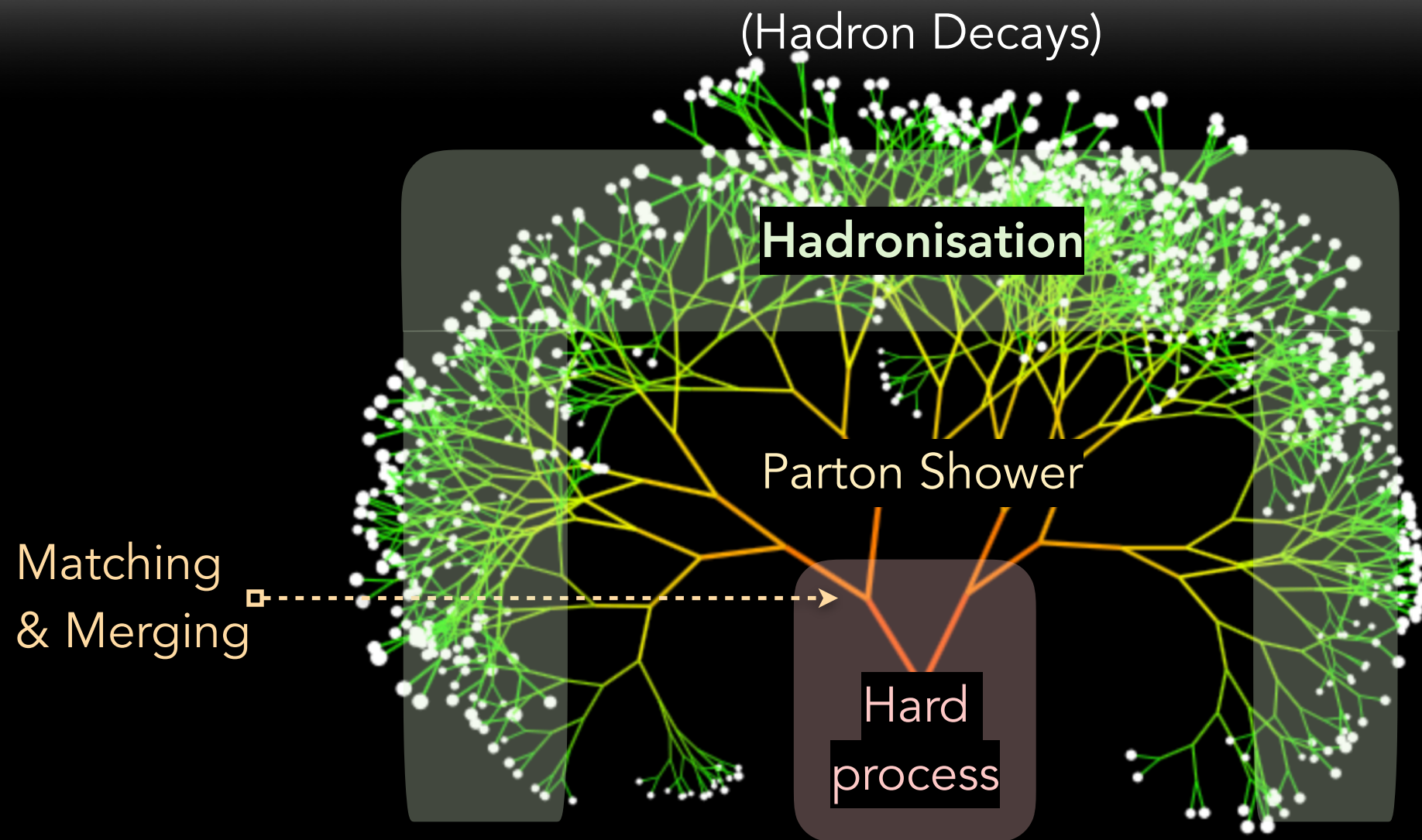


Parton Showers and Matching/Merging

Lecture 2 of 2: Matching/Merging & Non-Perturbative Corrections



Peter Skands (Monash University)
Feynrules/Madgraph School, Hefei 2018

SHOWERS VS MATRIX ELEMENTS

Showers. Nice to have all-orders solution

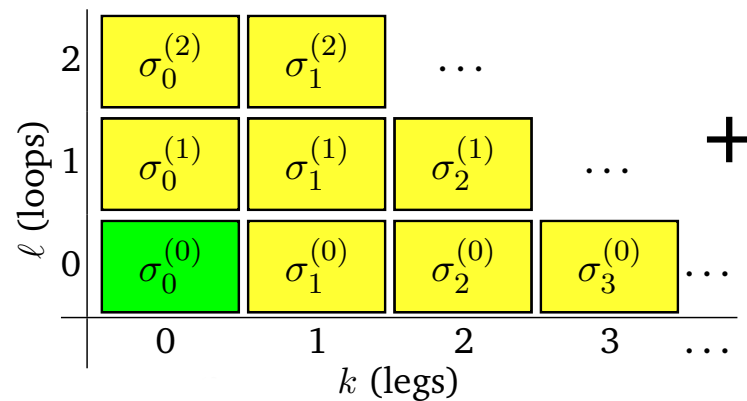
But only exact in singular (soft & collinear) limits

→ gets bulk of bremsstrahlung corrections right, but no precision for hard wide-angle radiation: **visible, extra jets**

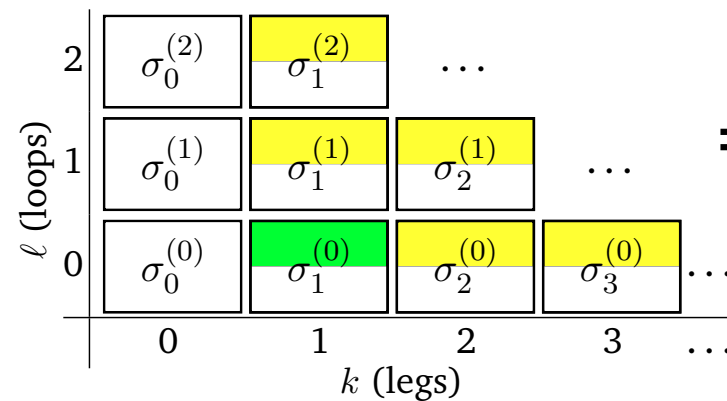
... which is exactly where fixed-order (ME) calculations work!

So combine them!

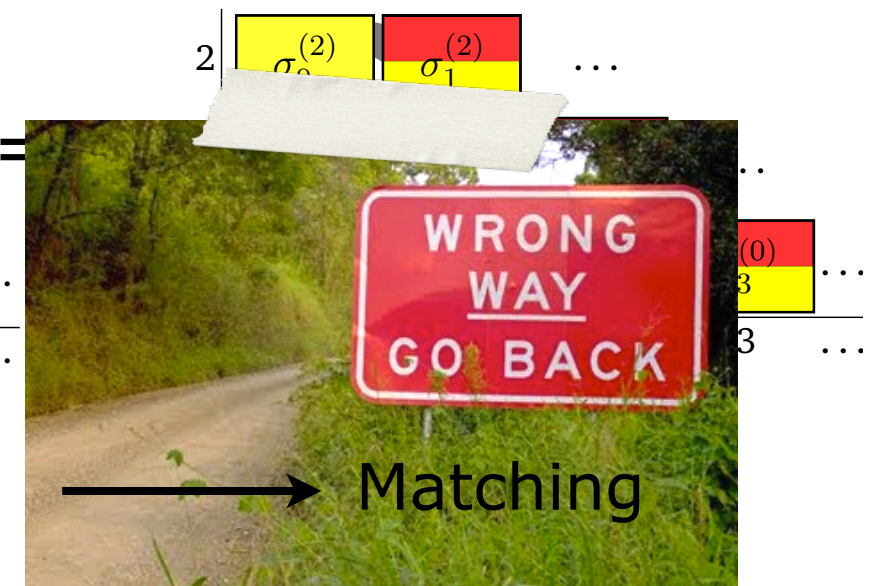
F @ LO×LL



F+1 @ LO×LL



F & F+1 @ LO×LL



See also: PS, *Introduction to QCD*, TASI 2012, [arXiv:1207.2389](https://arxiv.org/abs/1207.2389)

HOW NOT TO DO IT ... IN MORE DETAIL

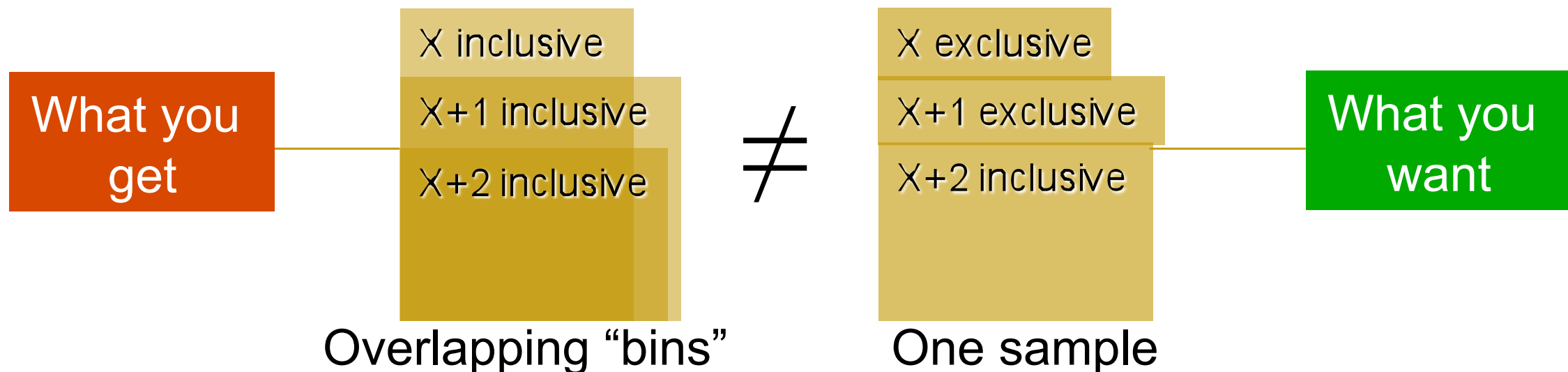
► A (Complete Idiot's) Solution – Combine

1. $[X]_{ME}$ + showering
2. $[X + 1 \text{ jet}]_{ME}$ + showering
3. ...

Run generator for X (+ shower)
Run generator for $X+1$ (+ shower)
Run generator for ... (+ shower)
Combine everything into one sample

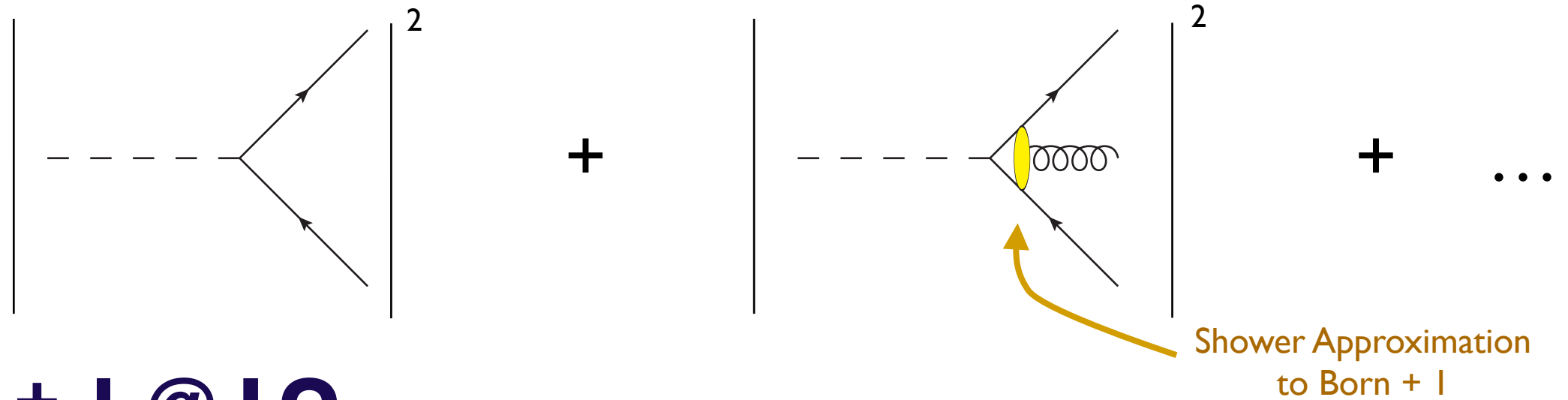
► Doesn't work

- $[X]$ + shower is inclusive
- $[X+1]$ + shower is also inclusive

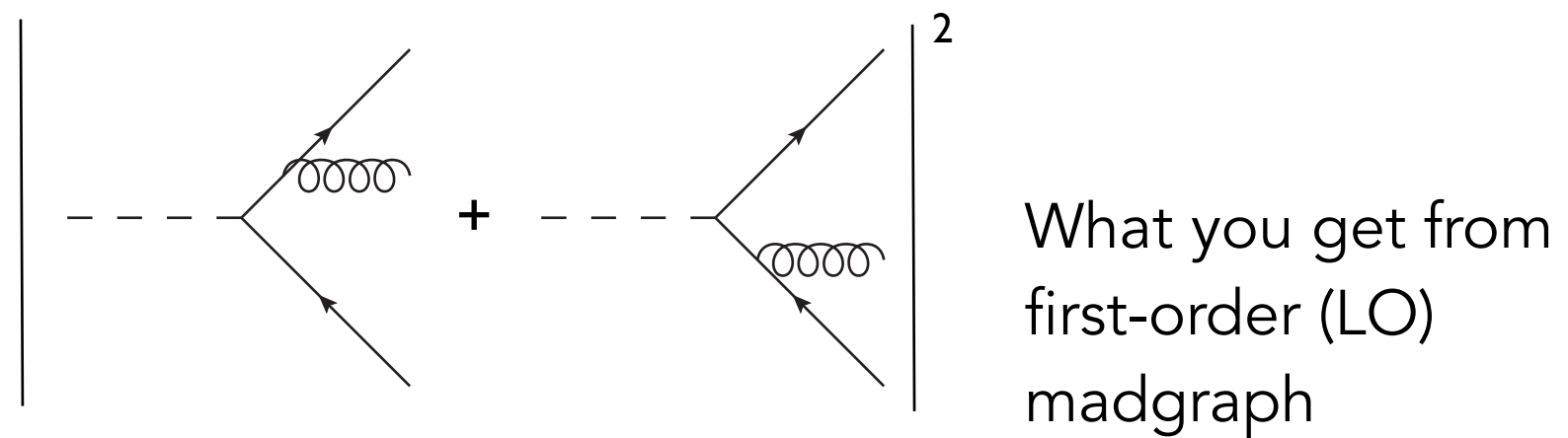


EXAMPLE: $H^0 \rightarrow b\bar{b}$.

Born + Shower

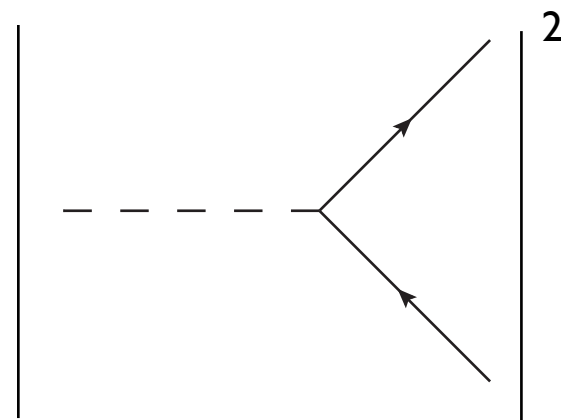


Born + 1 @ LO



EXAMPLE: $H^0 \rightarrow b\bar{b}$.

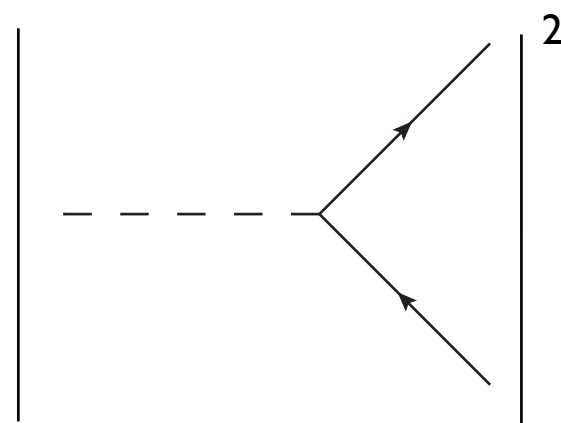
Born + Shower



$$\left(\mathbf{1} + \frac{g_s^2 2C_F \left[\frac{2s_{ik}}{s_{ij}s_{jk}} + \frac{1}{s_{IK}} \left(\frac{s_{ij}}{s_{jk}} + \frac{s_{jk}}{s_{ij}} \right) \right]}{\dots} + \dots \right)$$

Example of shower kernel
(here, used an "antenna function" for coherent gluon emission from a quark pair)

Born + 1 @ LO



$$\left(\frac{g_s^2 2C_F \left[\frac{2s_{ik}}{s_{ij}s_{jk}} + \frac{1}{s_{IK}} \left(\frac{s_{ij}}{s_{jk}} + \frac{s_{jk}}{s_{ij}} + 2 \right) \right]}{\dots} \right)$$

Example of matrix element;
what MG would give you

Total Overkill to add these two. All we really need is just that **+2** ...

1. MATRIX-ELEMENT CORRECTIONS

Bengtsson, Sjöstrand,
PLB 185 (1987) 435

Exploit freedom to choose non-singular terms

Modify parton shower to use process-dependent radiation functions for first emission → absorb real correction

$$\text{Parton Shower } \frac{P(z)}{Q^2} \rightarrow \frac{P'(z)}{Q^2} = \frac{P(z)}{Q^2} \underbrace{\frac{|M_{n+1}|^2}{\sum_i P_i(z)/Q_i^2 |M_n|^2}}_{\text{MEC}}$$

(suppressing α_s and Jacobian factors)

Process-dependent MEC → P' different for each process

Done in PYTHIA for all SM decays and many BSM ones

Based on systematic classification of spin/colour structures

Also used to account for mass effects, and for a few $2 \rightarrow 2$ procs

Norrbin, Sjöstrand,
NPB 603 (2001) 297

Difficult to generalise beyond one emission

Parton-shower expansions complicated & can have “dead zones”

Achieved in VINCIA (by devising showers that have simple expansions)

Only recently done for hadron collisions

Giele, Kosower, Skands, PRD 84 (2011) 054003
Fischer et al, arXiv:1605.06142

MECS WITH LOOPS: POWHEG

Acronym stands for: **P**ositive **W**eight **H**ardest **E**mission **G**enerator.

Start at Born level

$$|M_F|^2$$

Generate "shower" emission

$$|M_{F+1}|^2 \stackrel{LL}{\sim} \sum_{i \in \text{ant}} a_i |M_F|^2$$

Correct to Matrix Element

$$a_i \rightarrow \frac{|M_{F+1}|^2}{\sum a_i |M_F|^2} a_i$$

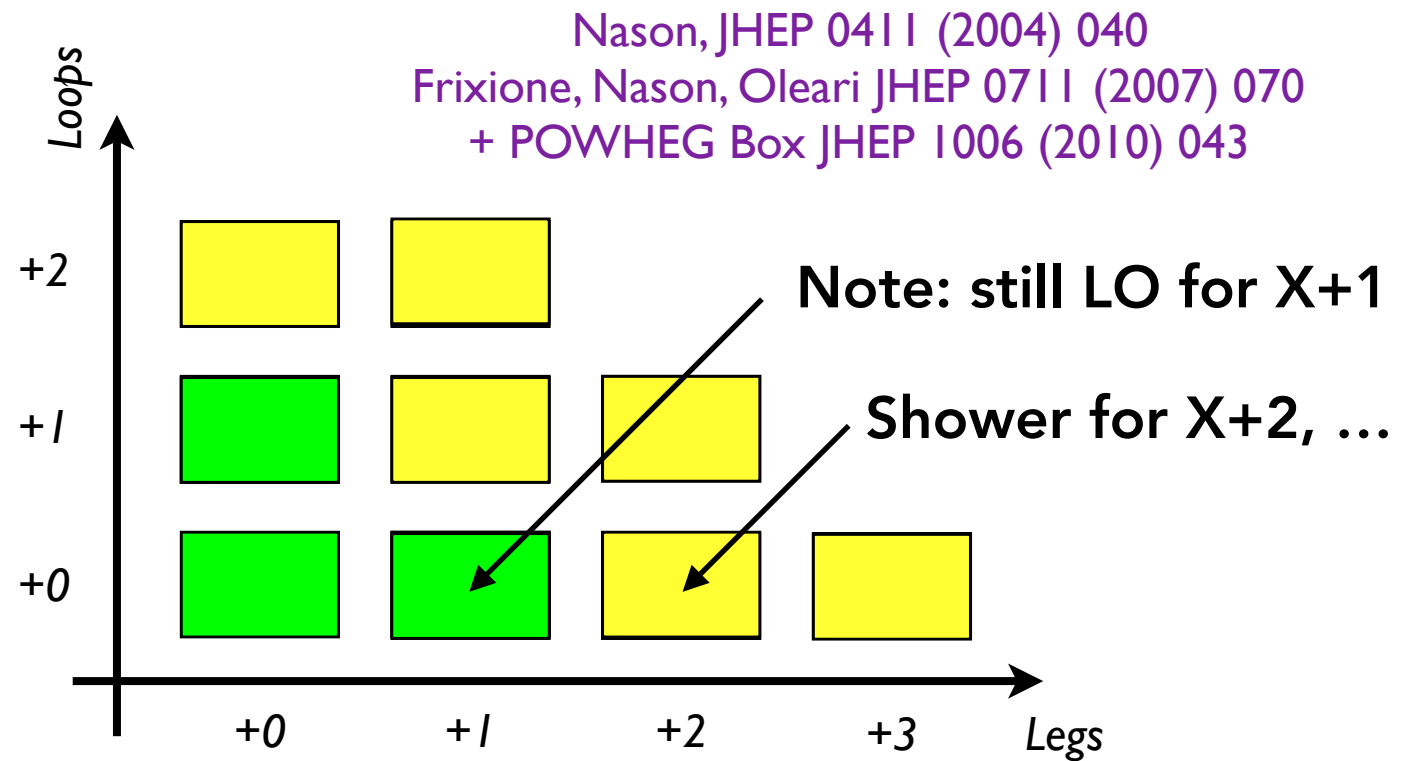
Unitarity of Shower

$$\text{Virtual} = - \int \text{Real}$$

Correct to Matrix Element

$$|M_F|^2 \rightarrow |M_F|^2 + 2\text{Re}[M_F^1 M_F^0] + \int \text{Real}$$

Repeat: ordinary parton shower



Method is widely applied/available, can be used with PYTHIA, HERWIG, SHERPA

Subtlety 1: Connecting with parton shower

Truncated Showers & Vetoed Showers

Subtlety 2: Avoiding (over)exponentiation of hard radiation

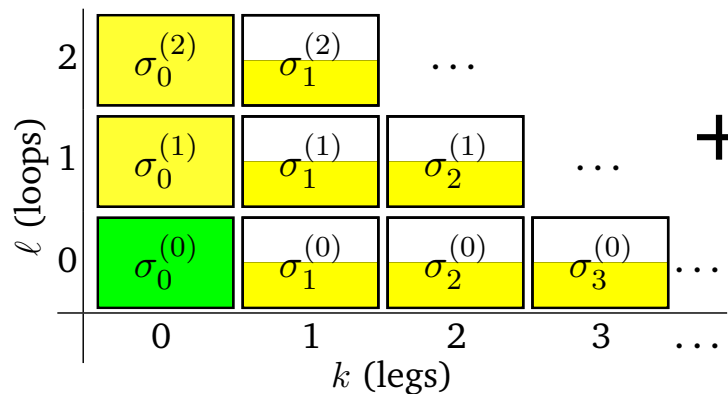
Controlled by "hFact" parameter (POWHEG)

2: SLICING (MLM & CKKW-L)

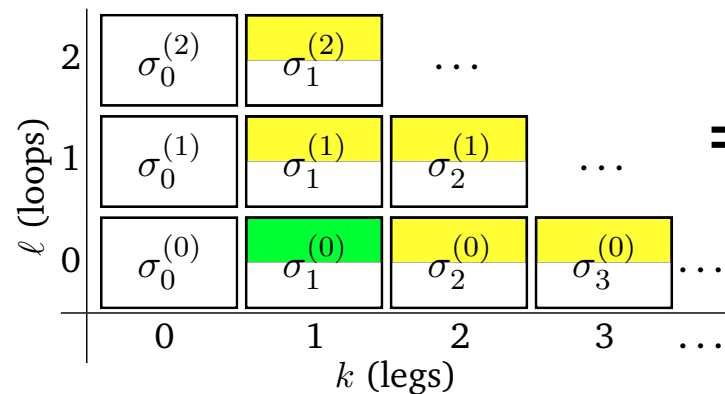
First emission: “the HERWIG correction”

Use the fact that the angular-ordered HERWIG parton shower has a “dead zone” for hard wide-angle radiation (Seymour, 1995)

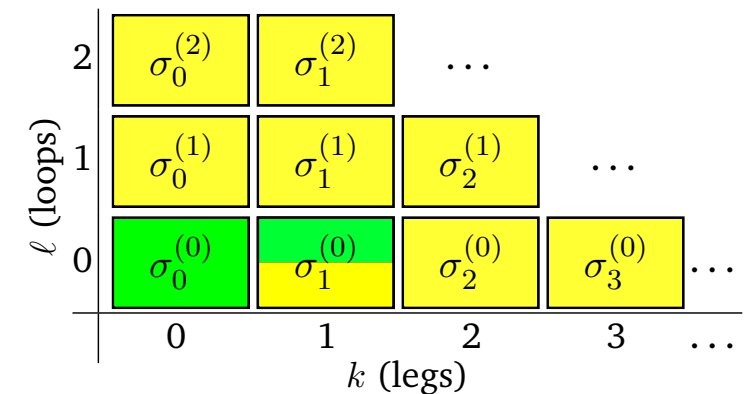
F @ LO×LL-Soft (HERWIG Shower)



F+1 @ LO×LL (HERWIG Corrections)

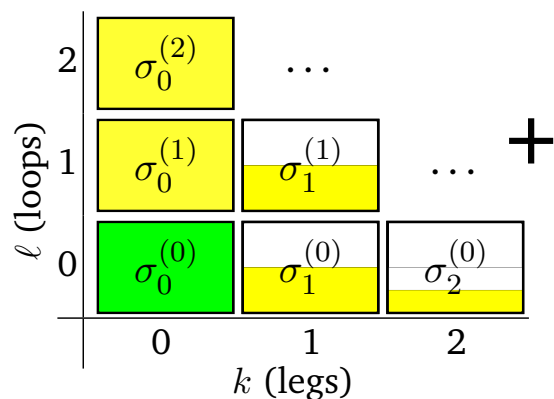


F @ LO₁×LL (HERWIG Matched)

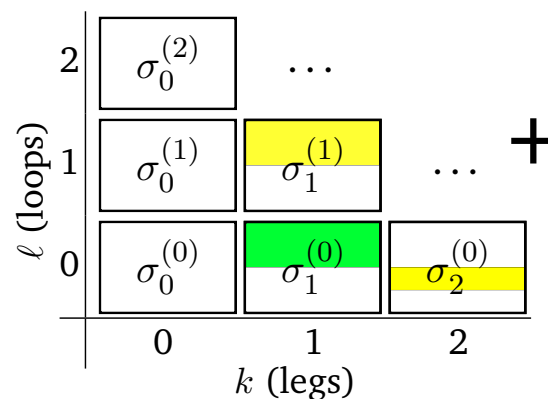


Many emissions: the MLM & CKKW-L prescriptions

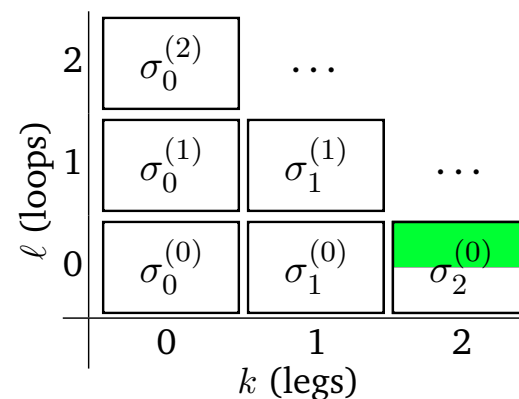
F @ LO×LL-Soft (excl)



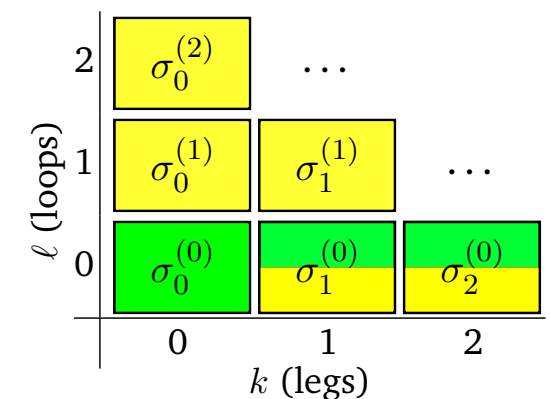
F+1 @ LO×LL-Soft (excl)



F+2 @ LO×LL (incl)



F @ LO₂×LL (MLM & (L)-CKKW)



(CKKW & Lönnblad, 2001)

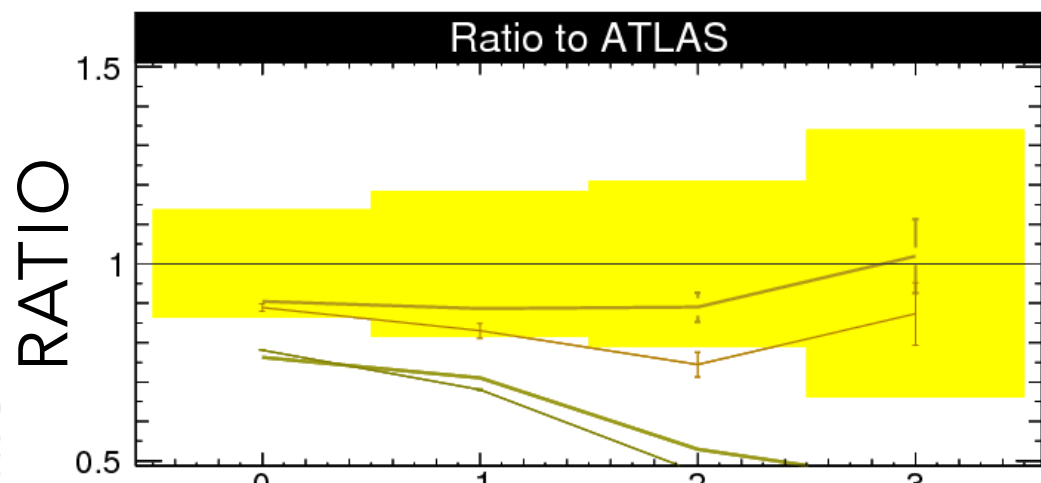
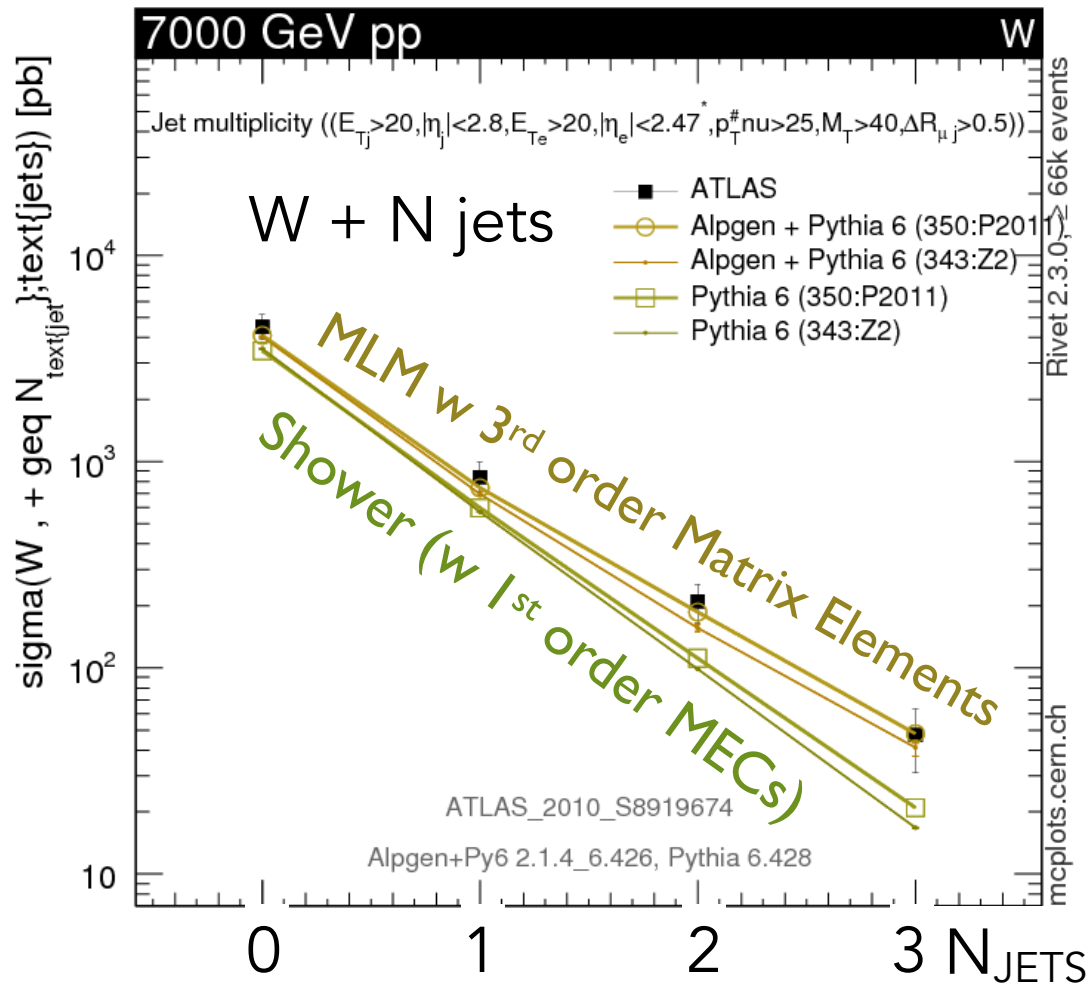
(Mangano, 2002)

(+many more recent; see Alwall et al., EPJC53(2008)473)

THE GAIN

THE COST

Example: LHC₇ : W + 20-GeV Jets

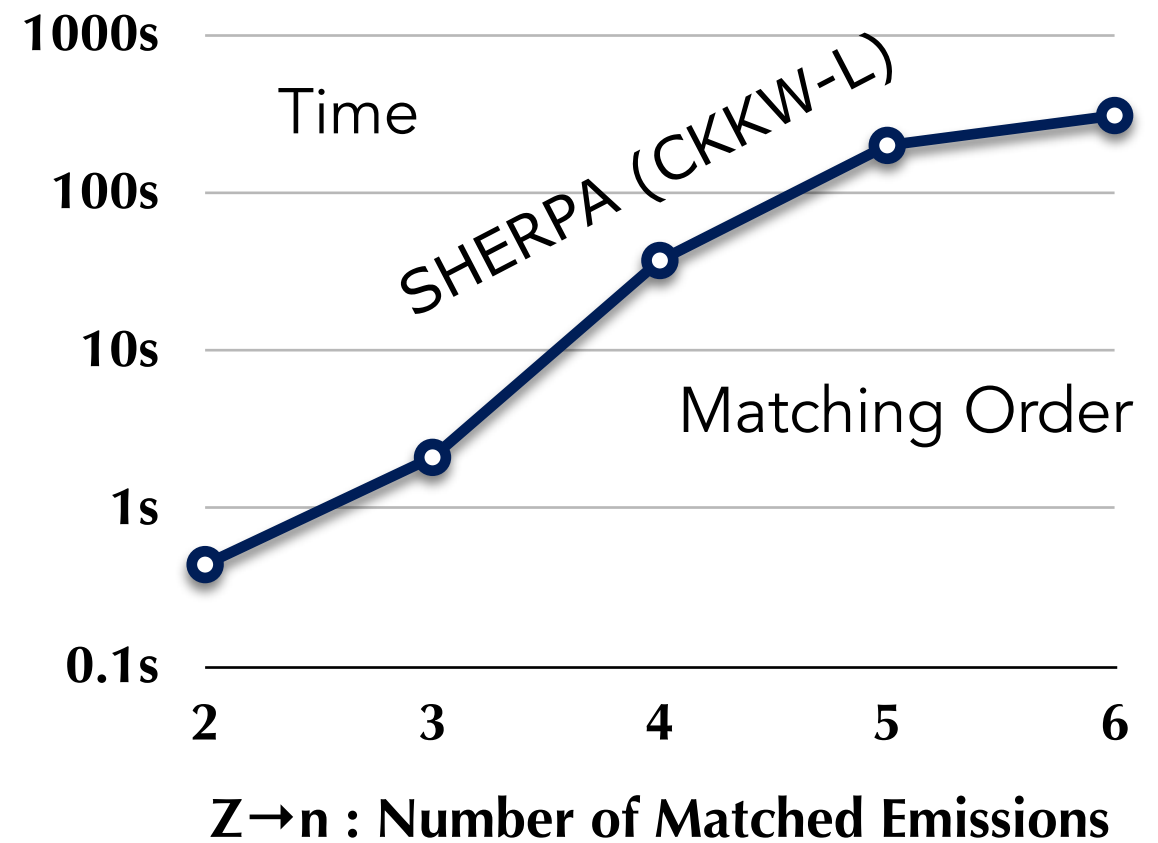


Plot from mcplots.cern.ch; see arXiv:1306.3436

Example: $e^+e^- \rightarrow Z \rightarrow$ Jets

2. Time to generate 1000 events
(Z → partons, fully showered & matched. No hadronization.)

1000 SHOWERS

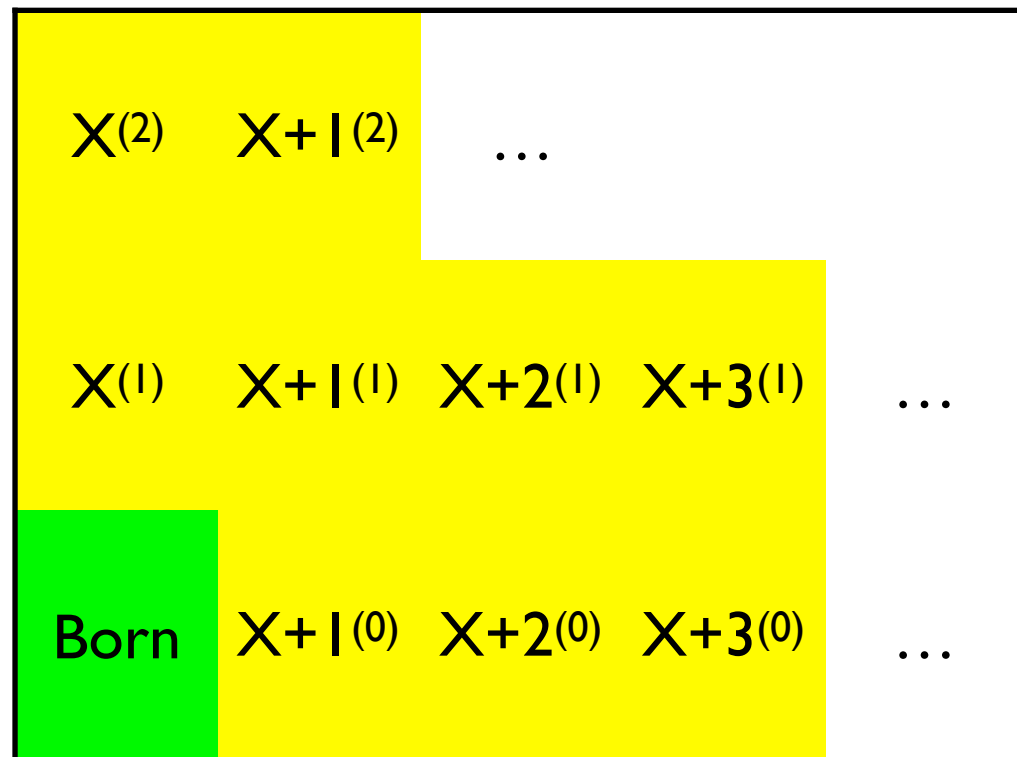


See e.g. Lopez-Villarejo & Skands, arXiv:1109.3608

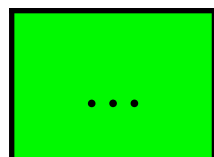
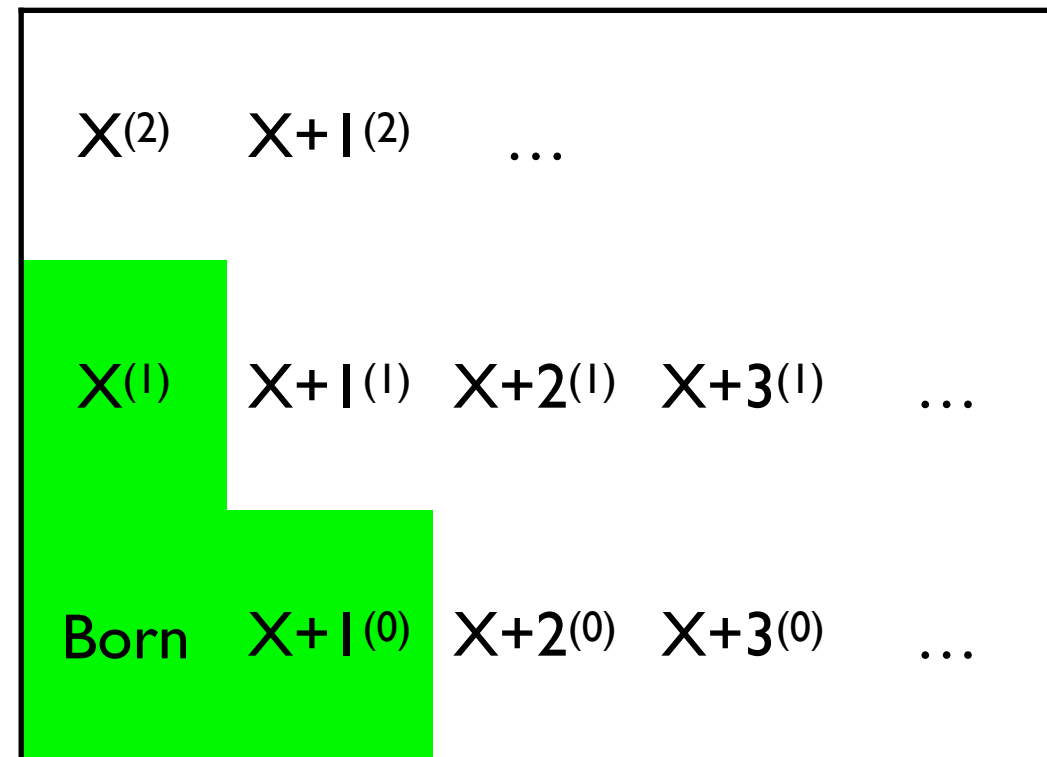
3: SUBTRACTION

Examples: MC@NLO, aMC@NLO

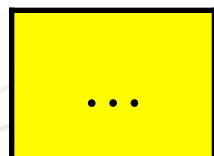
LO × Shower



NLO



Fixed-Order Matrix Element

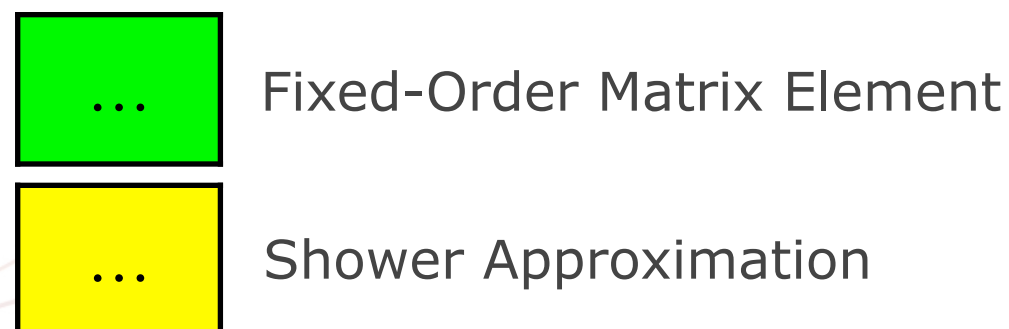
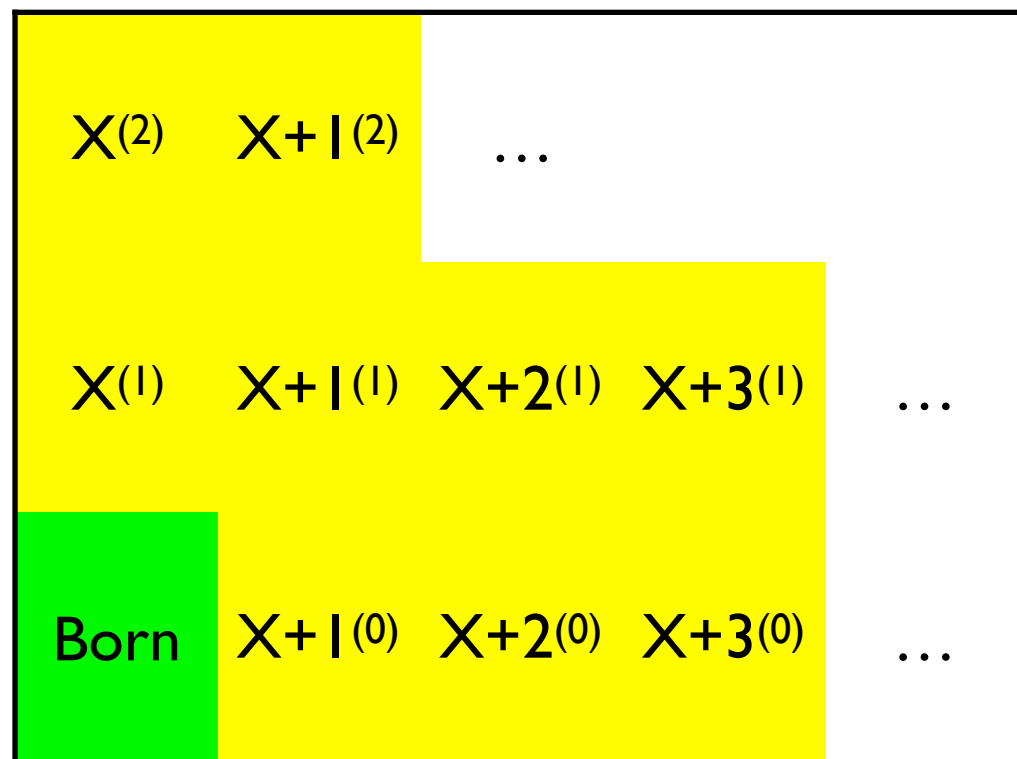


Shower Approximation

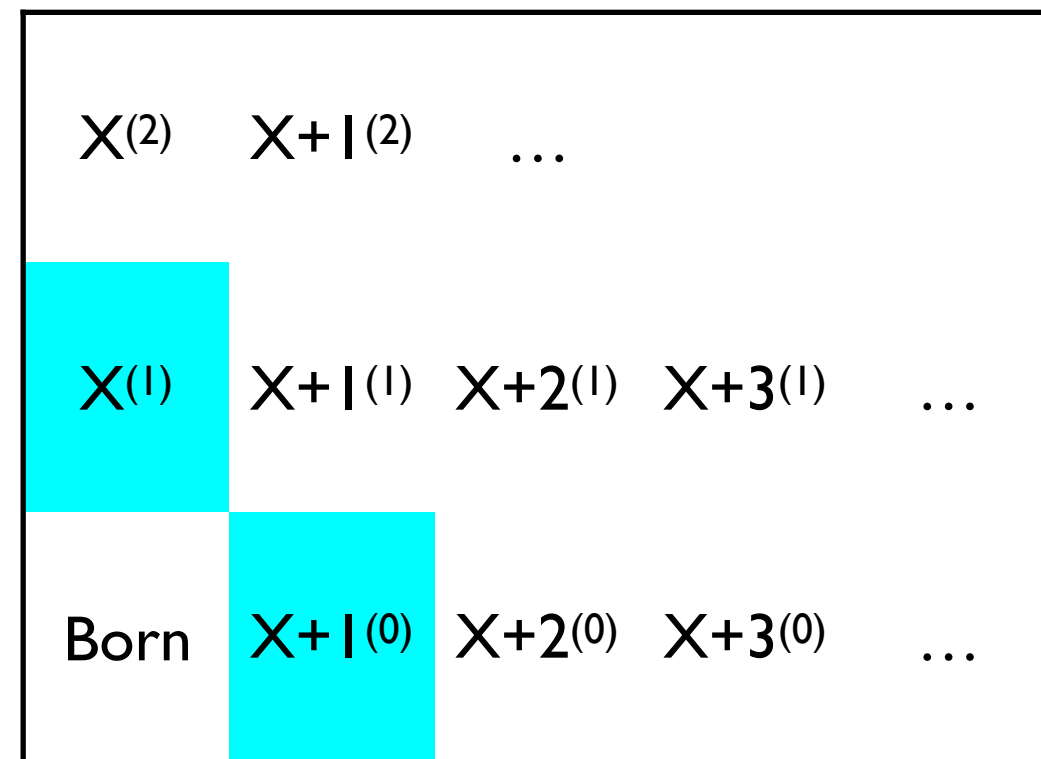
MATCHING 3: SUBTRACTION

Examples: MC@NLO, aMC@NLO

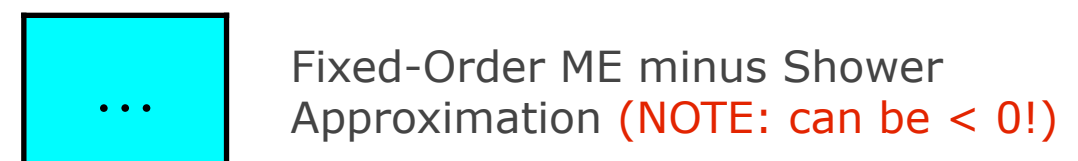
LO \times Shower



NLO - Shower_{NLO}



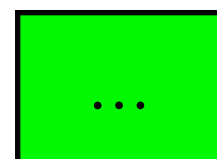
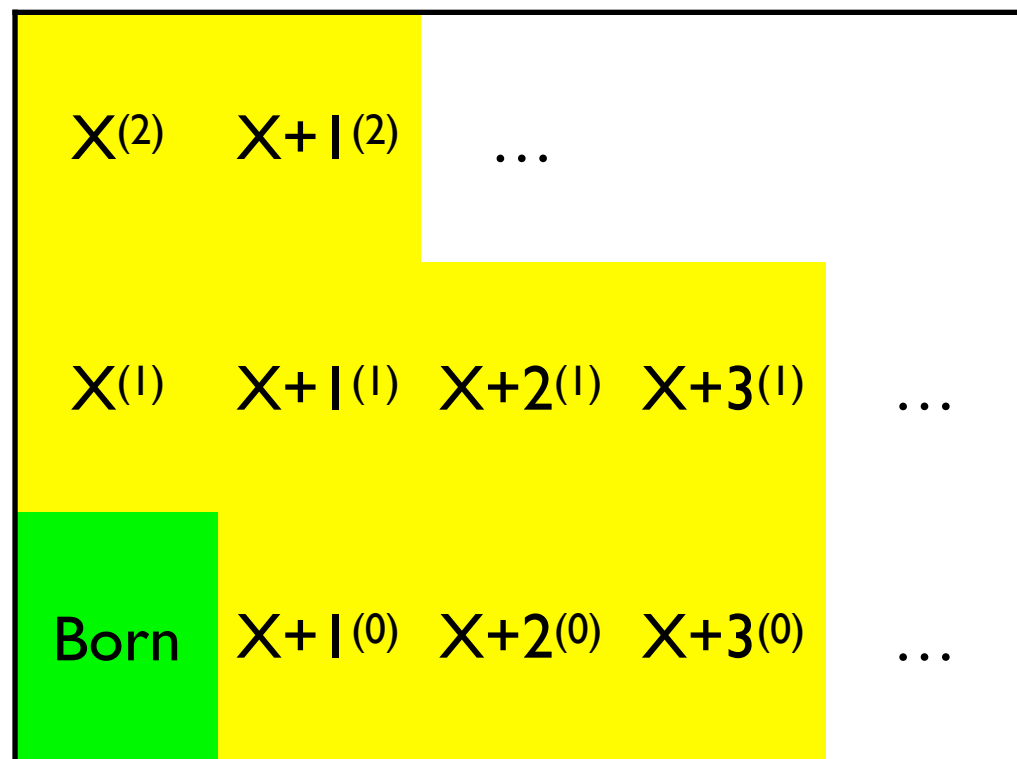
Expand shower approximation to NLO analytically, then subtract:



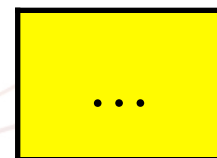
MATCHING 3: SUBTRACTION

Examples: MC@NLO, aMC@NLO

LO \times Shower

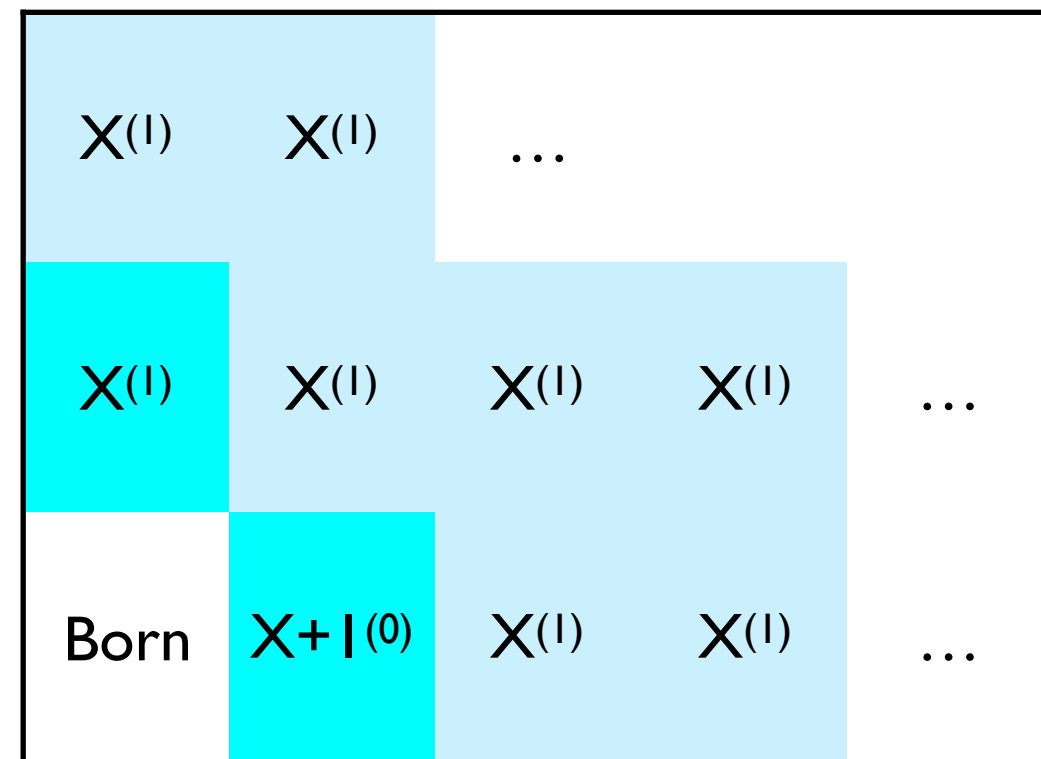


Fixed-Order Matrix Element

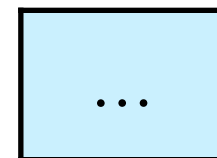


Shower Approximation

(NLO - Shower_{NLO}) \times Shower



Fixed-Order ME minus Shower Approximation (NOTE: can be < 0!)



Subleading corrections generated by shower off subtracted ME

MATCHING 3: SUBTRACTION

Combine ➤ MC@NLO

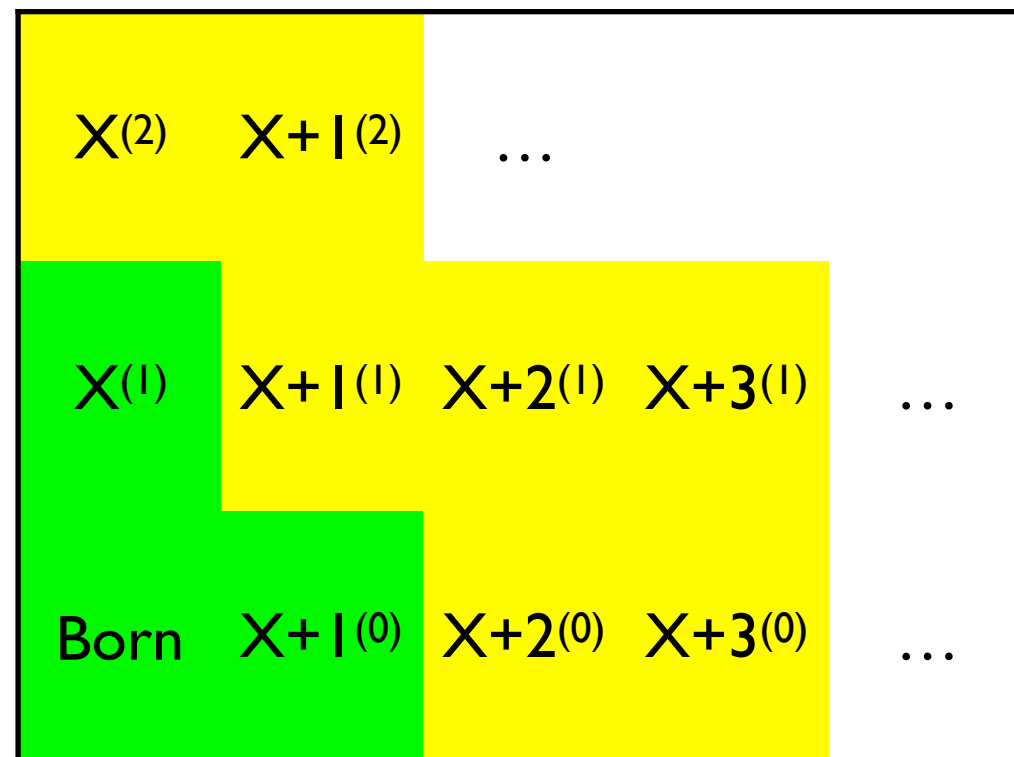
Examples: MC@NLO, aMC@NLO

Frixione, Webber, JHEP 0206 (2002) 029

Consistent NLO + parton shower (though correction events can have $w < 0$)

Recently, has been fully automated in **aMC@NLO**

Frederix, Frixione, Hirschi, Maltoni, Pittau, Torrielli, JHEP 1202 (2012) 048



NB: $w < 0$ are a problem because they kill efficiency:

Extreme example: 1000 positive-weight - 999 negative-weight events → statistical precision of 1 event, for 2000 generated (for comparison, normal MC@NLO has ~ 10% neg-weights)

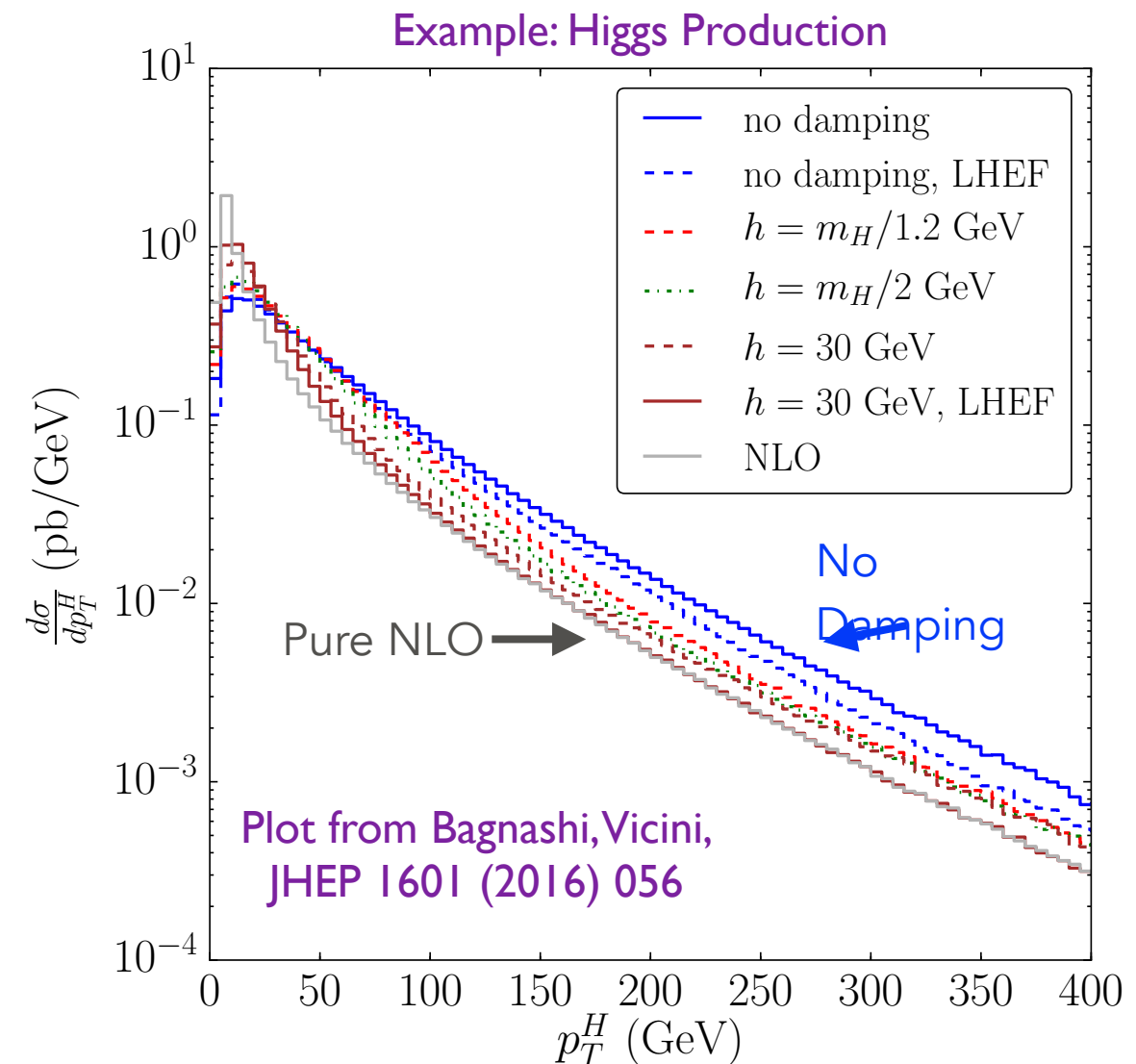
POWHEG VS MC@NLO

Both methods include the complete first-order (NLO) matrix elements.

Difference is in whether **only** the shower kernels are exponentiated (MC@NLO) or whether part of the matrix-element corrections are too (POWHEG)

In POWHEG, how much of the MEC you exponentiate can be controlled by the "hFact" parameter

Variations basically span range between MC@NLO-like case, and original (hFact=1) POWHEG case (~PYTHIA-style MECs)



$$D_h = \frac{h^2}{h^2 + (p_{\perp}^H)^2}$$

$$R^s = D_h R_{\text{div}} \quad R^f = (1 - D_h) R_{\text{div}}$$

exponentiated not exponentiated

(MULTI-LEG MERGING AT NLO)

Currently, much activity on how to combine several NLO matrix elements for the same process: NLO for $X, X+1, X+2, \dots$

Unitarity is a common main ingredient for all of them

Most also employ **slicing** (separating phase space into regions defined by one particular underlying process)

Methods

UNLOPS, generalising CKKW-L/UMEPS: [Lonnblad, Prestel, arXiv:1211.7278](#)

MiNLO, based on POWHEG: [Hamilton, Nason, Zanderighi \(+more\)](#)

[arXiv:1206.3572,](#)
[arXiv:1512.02663](#)

FxFx, based on MC@NLO: [Frederix & Frixione, arXiv:1209.6215](#)

(VINCIA, based on NLO MECs): [Hartgring, Laenen, Skands, arXiv:1303.4974](#)

Most (all?) of these also allow NNLO on total inclusive cross section

Will soon define the state-of-the-art for SM processes

For BSM, the state-of-the-art is generally one order less than SM

SUMMARY: MATCHING AND MERGING

The Problem:

Showers generate singular parts of (all) higher-order matrix elements

Those terms are of course also present in $X + \text{jet(s)}$ matrix elements

To combine, must be careful not to count them twice! (double counting)

3 Main Methods

1. Matrix-Element Corrections (MECs): **multiplicative correction factors**

Pioneered in PYTHIA (mainly for real radiation \Rightarrow LO MECs)

Similar method used in POWHEG (with virtual corrections \Rightarrow NLO)

Generalised to multiple branchings: VINCIA

2. Slicing: **separate phase space** into two regions: ME populates high- Q region, shower populates low- Q region (and calculates Sudakov factors)

CKKW-L (pioneered by SHERPA) & **MLM** (pioneered by ALPGEN)

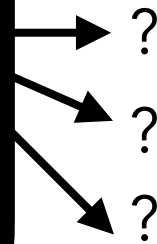
3. **Subtraction:** MC@NLO, now automated: aMC@NLO

State-of-the-art \blacktriangleright **Multi-Leg NLO** (UNLOPS, MiNLO, FxFx)

QUIZ: CONNECT THE BOXES

1

Ambiguity about how much of the nonsingular parts of the ME that get exponentiated; controlled by:
hFact



A

Matrix-Element Corrections (MECs)

2

Procedure can lead to a substantial fraction of events having:
Negative Weights

B

CKKW-L & MLM

3

Ambiguity about definition of which events "count" as hard N-jet events; controlled by:
Merging Scale

C

MC@NLO

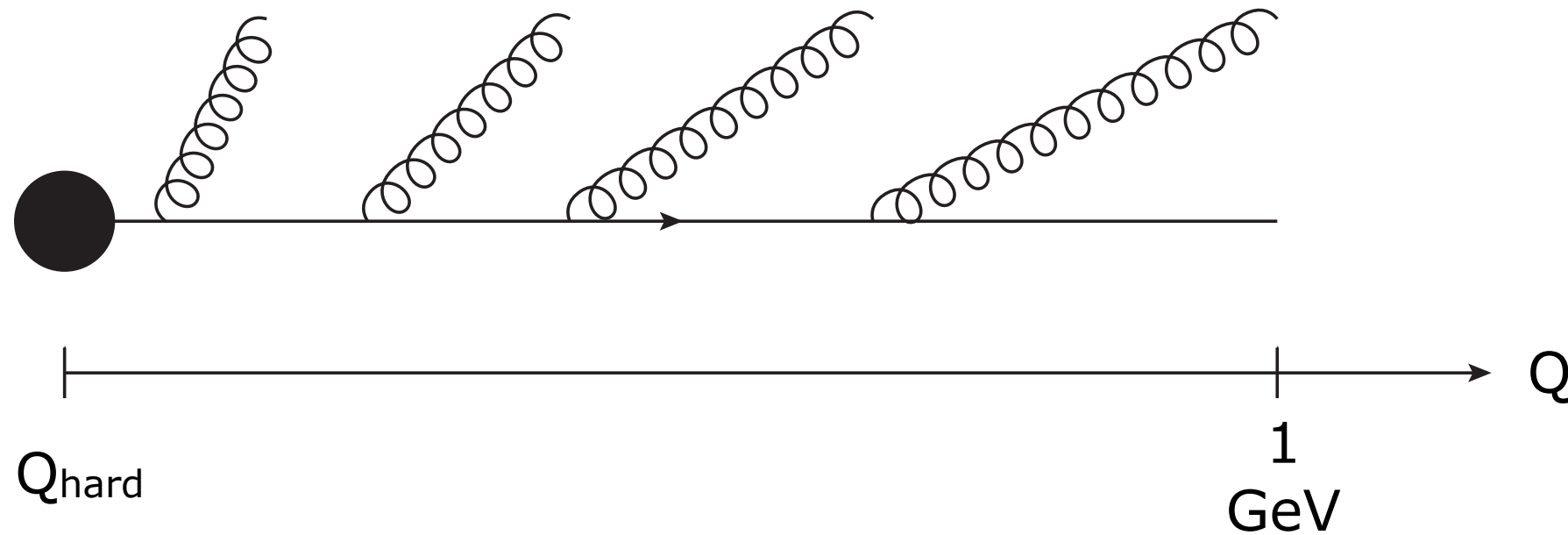
FROM PARTONS TO PIONS

Here's a fast parton

Fast: It starts at a high factorization scale
 $Q = Q_F = Q_{\text{hard}}$

It showers
(bremsstrahlung)

It ends up
at a low effective
factorization scale
 $Q \sim m_p \sim 1 \text{ GeV}$



FROM PARTONS TO PIONS

Here's a fast parton

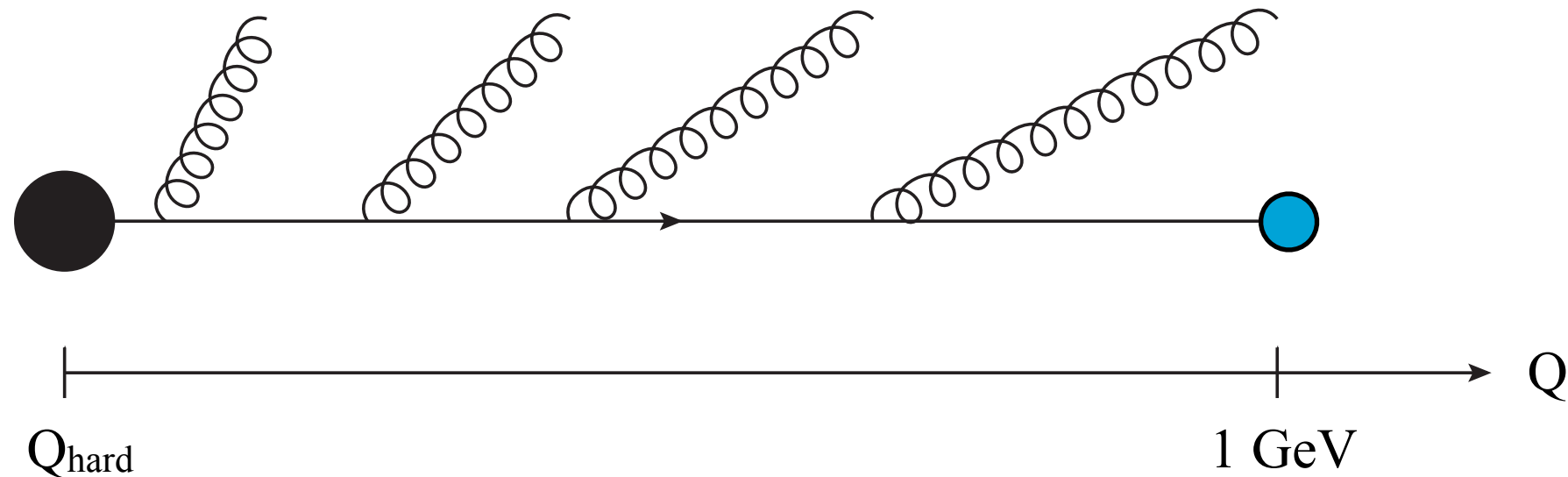
Fast: It starts at a high factorization scale

$$Q = Q_F = Q_{\text{hard}}$$

It showers
(bremsstrahlung)

It ends up
at a low effective
factorization scale

$$Q \sim m_p \sim 1 \text{ GeV}$$



How about I just call it a hadron?

→ "Local Parton-Hadron Duality"

PARTON → HADRONS?

Early models: “Independent Fragmentation”

Local Parton Hadron Duality (LPHD) can give useful results for inclusive quantities in collinear fragmentation

Motivates a simple model:



But ...

The point of confinement is that partons are coloured

Hadronisation = the process of **colour neutralisation**

→ Unphysical to think about independent fragmentation of a single parton into hadrons

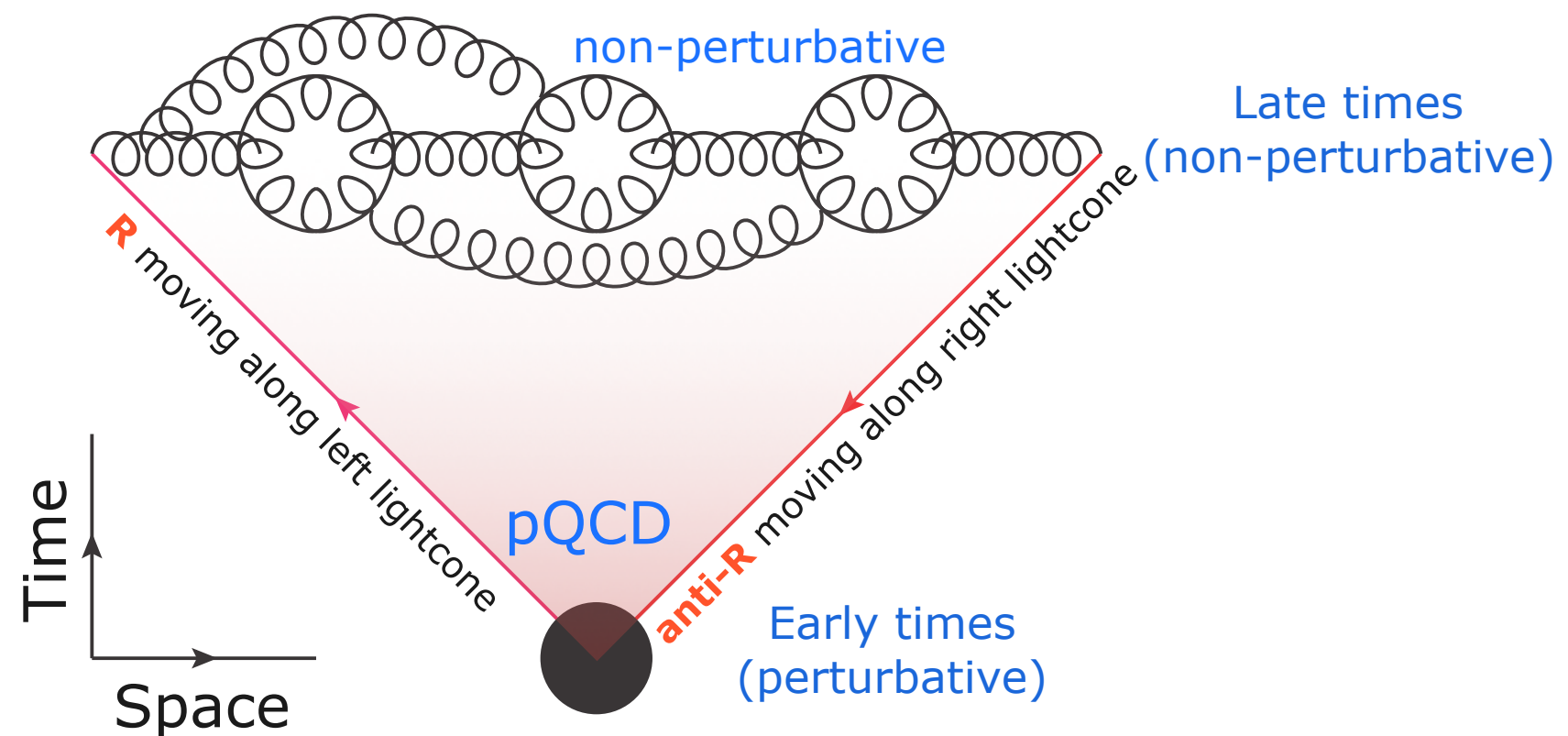
→ Too naive to see LPHD (inclusive) as a justification for Independent Fragmentation (exclusive)

→ More physics needed

COLOUR NEUTRALISATION

A physical hadronization model

Should involve at least TWO partons, with opposite color charges (e.g., **R** and **anti-R**)



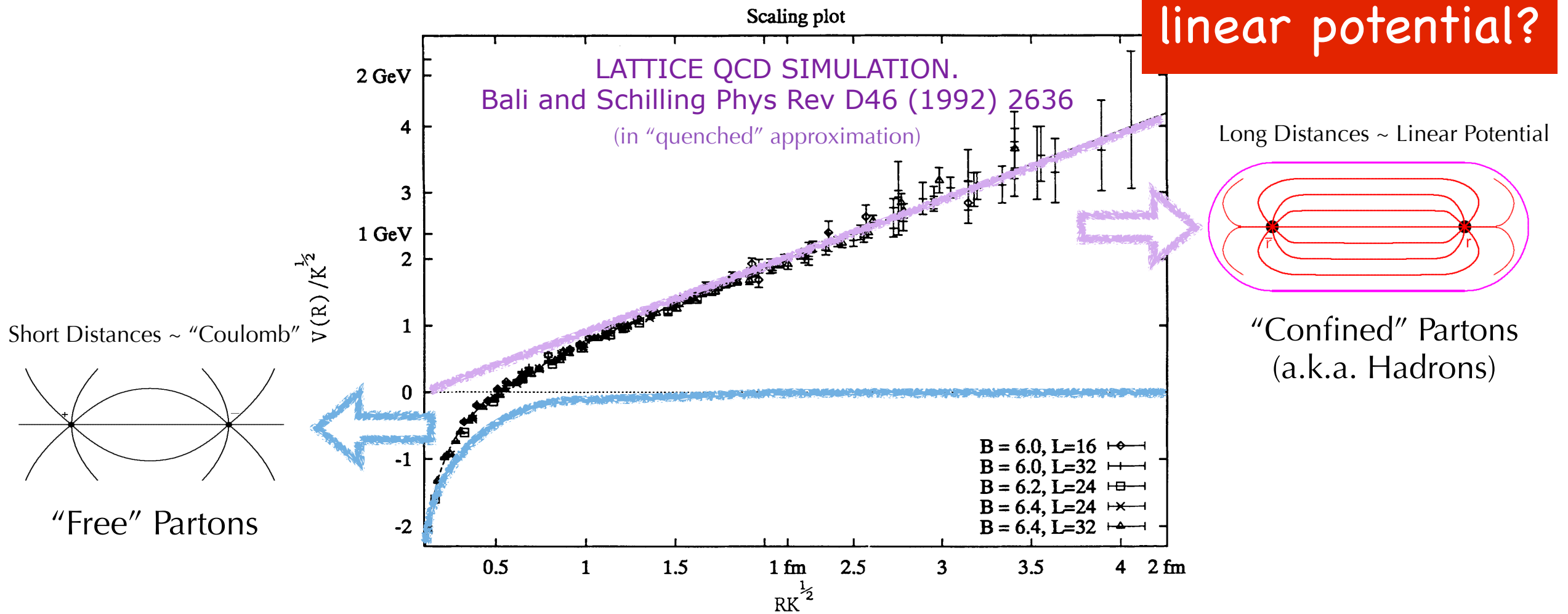
Strong “confining” field emerges between the two charges when their separation $> \sim 1\text{fm}$

THE ULTIMATE LIMIT: WAVELENGTHS $> 10^{-15}$ M

Quark-Antiquark Potential

As function of separation distance

What physical system has a linear potential?



$$F(r) \approx \text{const} = \kappa \approx 1 \text{ GeV/fm} \iff V(r) \approx \kappa r$$

~ Force required to lift a 16-ton truck

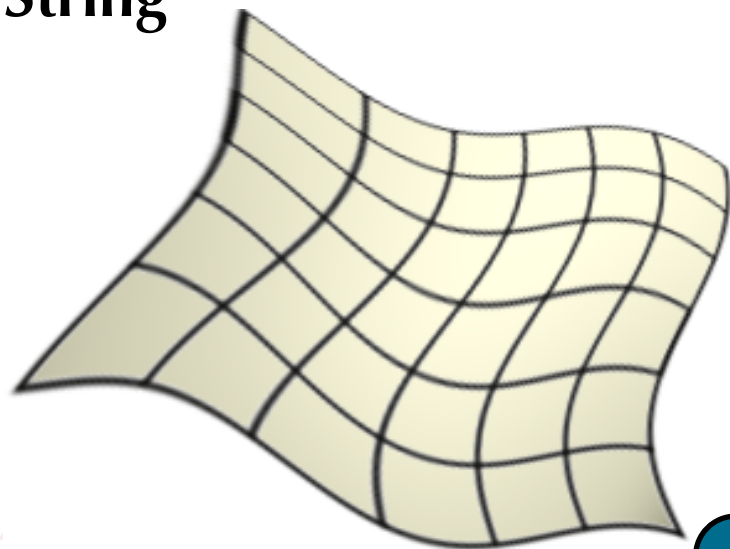
FROM PARTONS TO STRINGS

Motivates a model:

Let color field collapse into a (infinitely) narrow flux tube of uniform energy density $\kappa \sim 1 \text{ GeV / fm}$

→ Relativistic 1+1 dimensional worldsheet

String



Pedagogical Review: B. Andersson, *The Lund model*. Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol., 1997.

In “unquenched” QCD

$g \rightarrow qq \rightarrow$ The strings will break

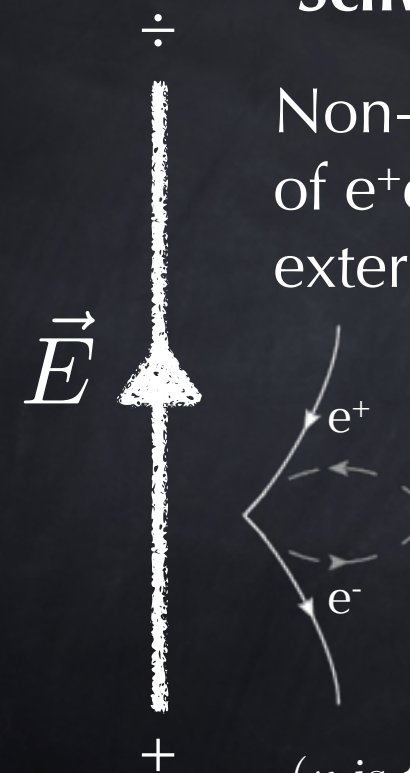
Schwinger Effect

Non-perturbative creation of e^+e^- pairs in a strong external Electric field

Probability from Tunneling Factor

$$\mathcal{P} \propto \exp\left(\frac{-m^2 - p_{\perp}^2}{\kappa/\pi}\right)$$

(κ is the string tension equivalent)



→ Gaussian p_T spectrum

Heavier quarks suppressed. Prob($q=d,u,s,c$) $\approx 1 : 1 : 0.2 : 10^{-11}$

(NOTE ON THE LENGTH OF STRINGS)

In Space:

String tension $\approx 1 \text{ GeV/fm} \rightarrow$ a 5-GeV quark can travel 5 fm before all its kinetic energy is transformed to potential energy in the string.

Then it must start moving the other way. String breaks will have happened behind it \rightarrow yo-yo model of mesons

In Rapidity :

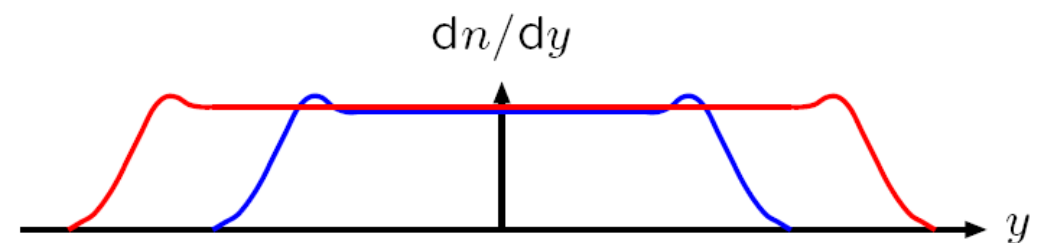
$$y = \frac{1}{2} \ln \left(\frac{E + p_z}{E - p_z} \right) = \frac{1}{2} \ln \left(\frac{(E + p_z)^2}{E^2 - p_z^2} \right)$$

For a pion with $z=1$ along string direction
(For beam remnants, use a proton mass):

$$y_{\text{max}} \sim \ln \left(\frac{2E_q}{m_\pi} \right)$$

Note: Constant average hadron multiplicity per unit $y \rightarrow$ logarithmic growth of total multiplicity

Scaling in lightcone $p_\pm = E \pm p_z$ (for $q\bar{q}$ system along z axis) implies flat central rapidity plateau + some endpoint effects:

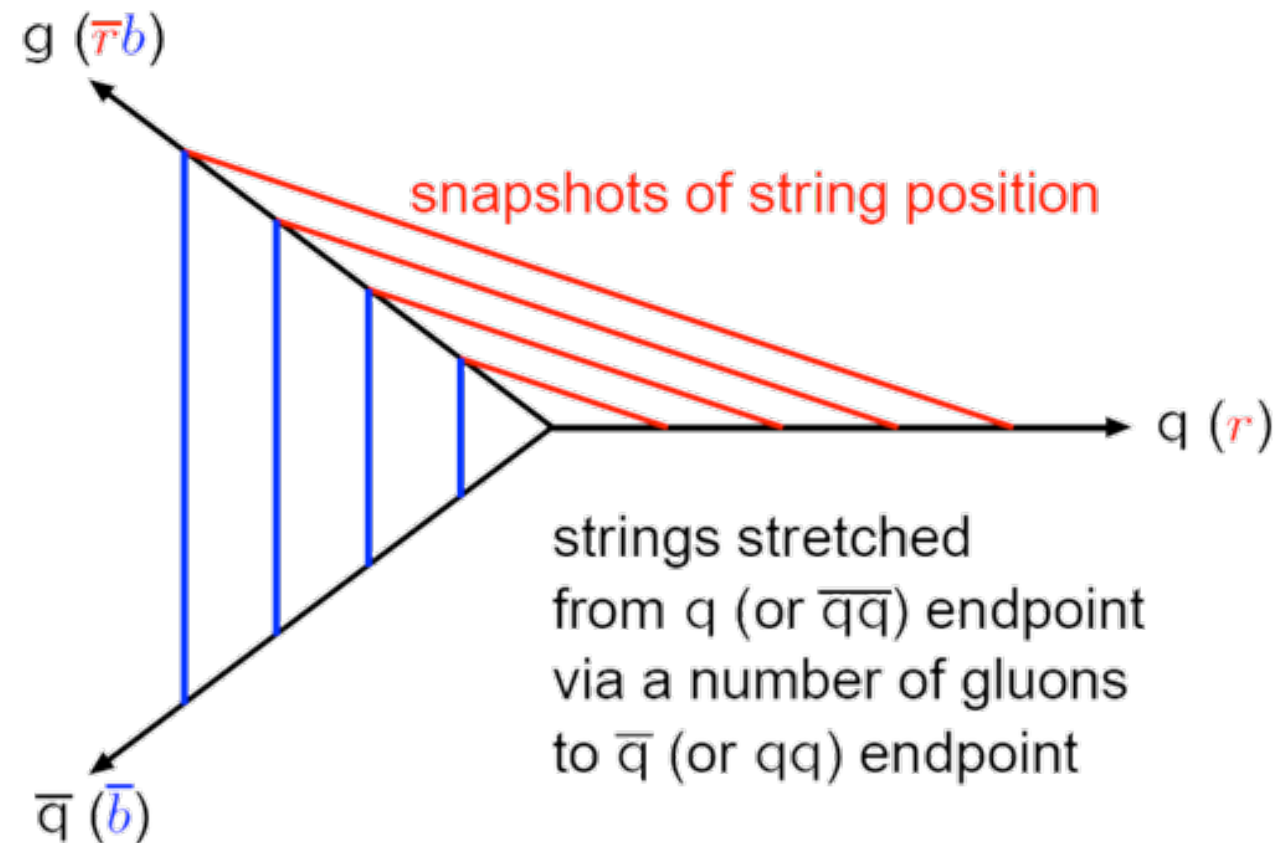


$$\langle n_{\text{ch}} \rangle \approx c_0 + c_1 \ln E_{\text{cm}}, \sim \text{Poissonian multiplicity distribution}$$

THE (LUND) STRING MODEL

Map:

- **Quarks** → String Endpoints
- **Gluons** → Transverse Excitations (kinks)
- Physics then in terms of string worldsheet evolving in spacetime
- Probability of string break (by quantum tunneling) constant per unit area → **AREA LAW**



Gluon = kink on string, carrying energy and momentum
→ **STRING EFFECT**

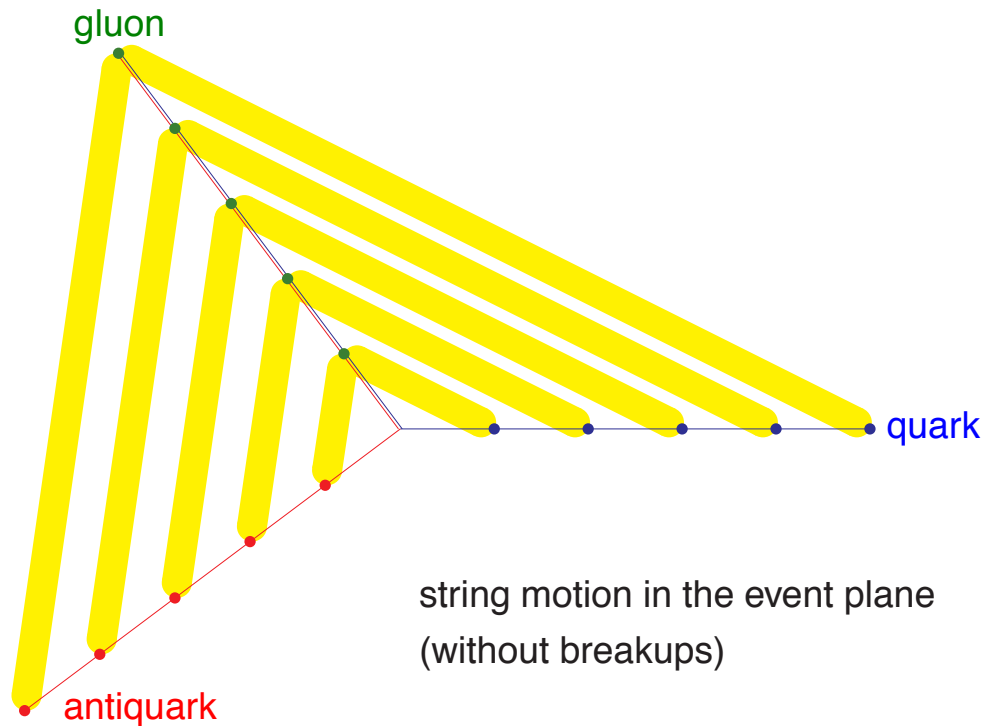
Simple space-time picture

Details of string breaks more complicated (e.g., baryons, spin multiplets)

DIFFERENCES BETWEEN QUARK AND GLUON JETS

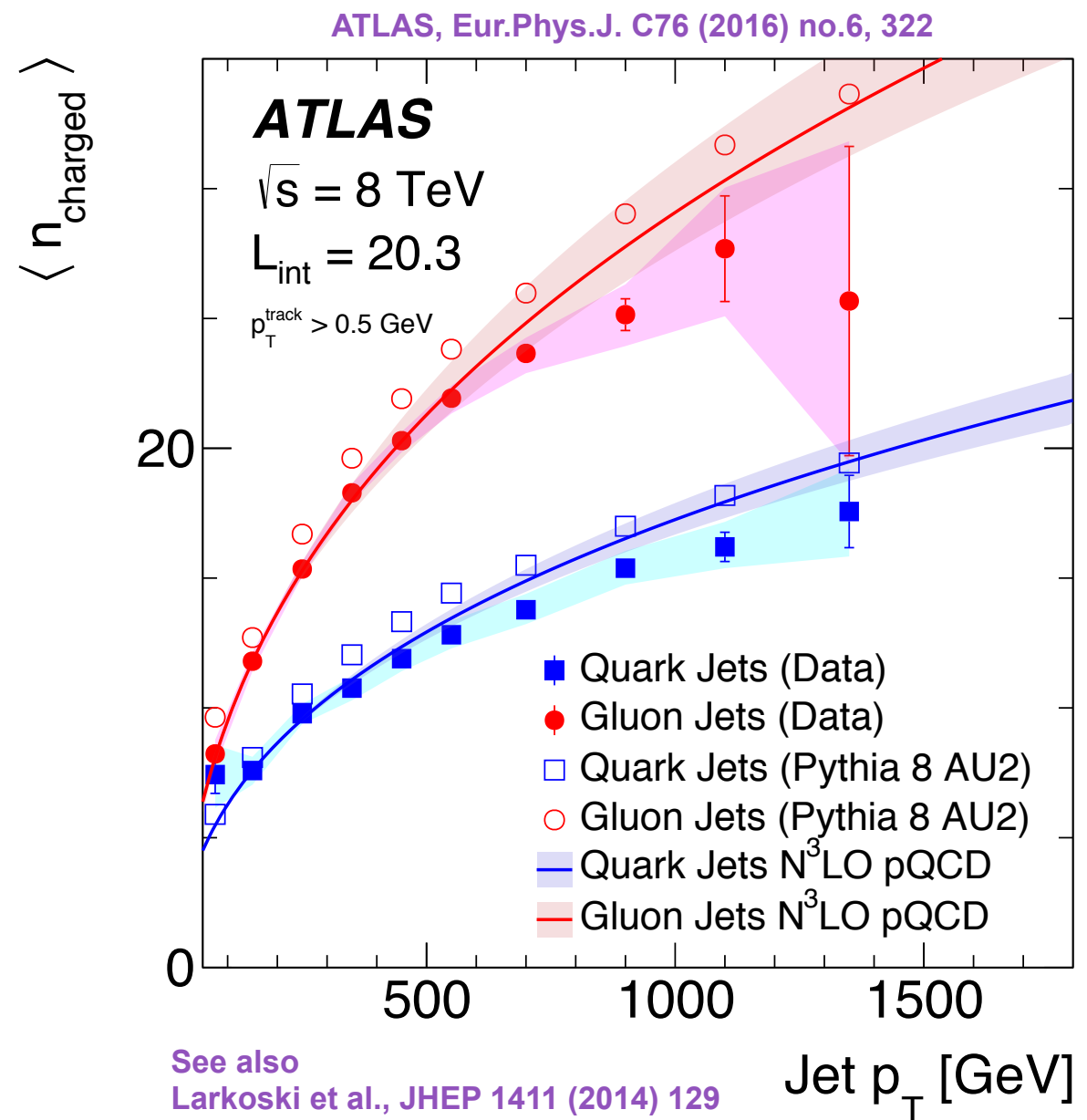
Recent "hot topic": Q/G Discrimination

Gluon connected to two string pieces



Each quark connected to one string piece

→ expect factor $2 \sim C_A/C_F$ larger particle multiplicity in gluon jets vs quark jets



See also

Larkoski et al., JHEP 1411 (2014) 129

Thaler et al., Les Houches, arXiv:1605.04692

Can be hugely important for discriminating new-physics signals (decays to quarks vs decays to gluons, vs composition of background and bremsstrahlung combinatorics)

➤ EVENT GENERATORS

Aim: generate events in as much detail as mother nature

→ Make stochastic choices ~ as in Nature (Q.M.) → Random numbers

Factor complete event probability into separate universal pieces, treated independently and/or sequentially (Markov-Chain MC)

Improve lowest-order (perturbation) theory by including 'most significant' corrections

Resonance decays (e.g., $t \rightarrow bW^+$, $W \rightarrow qq'$, $H^0 \rightarrow \gamma^0 \gamma^0$, $Z^0 \rightarrow \mu^+ \mu^-$, ...)

Bremsstrahlung (FSR and ISR, exact in collinear and soft* limits)

Hard radiation (matching & merging)

Hadronization (strings / clusters)

Additional Soft Physics: multiple parton-parton interactions, Bose-Einstein correlations, colour reconnections, hadron decays, ...

Coherence*

Soft radiation → Angular ordering or Coherent Dipoles/Antennae