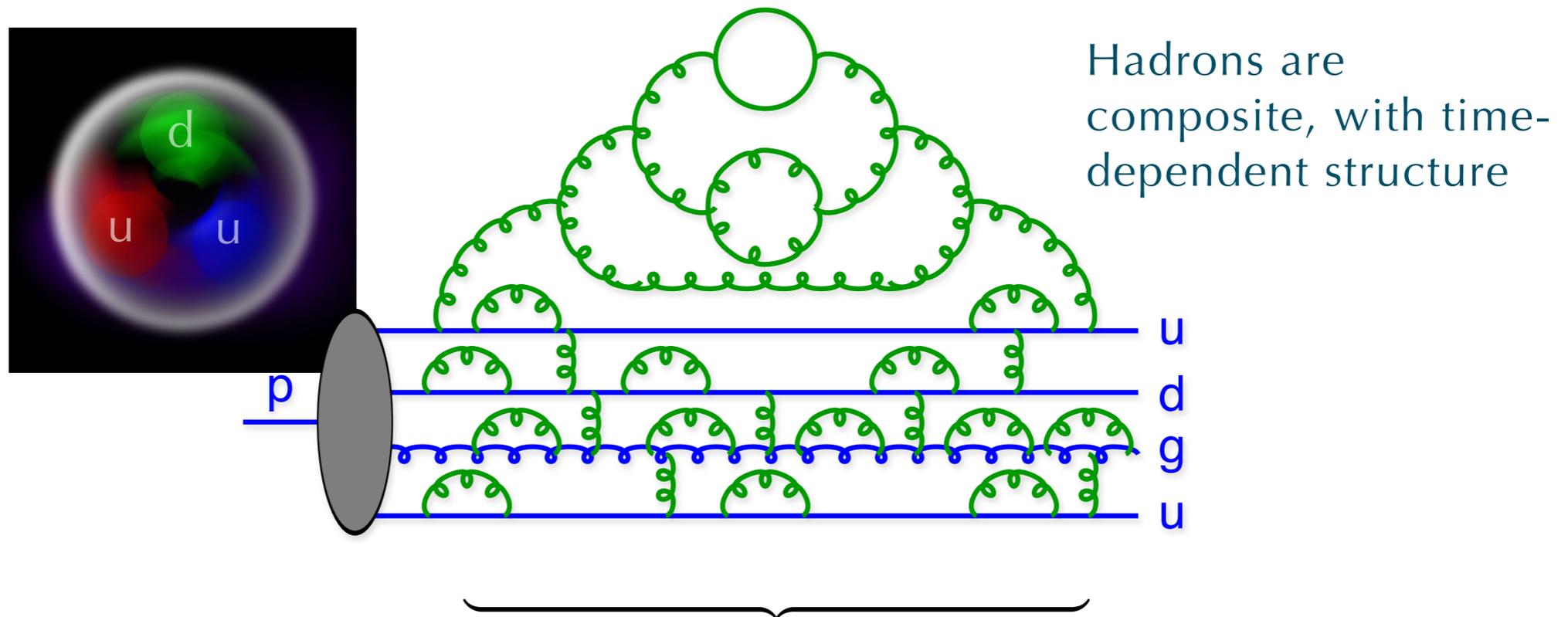
What can our (incoming and outgoing) states be?



Colliding Protons

What are we really colliding?

Take a look at the quantum level



Describe this mess **statistically** → **parton distribution functions (PDFs)**

PDFs: $f_i(x, Q_F^2)$ $i \in [u, d, s, c, b, g]$

Probability to find parton of flavour i with momentum fraction x , as function of "resolution scale" $Q_F \sim$ virtuality / inverse lifetime of fluctuation

Why PDFs work 1: heuristic explanation

Lifetime of typical fluctuation $\sim r_p/c$ (=time it takes light to cross a proton)

$\sim 10^{-23}$ s; Corresponds to a frequency of ~ 500 billion THz

To the LHC, that's slow! (reaches "shutter speeds" thousands of times faster)

Planck-Einstein: $E=hf \rightarrow \nu_{\text{LHC}} = 13 \text{ TeV}/h = 3.14$ million billion THz

→ Protons look "frozen" at moment of collision

But they have a lot more than just two "u" quarks and a "d" inside

Hard/impossible to calculate, so use statistics to parametrise the structure: **parton distribution functions (PDFs)**

Every so often I will pick a gluon, every so often a quark (antiquark)

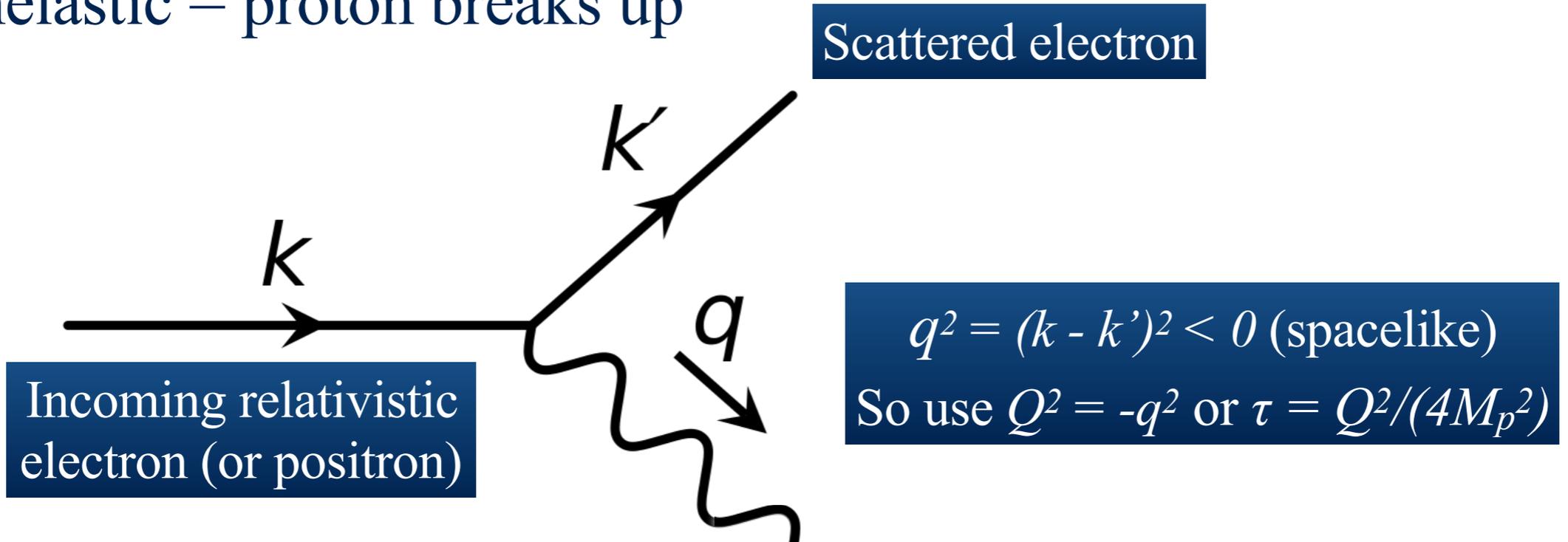
Measured at previous colliders

Expressed as functions of energy fractions, x , and resolution scale, Q^2

+ obey known scaling laws df/dQ^2 : "DGLAP equations".

Why PDFs work 2: Deep Inelastic Scattering

Inelastic = proton breaks up



Leptonic
part ~ clean



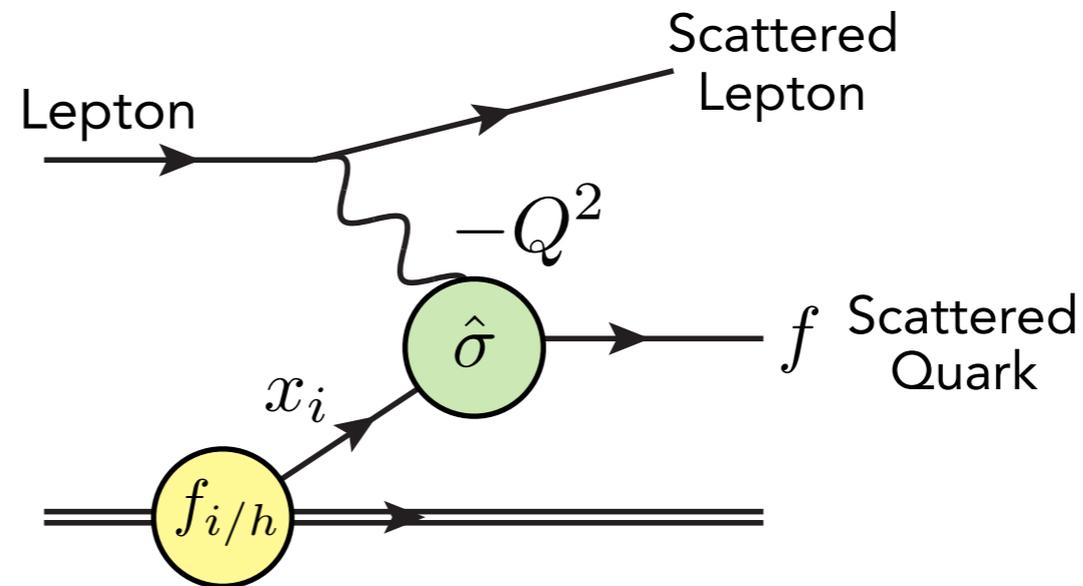
Hadronic
part : messy

Deep = invariant mass of final hadronic system $\gg M_{\text{proton}}$

Why PDFs work 2: factorisation in DIS

Collins, Soper (1987): Factorisation in Deep Inelastic Scattering

Deep Inelastic Scattering (DIS)



→ The cross section can be written in **factorised** form :

$$\sigma^{\ell h} = \sum_i \sum_f \int dx_i \int d\Phi_f f_{i/h}(x_i, Q_F^2) \frac{d\hat{\sigma}^{\ell i \rightarrow f}(x_i, \Phi_f, Q_F^2)}{dx_i d\Phi_f}$$

Sum over
Initial (i)
and final (f)
parton flavors

Φ_f
= Final-state
phase space

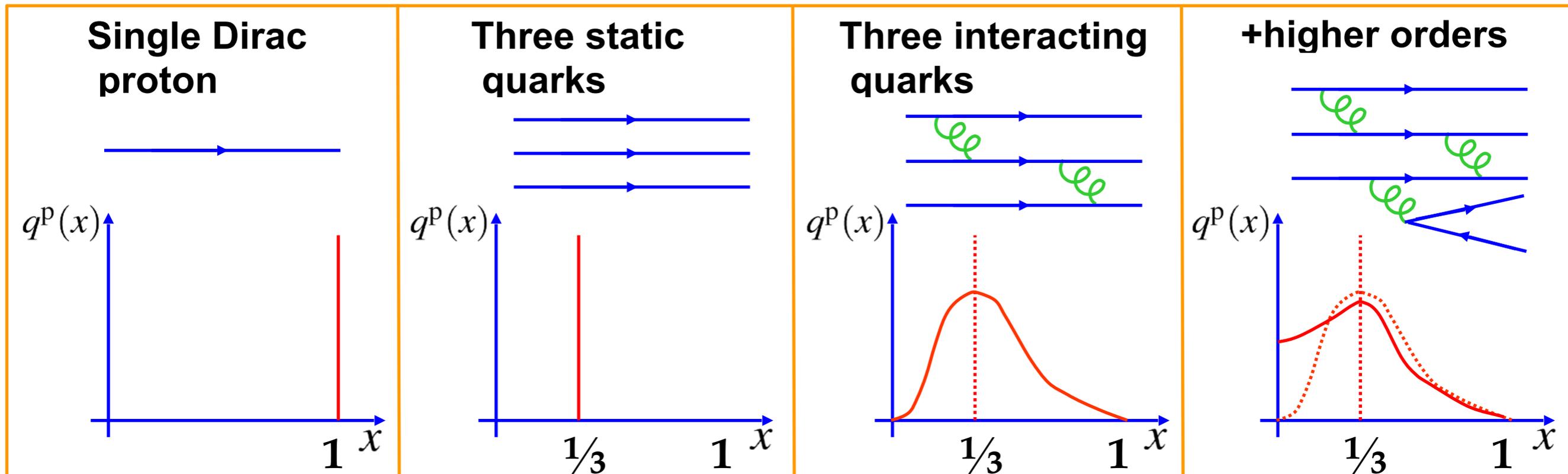
$f_{i/h}$
= PDFs
Assumption:
 $Q^2 = Q_F^2$

Differential partonic
Hard-scattering
Matrix Element(s)

Distribution of quarks in x

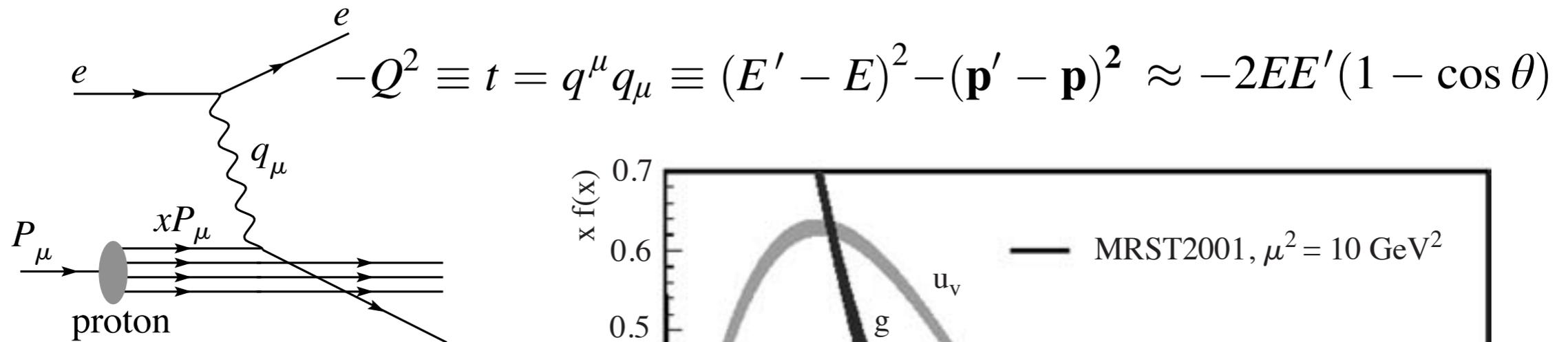
★ $q^P(x)dx$ is the number of quarks of type q within a proton with momentum fractions between $x \rightarrow x + dx$

★ Expected form of the parton distribution function ?



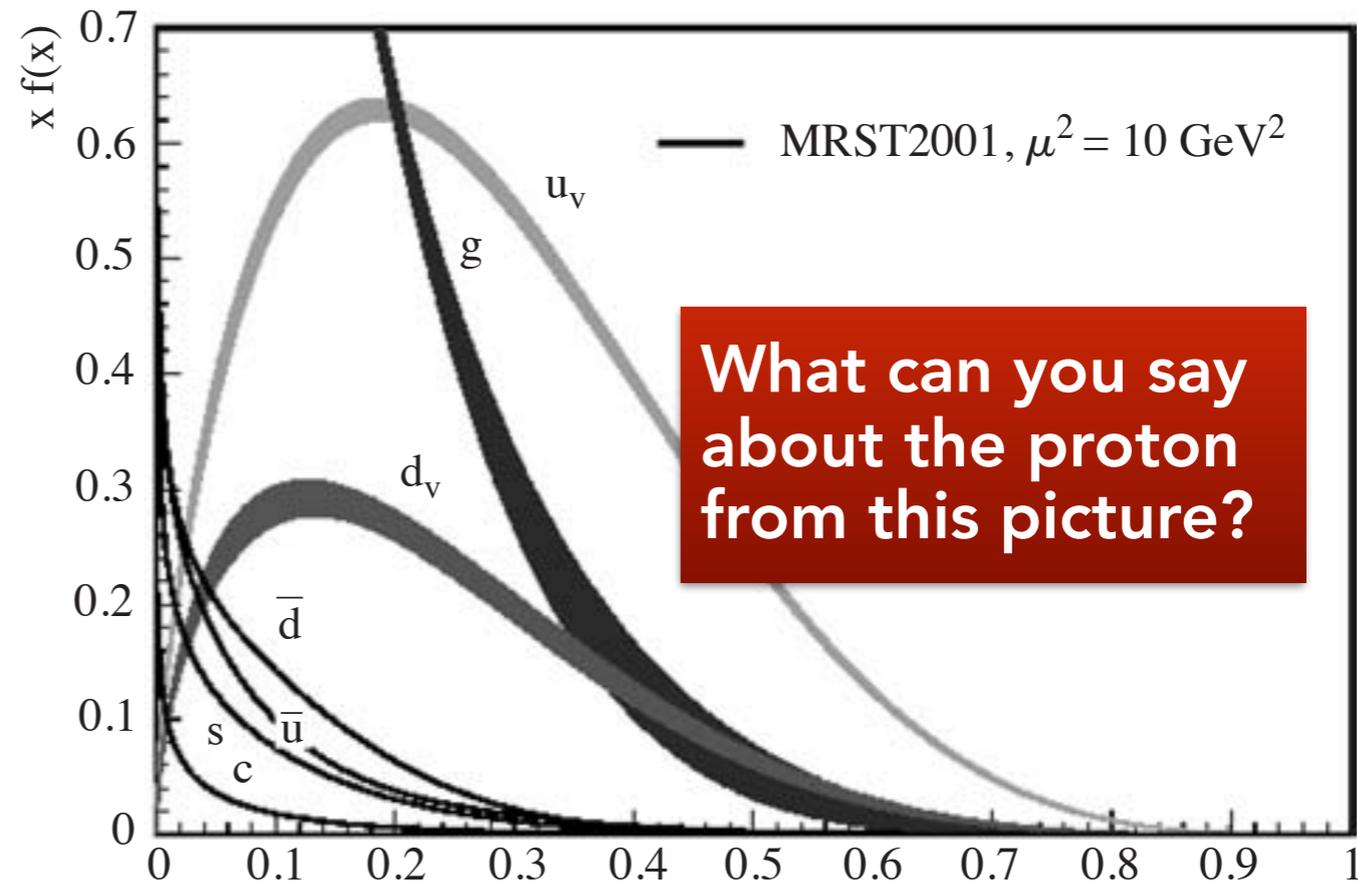
Parton Distribution Functions (PDFs)

$q_i(x, Q^2)$ = probability density to find quark of type i carrying momentum fraction x , when probing the proton at momentum transfer $t = q_\mu q^\mu = -Q^2$.



$$x = \frac{Q^2}{2P_\mu q^\mu} = \frac{Q^2}{2\nu m_p}$$

$$\begin{aligned} \Delta E_{\text{LAB}} &= (E - E')_{\text{LAB}} \\ &= \nu = p_\mu q^\mu / m_p \end{aligned}$$



What can you say about the proton from this picture?

Note: showing momentum density, $xf(x)$, rather than number density^x

Constraints on PDFs

PDFs not a priori calculable. Parametrised as functions of x and fit to data

But, similarly to the running of α_s , the “**scaling violation**” = **running of the PDFs with Q^2 is** calculable in perturbation theory (not covered in this course)

➤ a differential equation, called the “DGLAP equation” for $df_i(x, Q^2)/d\ln Q^2$

If you have measured the PDFs at one Q^2 , their form at some other Q^2 is calculable; measurements at different Q^2 values constrain the same f

There must also be some “sum rules”

Example: total number of up quarks must be 2. $\int_0^1 (u(x) - \bar{u}(x)) dx = 2$

Similarly, total number of down quarks must be = 1; other flavours = 0.

Total momentum fraction summed over all partons = 1. $\sum_{i \in \text{partons}} \int_0^1 x f_i(x) dx = 1$

Sum over quark x fractions with realistic (measured) PDFs:

$$\int_0^1 x [u(x) + d(x) + \bar{u}(x) + \bar{d}(x) + s(x) + \bar{s}(x)] dx \approx 0.50.$$

➔ Gluons carry \approx half of the proton momentum

Cross Sections at Fixed Order

OK, protons \sim bags of partons with distributions $f(x, Q^2)$

Now want to compute the distribution of some observable: \mathcal{O}

In “inclusive X production” (suppressing PDF factors)

$X + \text{anything}$

Fixed Order
(All Orders)

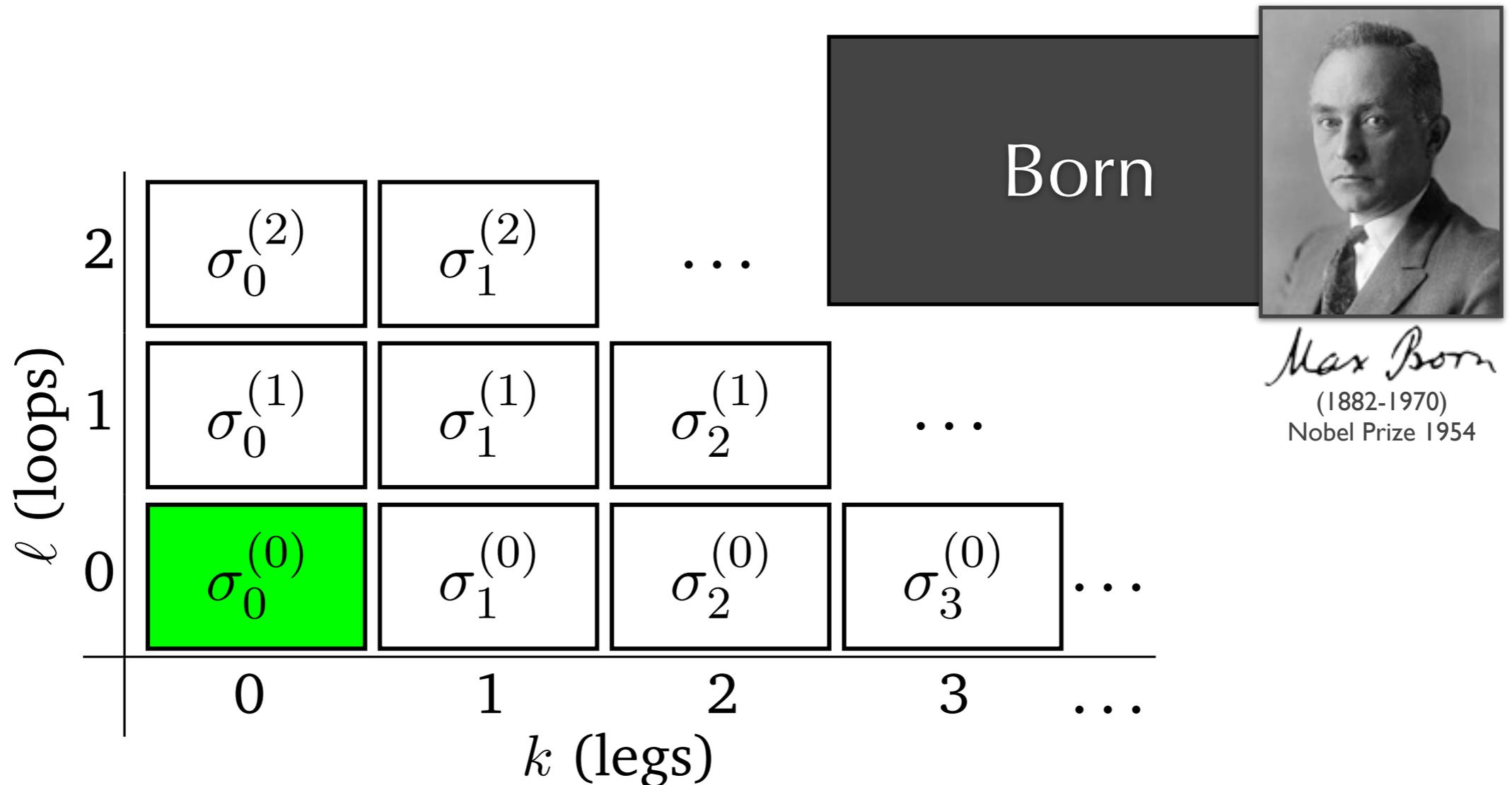
$$\left. \frac{d\sigma}{d\mathcal{O}} \right|_{\text{ME}} = \sum_{k=0} \int d\Phi_{X+k} \left| \sum_{\ell=0} M_{X+k}^{(\ell)} \right|^2 \delta(\mathcal{O} - \mathcal{O}(\{p\}_{X+k}))$$

↑ Cross Section differentially in \mathcal{O}
↑ Sum over “anything” \approx legs
Phase Space
↑ Matrix Elements for $X+k$ at (ℓ) loops
↑ Sum over identical amplitudes, then square
↑ Momentum configuration
↑ Evaluate observable \rightarrow differential in \mathcal{O}

Truncate at $k = 0, \ell = 0$,
 \rightarrow Born Level = First Term
 Lowest order at which X happens

Loops and Legs

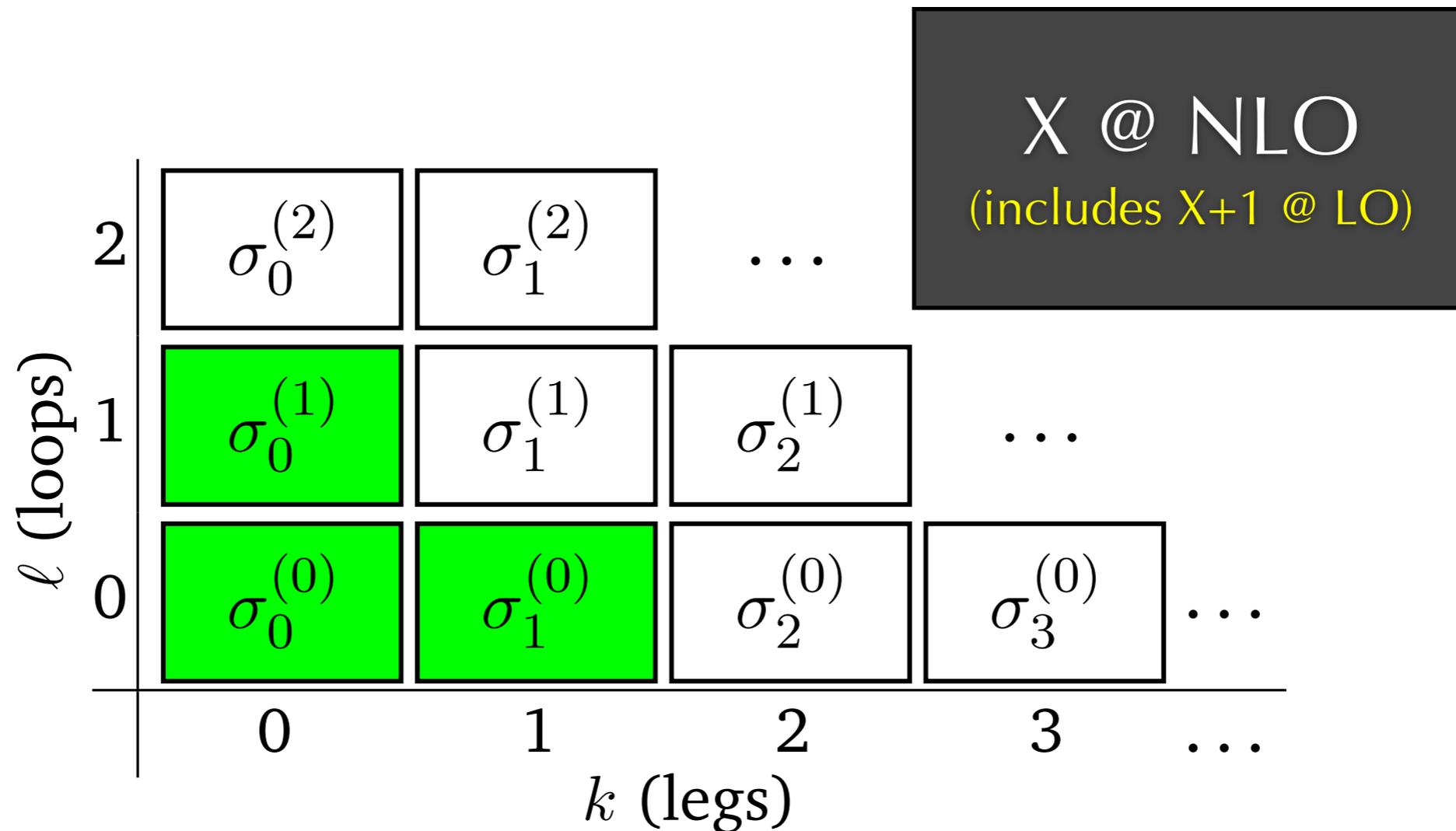
Another representation



Truncate at $k = 0, \ell = 0, \rightarrow$ **Born Level**
Lowest order at which X happens

Loops and Legs

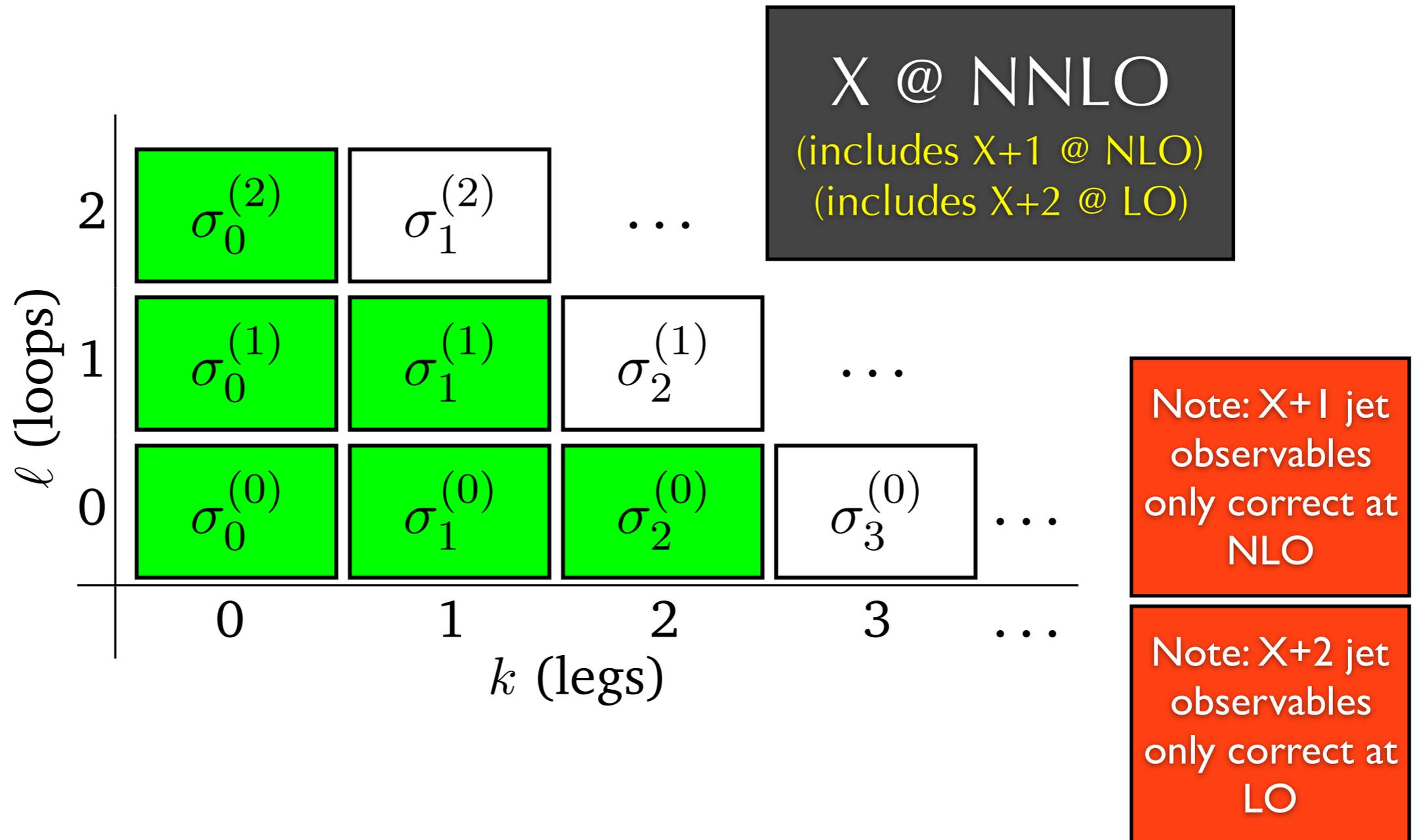
Another representation



Note: (X+1)-jet observables only correct at LO

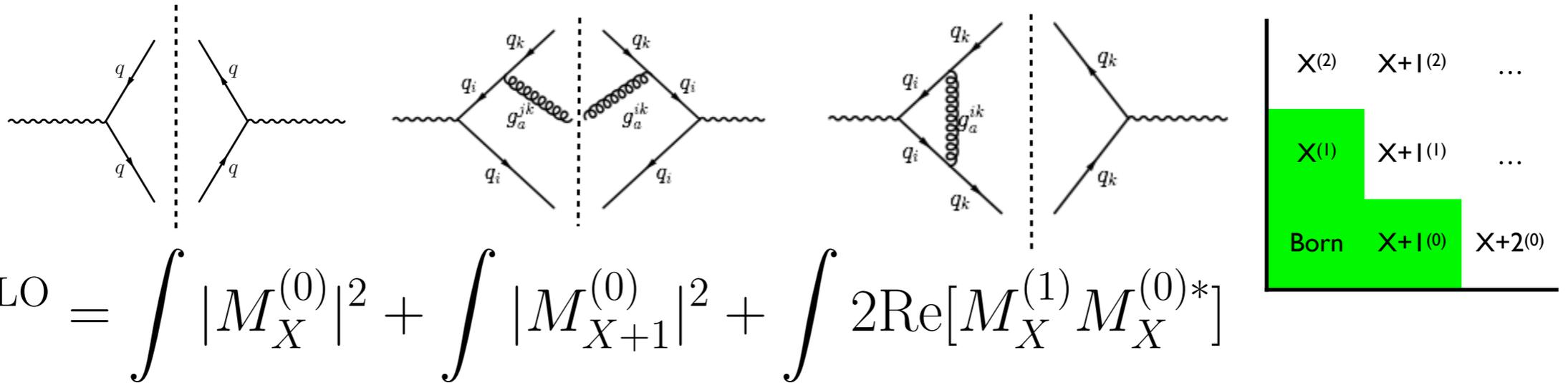
Loops and Legs

Another representation



Cross sections at NLO: a closer look

NLO:



$$\sigma_X^{\text{NLO}} = \int |M_X^{(0)}|^2 + \int |M_{X+1}^{(0)}|^2 + \int 2\text{Re}[M_X^{(1)} M_X^{(0)*}]$$

(note: this is *not* the 1-loop diagram squared)

KLN Theorem (Kinoshita-Lee-Nauenberg)

Sum over ‘degenerate quantum states’: **Singularities cancel at complete order** (only finite terms left over)

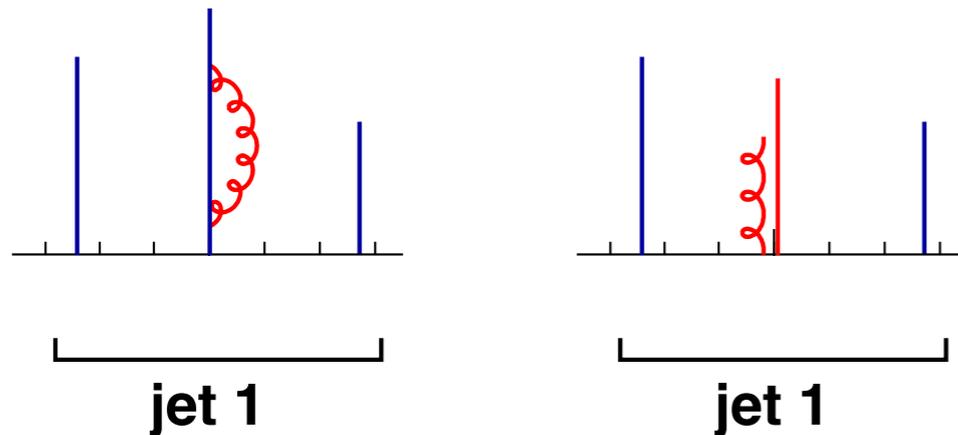
$$= \sigma_{\text{Born}} + \text{Finite} \left\{ \int |M_{X+1}^{(0)}|^2 \right\} + \text{Finite} \left\{ \int 2\text{Re}[M_X^{(1)} M_X^{(0)*}] \right\}$$

$$\sigma_{\text{NLO}}(e^+e^- \rightarrow q\bar{q}) = \sigma_{\text{LO}}(e^+e^- \rightarrow q\bar{q}) \left(1 + \frac{\alpha_s(E_{\text{CM}})}{\pi} + \mathcal{O}(\alpha_s^2) \right)$$

Not all observables can be computed perturbatively:

Collinear Safe

Virtual and Real go into **same bins!**



$$\alpha_s^n \times (-\infty)$$

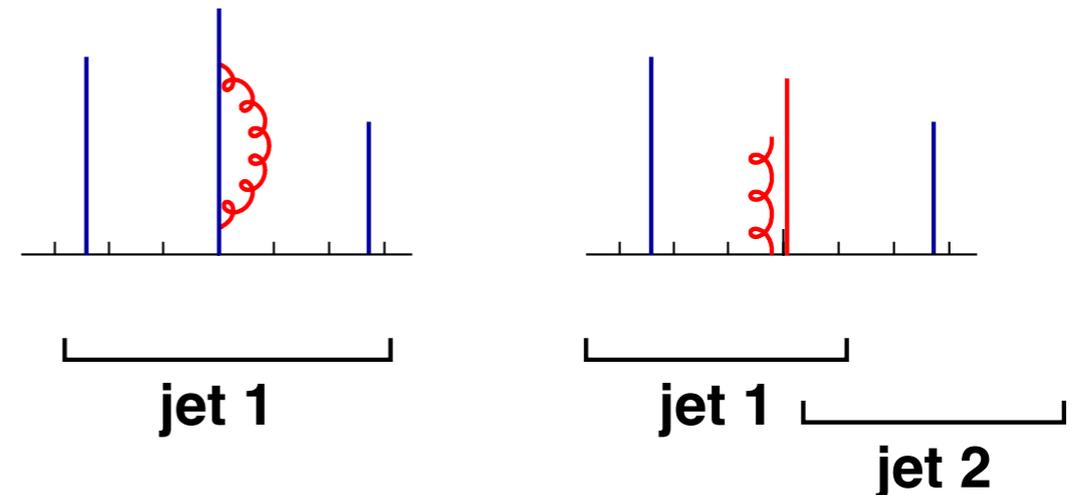
$$\alpha_s^n \times (+\infty)$$

Infinities cancel

(KLN: 'degenerate states')

Collinear Unsafe

Virtual and Real go into **different bins!**



$$\alpha_s^n \times (-\infty)$$

$$\alpha_s^n \times (+\infty)$$

Infinities do not cancel

Invalidates perturbation theory

Definition: an observable is infrared and collinear safe if it is *insensitive* to

SOFT radiation:

Adding any number of infinitely *soft* particles (zero-energy) should not change the value of the observable

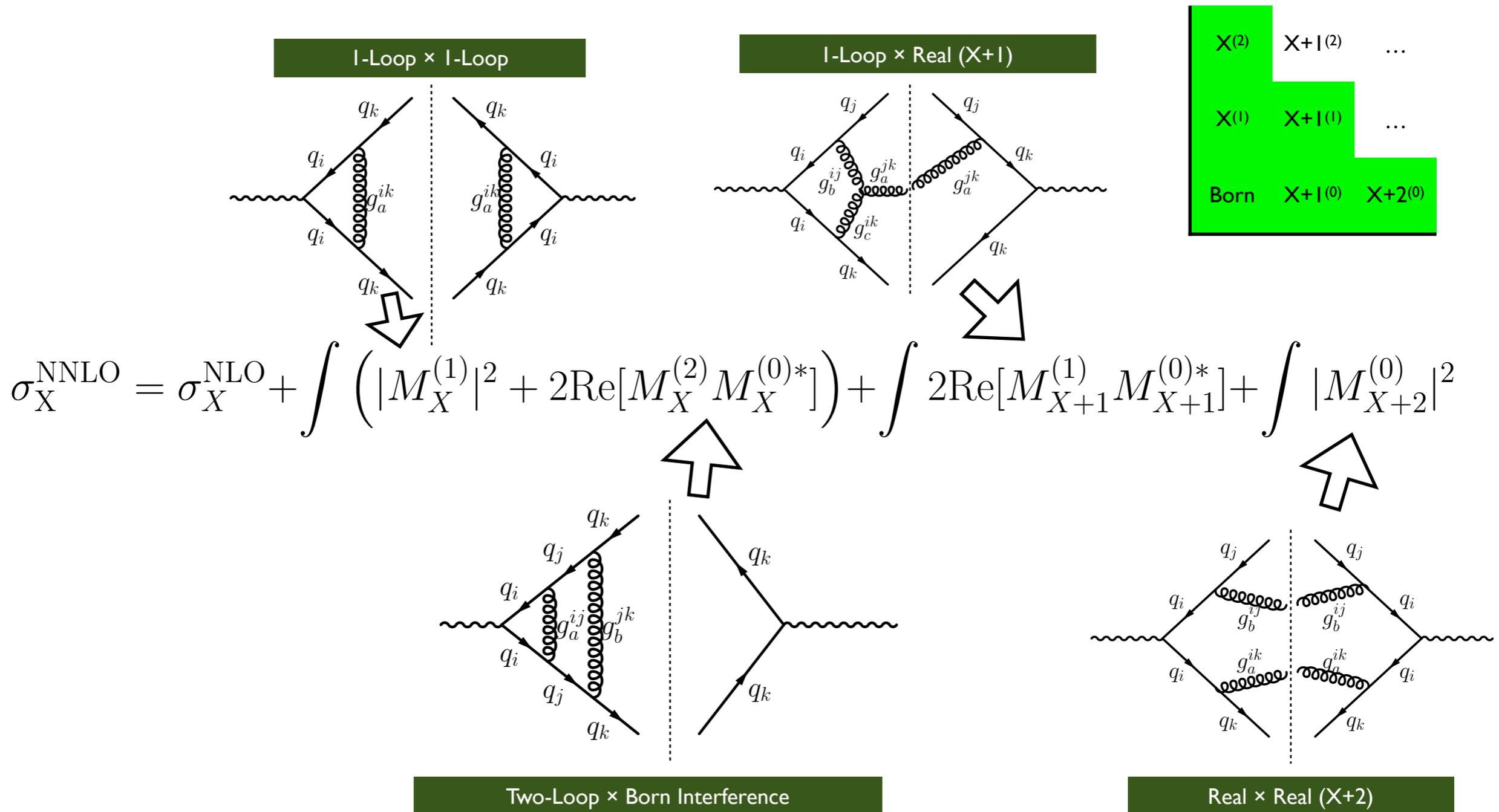
COLLINEAR radiation:

Splitting an existing particle up into two *comoving* ones (conserving the total momentum and energy) should not change the value of the observable

More on this tomorrow

Structure of an NNLO calculation

NNLO



Summary

Particle Physics Basics: Scattering Experiments

Natural units $\hbar = c = 1$ → everything in MeV, GeV, TeV, to some power

Event Rate = Luminosity * Cross Section

Cross section ~ effective area two colliding particles present to each other (longitudinally boost invariant); can be total or partial

Virtual Particles: Lorentz invariant version of Heisenberg uncertainty

Cast in terms of “off-shell” particles with $E^2 - p^2 \neq m^2$.

Can only appear as intermediate states in diagrams, not as in- or outgoing states.

Unstable Particles: Fourier transform of exponential decay → Breit-Wigner

Factorisation (assuming proof from DIS extends to pp):

$\sigma_{\text{hadron-hadron}} = \text{PDFs} \otimes \sigma_{\text{parton-parton}}$ ← can be calculated in perturbative QFT

“universal” : independent of parton-parton interaction

Instantaneous snapshots of “average” proton structure.

Cast as 2-dimensional functions, $f(x, Q^2)$, which must be fit to data.

Known evolution in Q^2 (DGLAP) → essentially a set of 1D functions, say at fixed Q_0