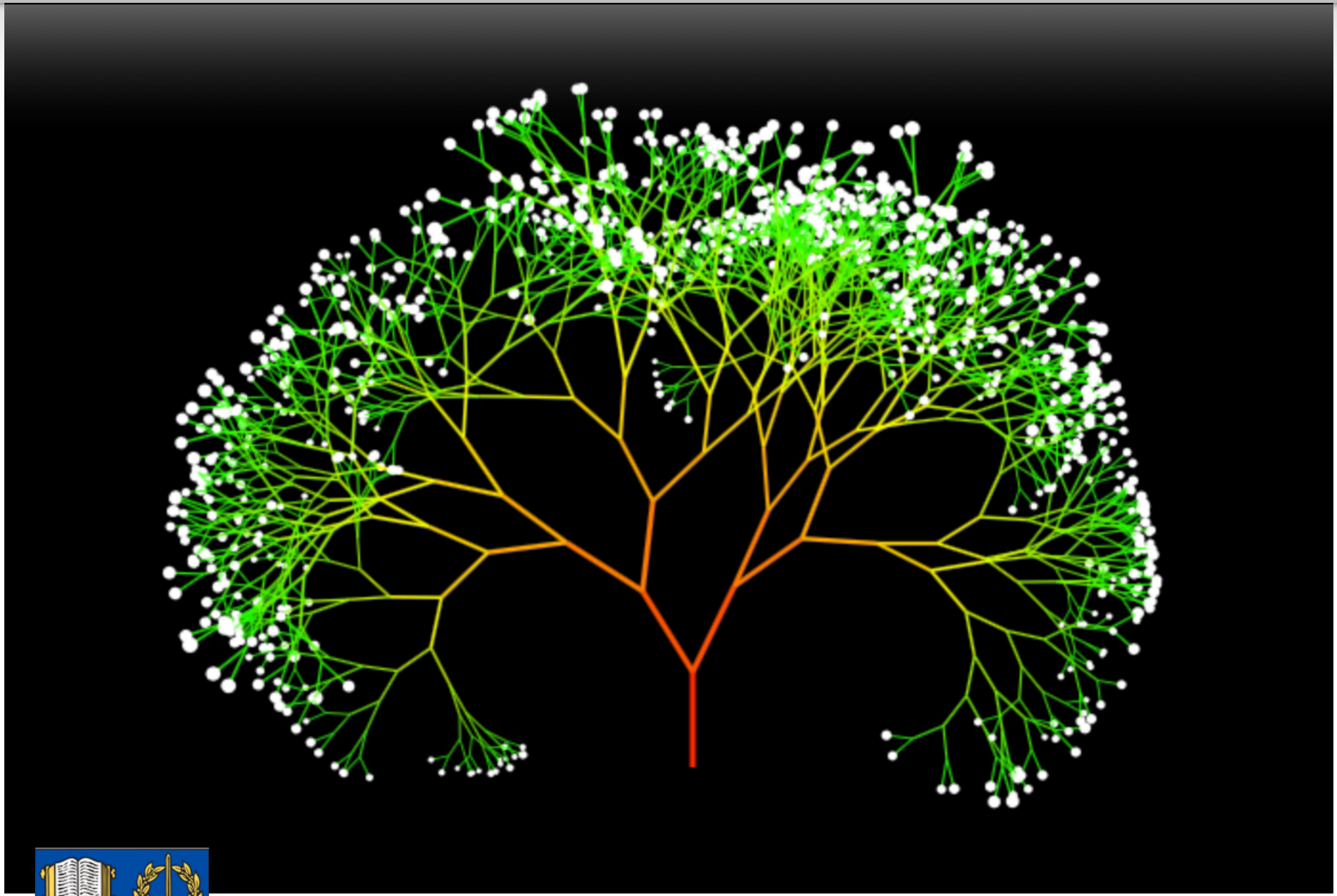
Introduction to Event Generators

Lecture 2: Parton Showers



Peter Skands (Monash University)

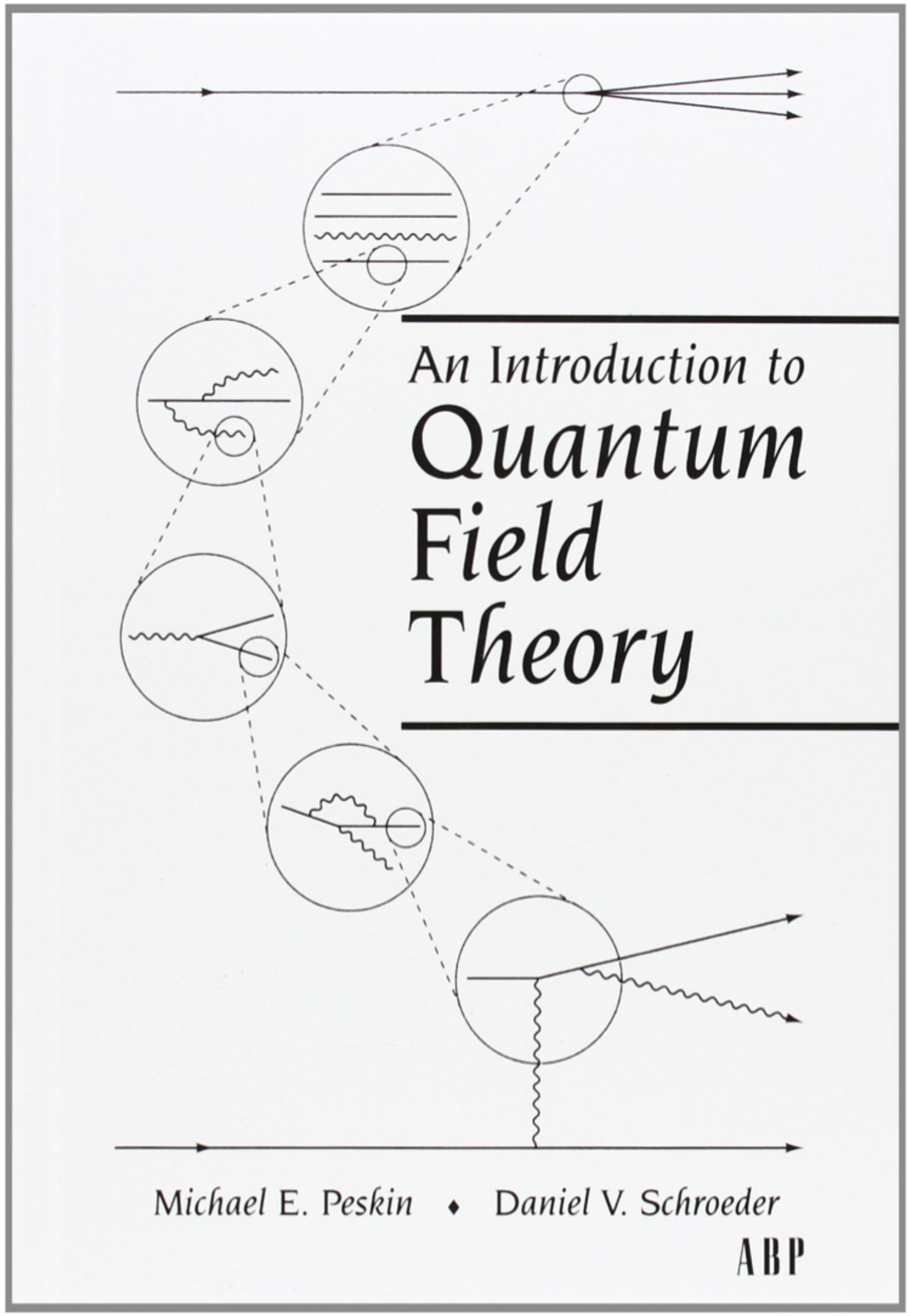
11th MCnet School, Lund 2017

What we actually see when we look at a "jet" (or inside a proton)

An ever-repeating self-similar pattern of quantum fluctuations

At increasingly smaller energies or distances : ***scaling*** (modulo $\alpha_s(Q)$ scaling violation)

To our best knowledge, this is what a fundamental ('elementary') particle really looks like



RECAP: THE STRUCTURE OF QUANTUM FIELDS

What we actually see when we look at a "jet" (or inside a proton)

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At increasingly smaller energies or distances : ***scaling*** (modulo $\alpha_s(Q)$ scaling violation)

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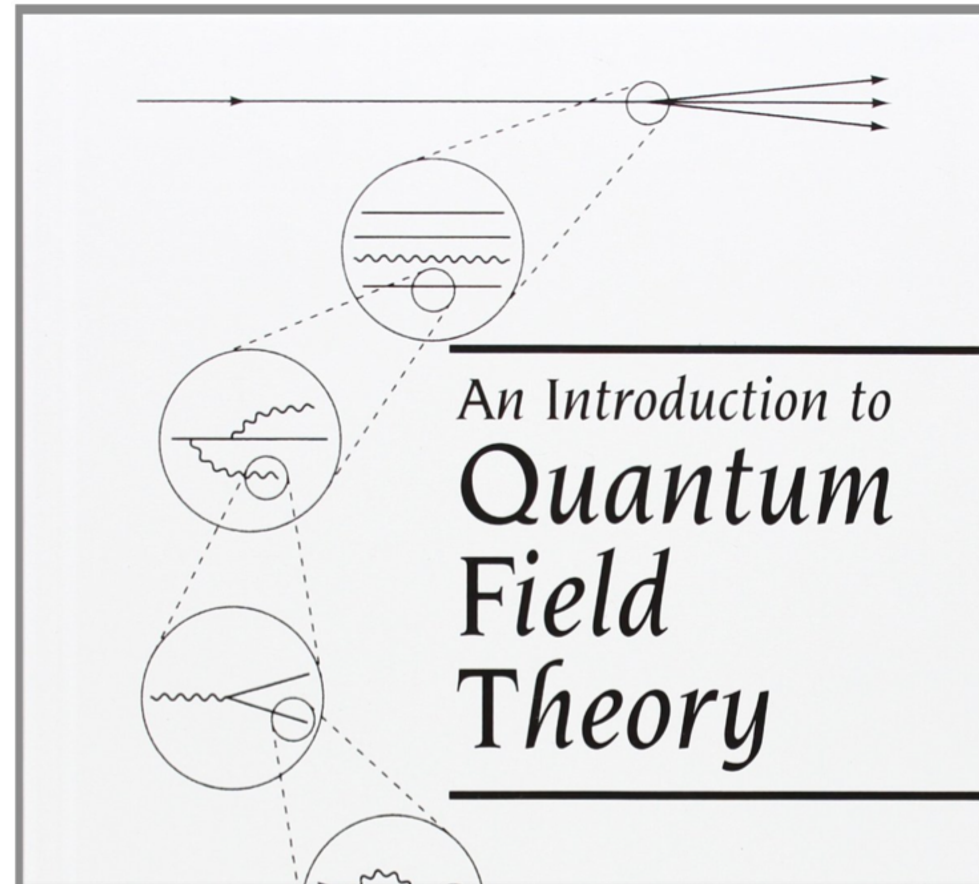
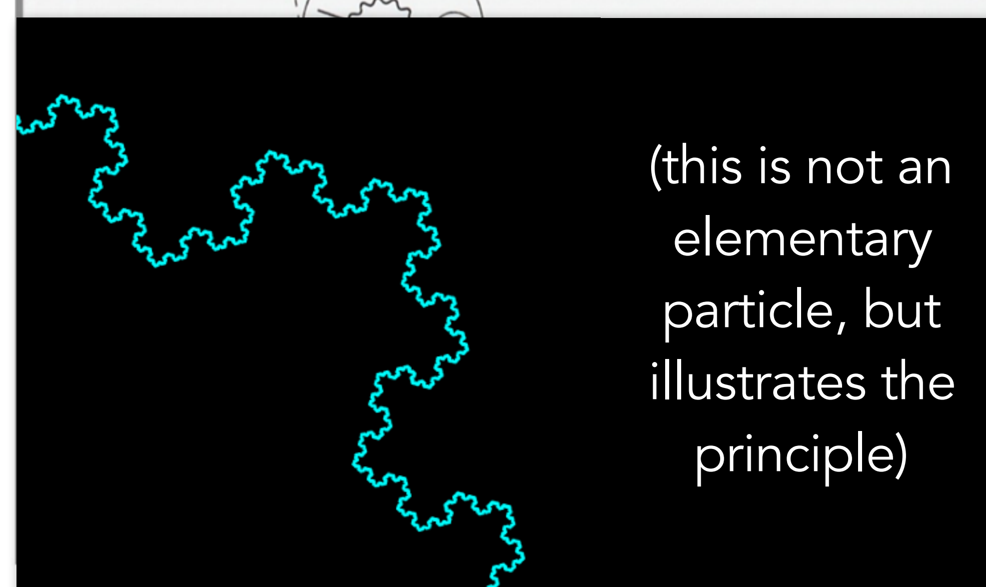
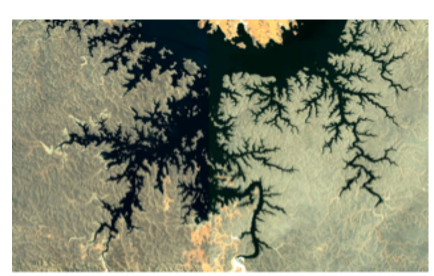




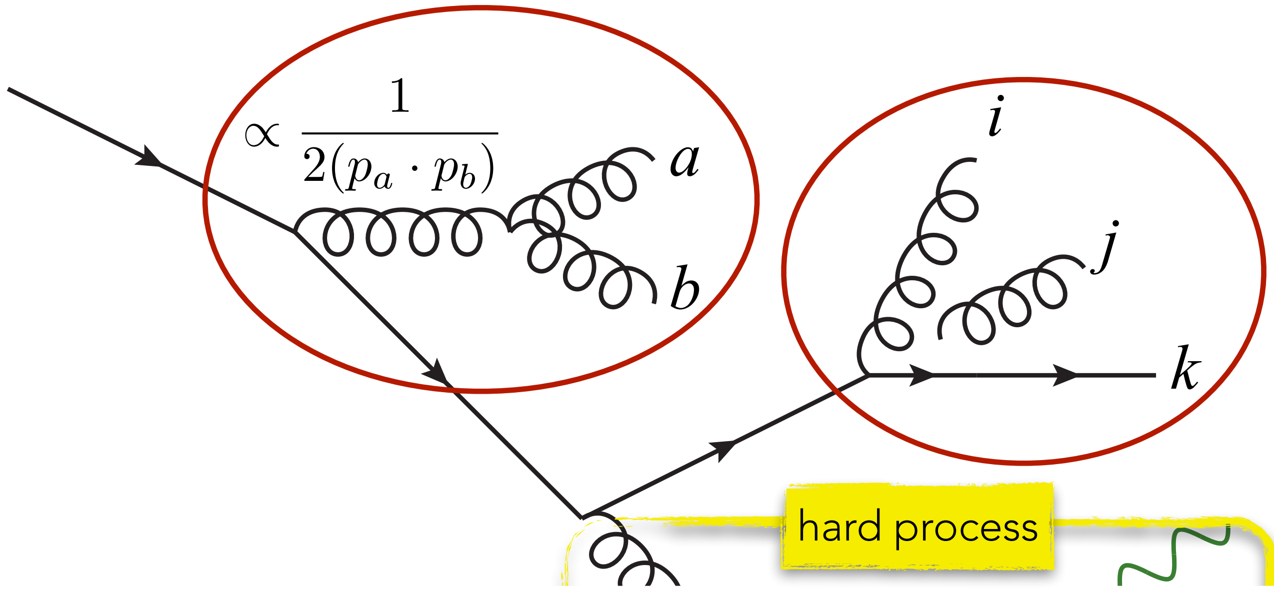
Nature makes copious use of such structures - **Fractals**



Most bremsstrahlung is driven by **divergent propagators** → simple structure

Amplitudes *factorise* in singular limits (→ universal "scale-invariant" or "conformal" structure)

Bremsstrahlung



Partons ab → "collinear":

P(z) = DGLAP splitting kernels, with z = energy fraction = $E_a/(E_a+E_b)$

$$|\mathcal{M}_{F+1}(\ldots,a,b,\ldots)|^2 \stackrel{a||b}{\to} g_s^2 \mathcal{C} \frac{P(z)}{2(p_a \cdot p_b)} |\mathcal{M}_F(\ldots,a+b,\ldots)|^2$$

Gluon j → "soft":

Coherence → Parton j really emitted by (i,k) "colour antenna"

$$|\mathcal{M}_{F+1}(\ldots,i,j,k\ldots)|^2 \stackrel{j_g\to 0}{\to} g_s^2 \mathcal{C} \frac{(p_i \cdot p_k)}{(p_i \cdot p_j)(p_j \cdot p_k)} |\mathcal{M}_F(\ldots,i,k,\ldots)|^2$$

+ scaling ***violation***: $g_s^2 \to 4\pi\alpha_s(Q^2)$

Can apply this many times
→ nested factorizations

SCALING QCD, IN ACTION

Naively, QCD radiation suppressed by $\alpha_s \approx 0.1$

→ Truncate at fixed order = LO, NLO, …

But beware the jet-within-a-jet-within-a-jet …

Example: 100 GeV can be "soft" at the LHC

SUSY pair production at LHC_{14}, with $M_{SUSY} \approx 600$ GeV

LHC - sps1a - m~600 GeV

Plehn, Rainwater, PS PLB645(2007)217

FIXED ORDER pQCD	σ_{tot}[pb]	$\tilde{g}\tilde{g}$	$\tilde{u}_L\tilde{g}$	$\tilde{u}_L\tilde{u}_L^*$	$\tilde{u}_L\tilde{u}_L$	TT
$p_{T,j} >$ 100 GeV	σ_{0j}	4.83	5.65	0.286	0.502	1.30
inclusive **X + 1 "jet"** →	σ_{1j}	2.89	2.74	0.136	0.145	0.73
inclusive **X + 2 "jets"** →	σ_{2j}	1.09	0.85	0.049	0.039	0.26
$p_{T,j} >$ 50 GeV	σ_{0j}	4.83	5.65	0.286	0.502	1.30
	σ_{1j}	5.90	5.37	0.283	0.285	1.50
	σ_{2j}	4.17	3.18	0.179	0.117	1.21

(Computed with SUSY-MadGraph)

σ for X + jets much larger than naive factor-α_s estimate

σ for 50 GeV jets ≈ larger than total cross section
→ what is going on?

All the scales are high, Q >> 1 GeV, so perturbation theory **should** be OK

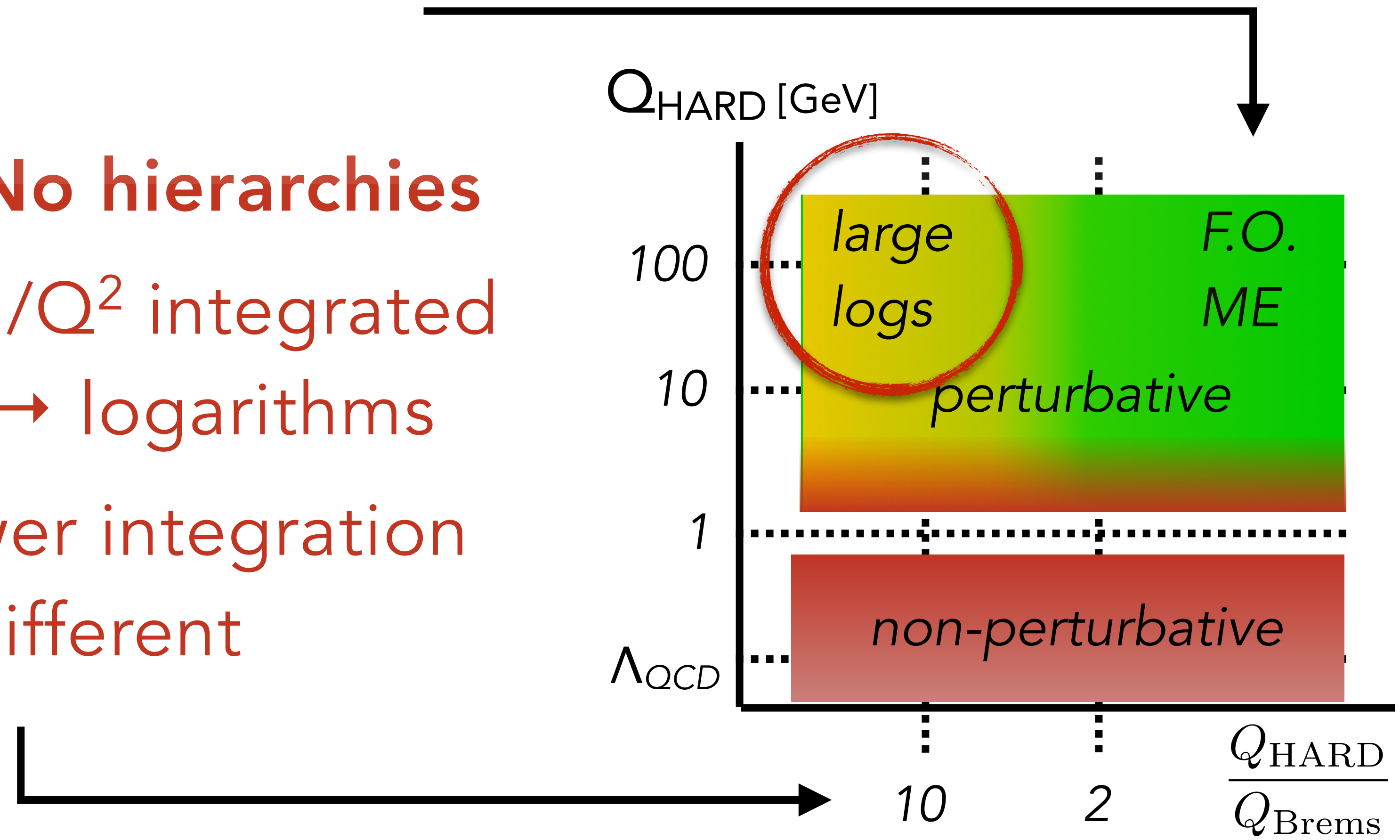
Why are Fixed-Order QCD matrix elements not enough?

F.O. QCD requires **Large scales** (α_s small enough to be perturbative → high-scale processes)

F.O. QCD also requires **No hierarchies**

Bremsstrahlung poles $\propto 1/Q^2$ integrated over phase space $\propto dQ^2$ → logarithms

→ large if upper and lower integration limits are hierarchically different



PARTON SHOWERS

So it's not like you can put a cut at X (e.g., 50, or even 100) GeV and say: "ok, now fixed-order matrix elements will be OK"

Harder Processes are Accompanied by Harder Jets

The hard scale Q_{HARD} of your process will "start off" the fractal

Sooner or later you **will** resolve bremsstrahlung structure (when $Q_{Resolved}/Q_{HARD} \ll 1$)

Extra radiation:

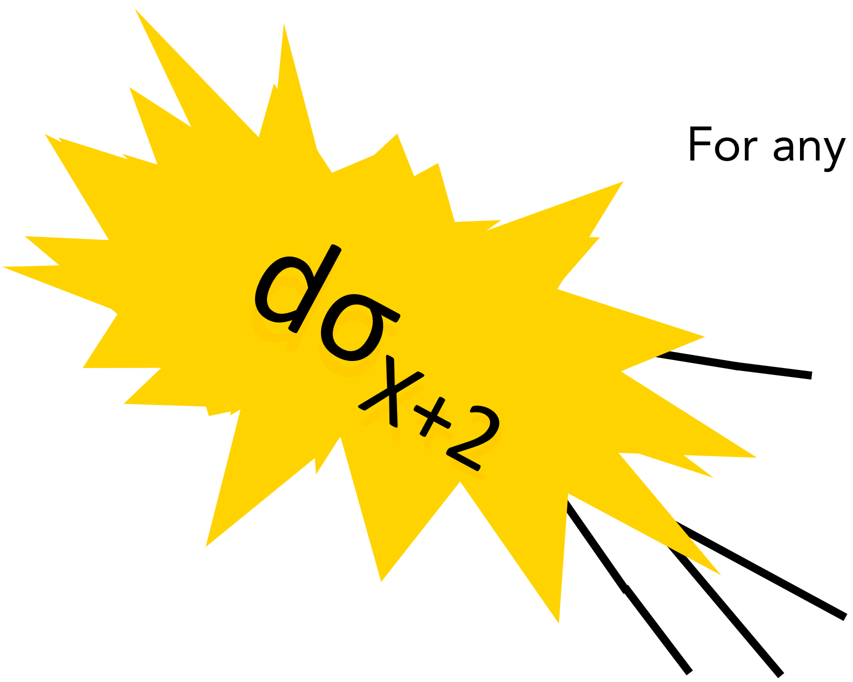
Will generate corrections to your kinematics

Is an unavoidable aspect of the quantum description of quarks and gluons (no such thing as a "bare" quark or gluon; they *always* depend on how you look at them)

Extra jets from bremsstrahlung can be important combinatorial background especially if you are looking for decay jets of similar p_T scales (often, $\Delta M \ll M$)

This is what parton showers are for

MCnet



For any basic process $d\sigma_X = \checkmark$ (calculated process by process)

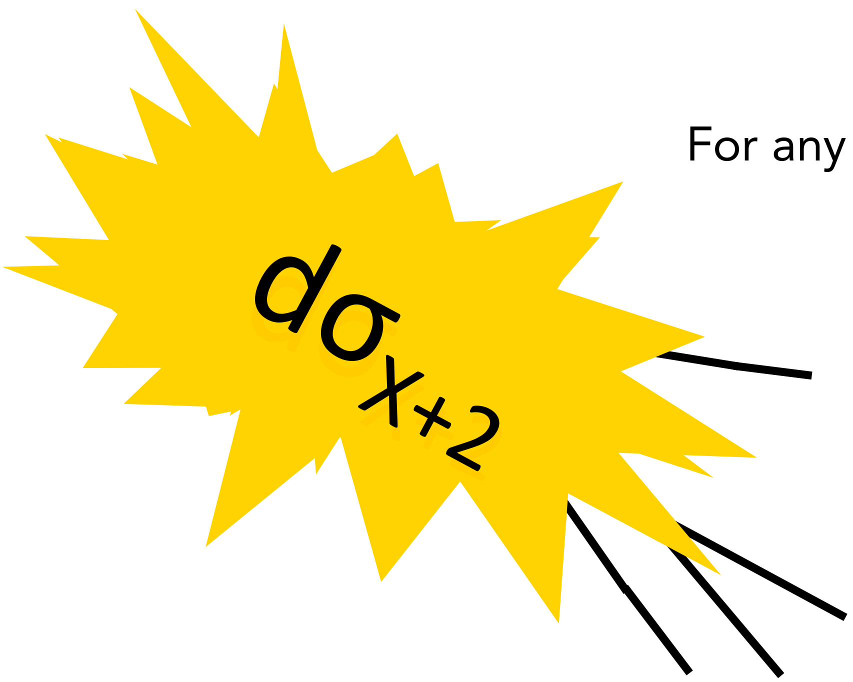
$$d\sigma_{X+1} \sim N_C 2 g_s^2 \frac{ds_{i1}}{s_{i1}} \frac{ds_{1j}}{s_{1j}} d\sigma_X \quad \checkmark$$

$$d\sigma_{X+2} \sim N_C 2 g_s^2 \frac{ds_{i2}}{s_{i2}} \frac{ds_{2j}}{s_{2j}} d\sigma_{X+1} \quad \checkmark$$

$$d\sigma_{X+3} \sim N_C 2 g_s^2 \frac{ds_{i3}}{s_{i3}} \frac{ds_{3j}}{s_{3j}} d\sigma_{X+2} \quad \ldots$$

NB: here just iterating a single eikonal emission; should really sum over all emitters.

Could also have built an approximation from iterating collinear emissions (DGLAP)



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Could also have built an approximation from iterating collinear emissions (DGLAP)

Singularities: universal (mandated by gauge theory)
Non-singular terms: process-dependent

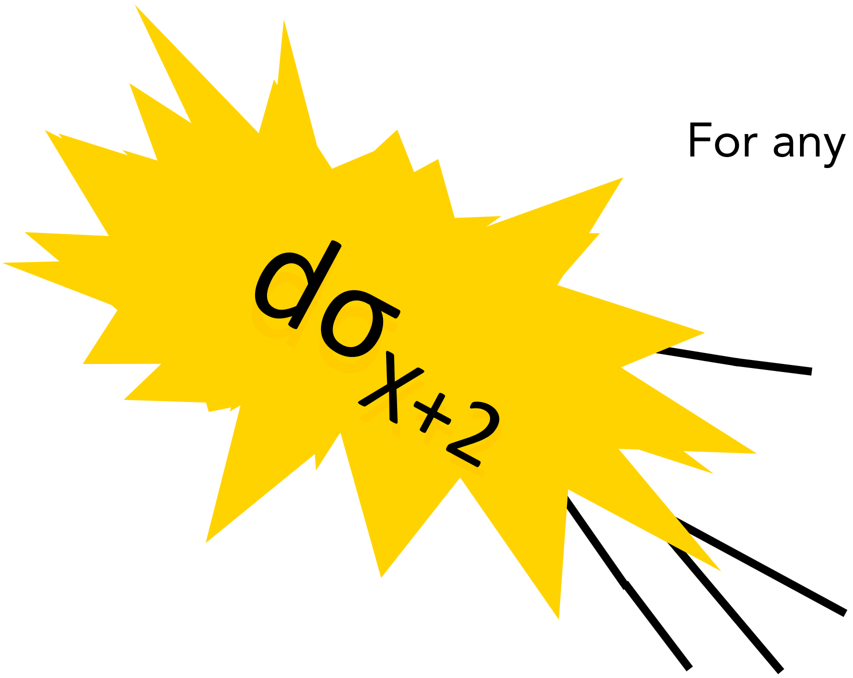
"SOFT" "COLLINEAR"

$$\frac{|\mathcal{M}(Z^0 \to q_i g_j \bar{q}_k)|^2}{|\mathcal{M}(Z^0 \to q_I \bar{q}_K)|^2} = g_s^2\, 2C_F \left[\frac{2 s_{ik}}{s_{ij} s_{jk}} + \frac{1}{s_{IK}} \left(\frac{s_{ij}}{s_{jk}} + \frac{s_{jk}}{s_{ij}} \right) \right]$$

$$\frac{|\mathcal{M}(H^0 \to q_i g_j \bar{q}_k)|^2}{|\mathcal{M}(H^0 \to q_I \bar{q}_K)|^2} = g_s^2\, 2C_F \left[\frac{2 s_{ik}}{s_{ij} s_{jk}} + \frac{1}{s_{IK}} \left(\frac{s_{ij}}{s_{jk}} + \frac{s_{jk}}{s_{ij}} + 2 \right) \right]$$

"SOFT" "COLLINEAR" +F

BREMSSTRAHLUNG



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$$d\sigma_{X+3} \sim N_C 2 g_s^2 \frac{ds_{i3}}{s_{i3}} \frac{ds_{3j}}{s_{3j}} d\sigma_{X+2} \quad \ldots$$

Iterated factorization

Gives us a universal approximation to ∞-order tree-level cross sections.

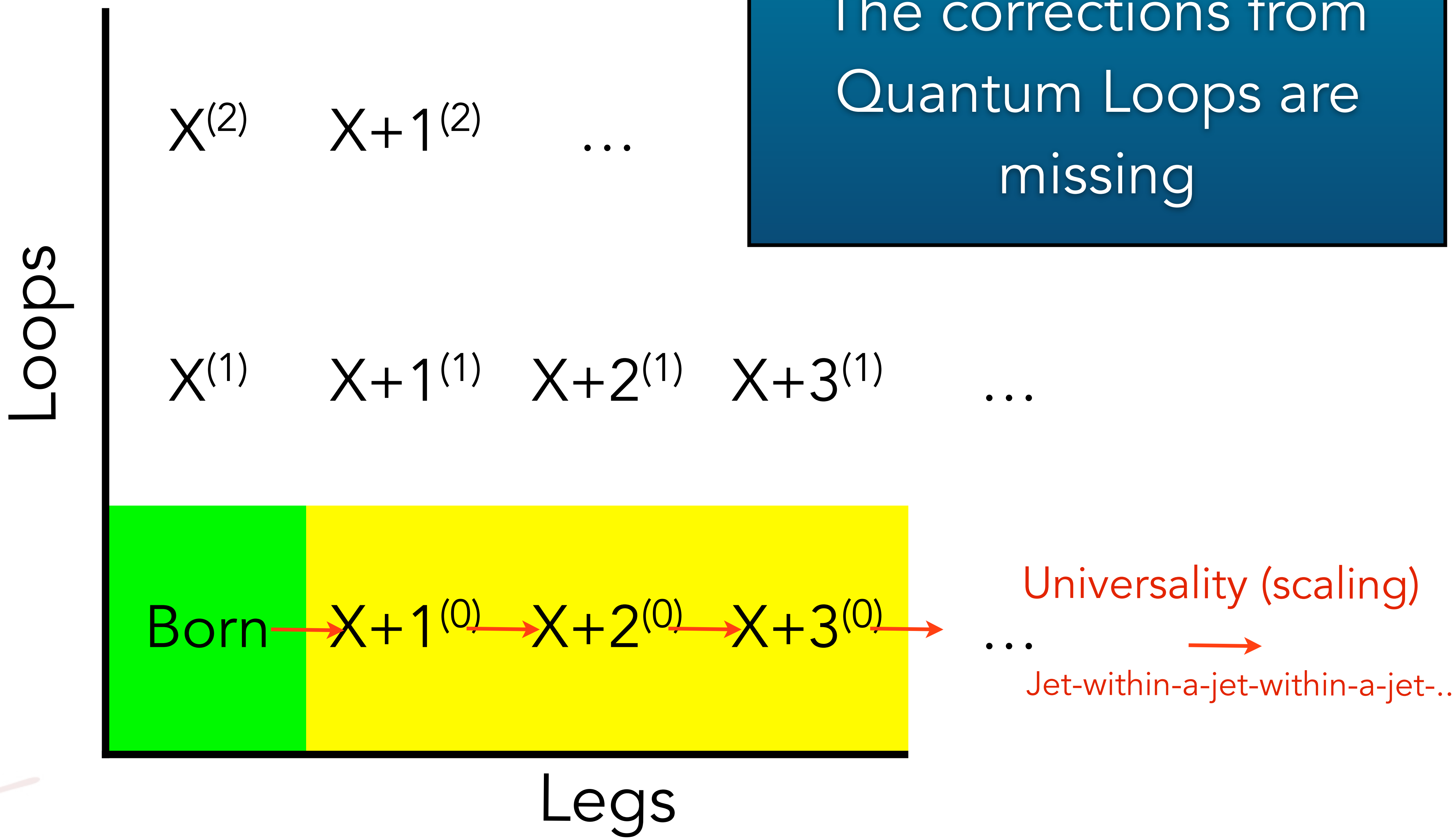
Exact in singular (strongly ordered) limit.

Non-singular terms (non-universal) → Uncertainties for hard radiation

But something is not right … Total σ would be infinite …

Coefficients of the Perturbative Series

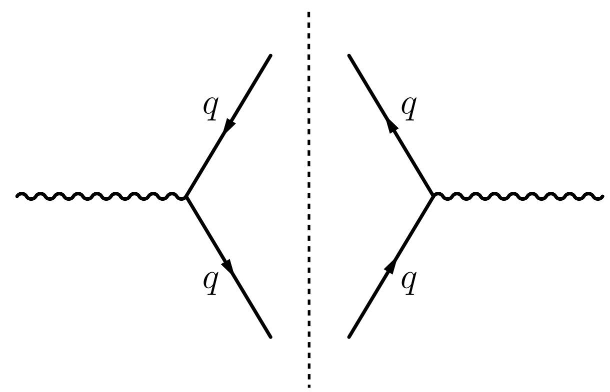
The corrections from Quantum Loops are missing



RECAP: ADDING JETS AT FIXED ORDER

Born @ LO

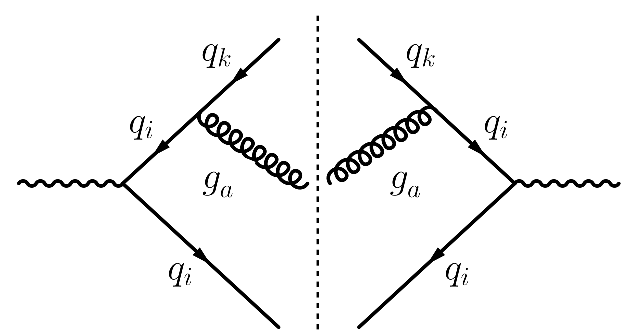
$$\sigma_{\mathrm{Born}} = \int |M_X^{(0)}|^2$$



$X^{(2)}$	$X+1^{(2)}$	...
$X^{(1)}$	$X+1^{(1)}$	...
Born	$X+1^{(0)}$	$X+2^{(0)}$

Born + n @ LO

$$\sigma^{\mathrm{LO}}_{\mathrm{X+1}}(R) = \int_R |M^{(0)}_{X+1}|^2$$



$X^{(2)}$	$X+1^{(2)}$	...
$X^{(1)}$	$X+1^{(1)}$	...
Born	$X+1^{(0)}$	$X+2^{(0)}$

$$\frac{|M_{X+1}|^2}{|M_X|^2} \propto \quad g_s^2\, 2C_F \left[\frac{2s_{ik}}{s_{ij}s_{jk}} + \frac{1}{s_{IK}}\left(\frac{s_{ij}}{s_{jk}} + \frac{s_{jk}}{s_{ij}}\right)\right]$$

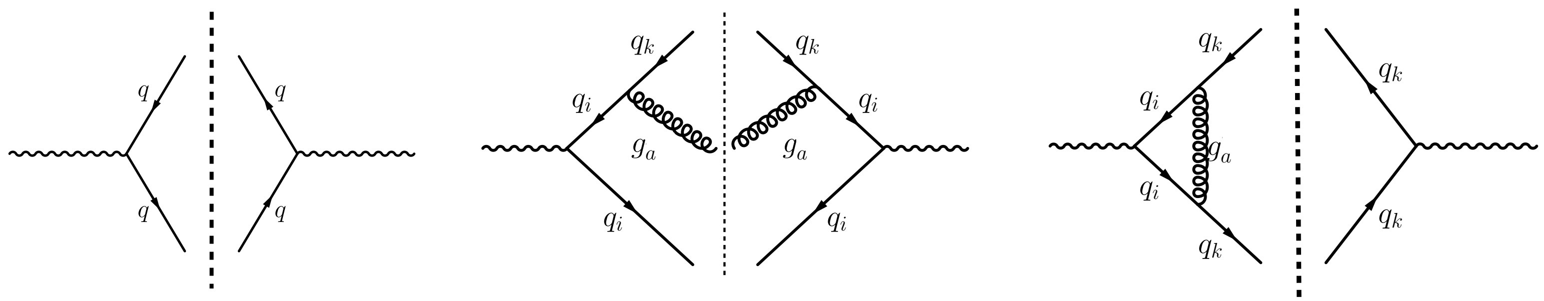
Divergent (when s_{ij} and/or s_{jk} → 0): Integral → **Logarithms**

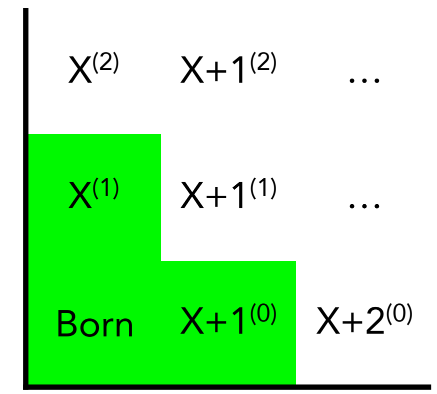
⇛ R = some "Infrared Safe" phase space region (E.g., cut on $p_\perp$, ΔR)

➜ Lecture 3

Careful not to take it too low!

NLO:





$$\sigma_{\mathrm{X}}^{\mathrm{NLO}} = \int |M_X^{(0)}|^2 + \int |M_{X+1}^{(0)}|^2 + \int 2\mathrm{Re}[M_X^{(1)} M_X^{(0)*}]$$

IR singularities

(from poles of propagators going on shell when integrating to $Q^2 \to 0$)

IR singularities

(from poles of propagators going on shell when integrating over gluon virtuality)

In IR limits, the X+1 final state is indistinguishable from an X+0 one

→ singularities must always* sum together (& they cancel!)

example:

$$\sigma_{\mathrm{NLO}}(e^+e^- \to q\bar{q}) = \sigma_{\mathrm{LO}}(e^+e^- \to q\bar{q}) \left(1 + \frac{\alpha_s(E_{\mathrm{CM}})}{\pi} + \mathcal{O}(\alpha_s^2)\right)$$

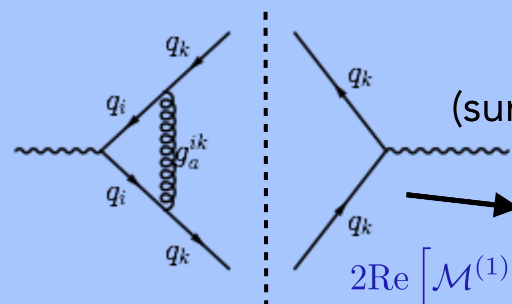
Sum of real and virtual $O(\alpha_s)$ nonsingular; no IR regulator dependence

*) for so-called IR safe observables; discussed in Lecture 3

UNITARITY → EVOLUTION (RESUMMATION)

Probability for nothing to happen (~virtual + unresolved-real) + Probability for something to happen (~ resolved real) = 1

Unitarity: sum(probability) = 1

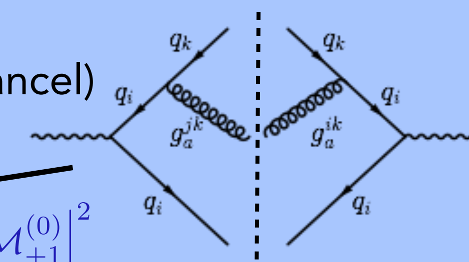


Kinoshita-Lee-Nauenberg

(sum over degenerate quantum states = finite; infinities must cancel)

$$\underset{2\mathrm{Re}\left[\mathcal{M}^{(1)}\mathcal{M}^{(0)*}\right]}{\text{Loop}} = -\int \text{Tree} + F$$

$\left|\mathcal{M}^{(0)}_{+1}\right|^2$



Parton Showers neglect F → "Leading-Logarithmic" (LL) Approximation

Imposed by Event *evolution*: "detailed balance"

When (X) branches to (X+1): **Gain** one (X+1). **Loose** one (X).

Differential equation with evolution kernel
(or, typically, a soft/collinear approximation thereof) $\dfrac{d\sigma_{X+1}}{d\sigma_X}$

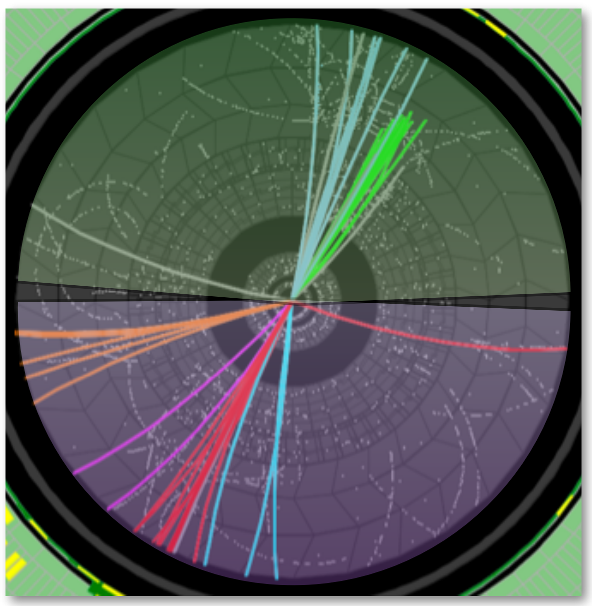
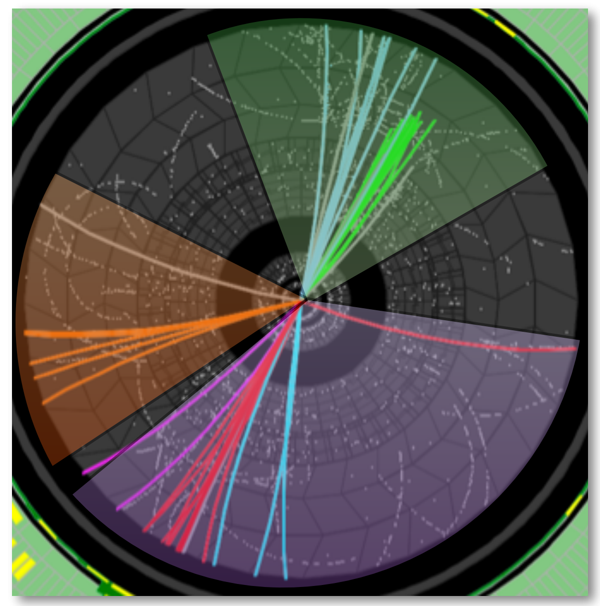
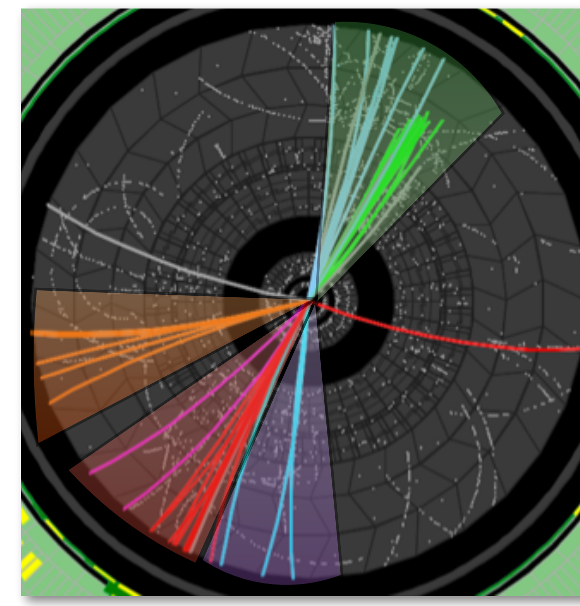
Evolve in some measure of *resolution* ~ hardness, 1/time … ~ fractal scale

+ account for scaling violation via quark masses and $g_s^2 \to 4\pi\alpha_s(Q^2)$

→ includes both real (tree) and virtual (loop) corrections, to arbitrary order

EVOLUTION ~ FINE-GRAINING

(E.g., starting from QCD 2→2)

$Q \sim Q_{\mathrm{HARD}}$	$Q_{\mathrm{HARD}}/Q <$ "A few"	$Q \ll Q_{\mathrm{HARD}}$ **Scale Hierarchy!**
		
At most inclusive level "Everything is 2 jets"	At (slightly) finer resolutions, some events have 3, or 4 jets	At high resolution, most events have >2 jets
Fixed order: $\sigma_{\mathrm{inclusive}}$	Fixed order: $\sigma_{X+n} \sim \alpha_s^n \, \sigma_X$	Fixed order diverges: $\sigma_{X+n} \sim \alpha_s^n \ln^{2n}(Q/Q_{\mathrm{HARD}})\sigma_X$

Unitarity: **Reinterpret** as *number of emissions diverging*, while cross section remains $\sigma_{\mathrm{inclusive}}$

EVOLUTION EQUATIONS

What we need is a **differential equation**

Boundary condition: a few partons defined at a high scale (Q_F)

Then evolves (or "runs") that parton system down to a low scale (the hadronization cutoff ~ 1 GeV) → It's an evolution equation in Q_F

Close analogue: **nuclear decay**

Evolve an unstable nucleus. Check if it decays + follow chains of decays.

Decay constant

$$\frac{\mathrm{d}P(t)}{\mathrm{d}t} = c_N$$

Probability to remain undecayed in the time interval [t_1,t_2]

$$\Delta(t_1,t_2) = \exp\left(-\int_{t_1}^{t_2} c_N \,\mathrm{d}t\right) = \exp\left(-c_N\,\Delta t\right)$$
$$= 1 - c_N \Delta t + \mathcal{O}(c_N^2)$$

Decay probability per unit time

$$\frac{\mathrm{d}P_{\mathrm{res}}(t)}{\mathrm{d}t} = \frac{-\mathrm{d}\Delta}{\mathrm{d}t} = c_N\,\Delta(t_1,t)$$

(respects that each of the original nuclei can only decay if not decayed already)

$\Delta(t_1,t_2)$: "Sudakov Factor"

THE SUDAKOV FACTOR

In nuclear decay, the Sudakov factor counts:

How many nuclei remain undecayed after a time t

Probability to remain undecayed in the time interval [t_1,t_2]

$$\Delta(t_1,t_2) = \exp\left(-\int_{t_1}^{t_2} c_N \, \mathrm{d}t\right) = \exp\left(-c_N \, \Delta t\right)$$

The Sudakov factor for a parton system "counts":

The probability that the parton system doesn't evolve (branch) when we run the factorization scale (~1/time) from a high to a low scale

(i.e., that there is no state change)

Evolution probability per unit "time"

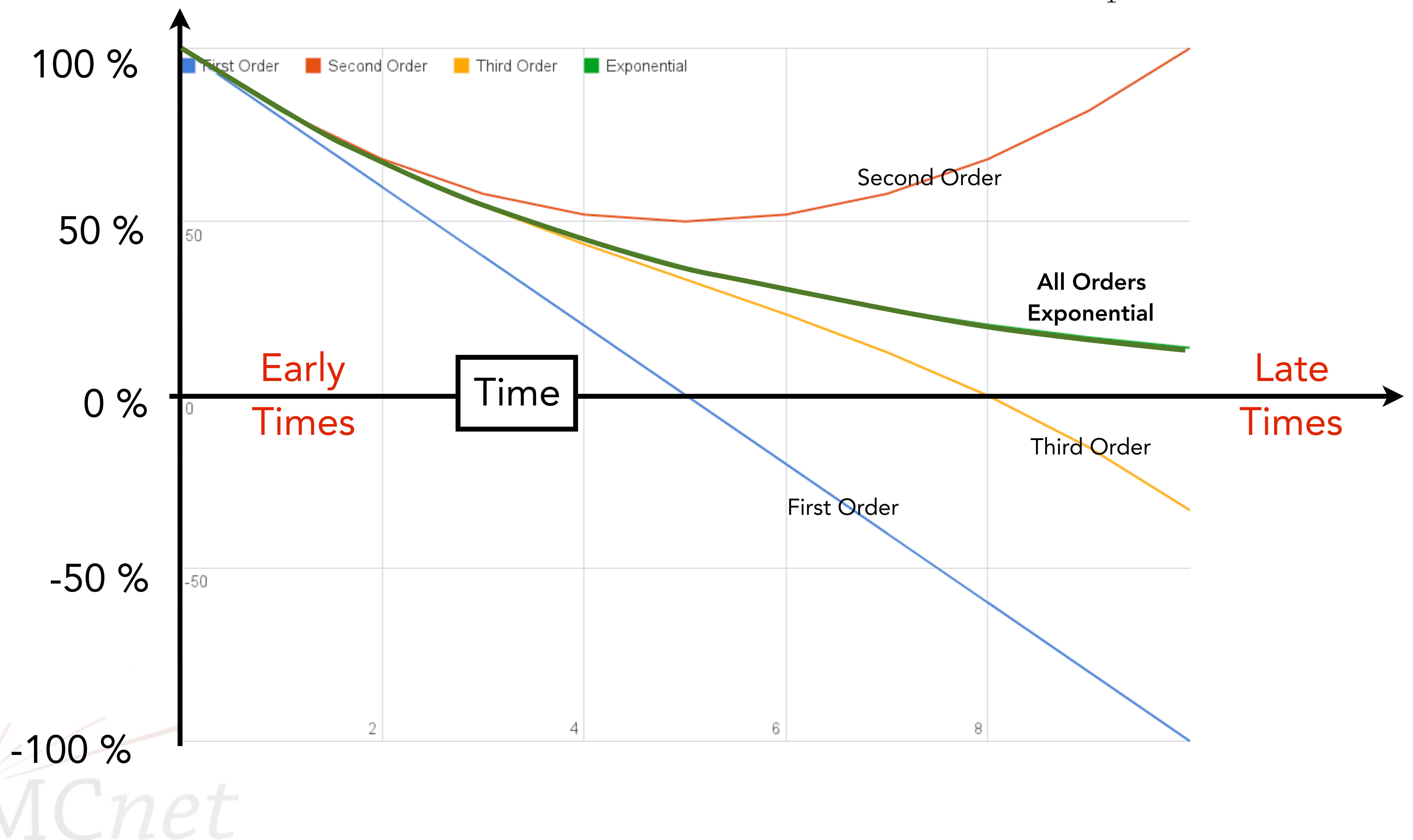
$$\frac{\mathrm{d}P_{\mathrm{res}}(t)}{\mathrm{d}t} = \frac{-\mathrm{d}\Delta}{\mathrm{d}t} = c_N \, \Delta(t_1,t)$$

(replace t by shower evolution scale)

(replace c_N by proper shower evolution kernels)

Nuclei remaining undecayed after time t

$$= \Delta(t_1, t_2) = \exp\left(-\int_{t_1}^{t_2} \mathrm{d}t \, \frac{\mathrm{d}\mathcal{P}}{\mathrm{d}t}\right)$$



A SHOWER ALGORITHM

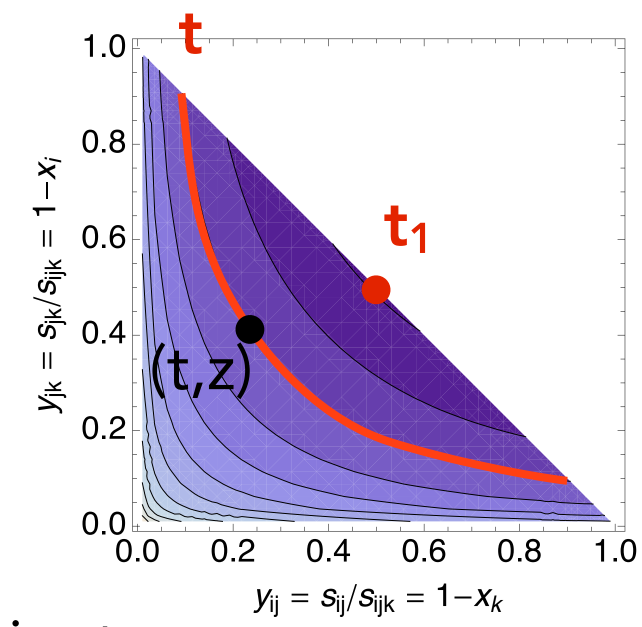
1. For each evolver, generate a random number R ∈ [0,1]

Solve equation $R = \Delta(t_1, t)$ for t (with starting scale t_1)

Analytically for simple splitting kernels,

else numerically and/or by trial+veto

→ t scale for next (trial) branching



2. Generate another Random Number, $R_z \in [0,1]$

To find second (linearly independent) phase-space invariant

Solve equation $R_z = \dfrac{I_z(z,t)}{I_z(z_{\max}(t),t)}$ for z (at scale t)

With the "primitive function" $$I_z(z,t) = \int_{z_{\min}(t)}^{z} \mathrm{d}z \left. \frac{\mathrm{d}\Delta(t')}{\mathrm{d}t'} \right|_{t'=t}$$

3. Generate a third Random Number, $R_\varphi \in [0,1]$

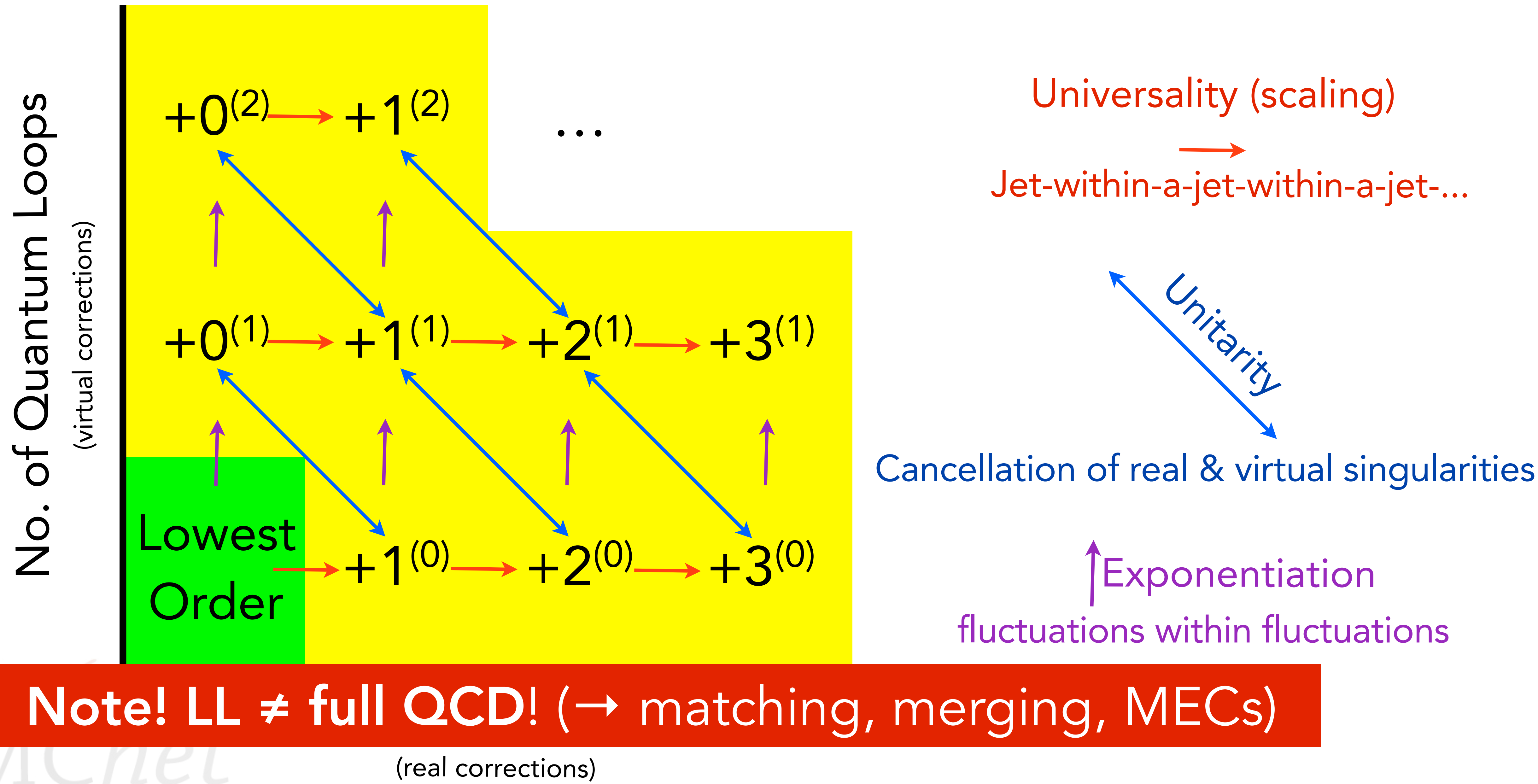
Solve equation $R_\varphi = \varphi/2\pi$ for **φ** → Can now do 3D branching

Accept/Reject based on full kinematics. Update t_1 = t. Repeat.

BOOTSTRAPPED PERTURBATION THEORY

Start from an **arbitrary lowest-order** process (green = QFT amplitude squared)

Parton showers generate the (LL) bremsstrahlung terms of the rest of the perturbative series (approximate infinite-order resummation)



Note! LL ≠ full QCD! (→ matching, merging, MECs)

WHAT ARE THE EVOLUTION KERNELS?

Recall: two universal (bremsstrahlung) limits:

Collinear (DGLAP) Limit: two partons becoming parallel

Partons ab → "collinear":

P(z) = DGLAP splitting kernels, with z = energy fraction = $E_a/(E_a+E_b)$

$$|\mathcal{M}_{F+1}(\ldots,a,b,\ldots)|^2 \stackrel{a||b}{\to} g_s^2\mathcal{C}\,\frac{P(z)}{2(p_a\cdot p_b)}|\mathcal{M}_F(\ldots,a+b,\ldots)|^2$$

Soft (eikonal) Limit: an emitted gluon having vanishing energy

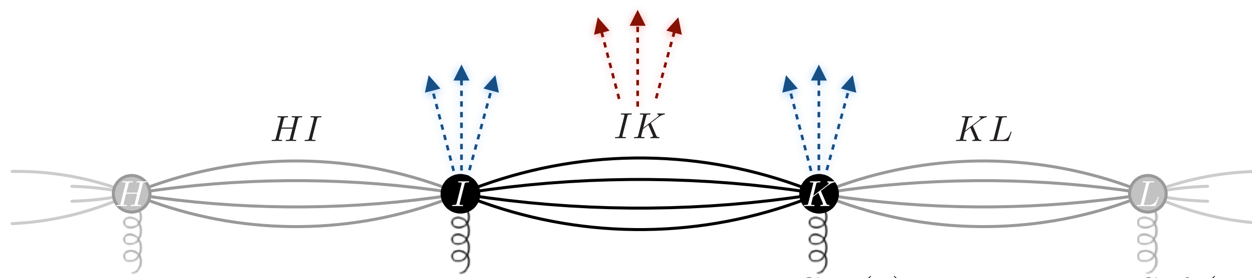
Gluon j → "soft":

Coherence → Parton j really emitted by (i,k) "colour antenna"

$$|\mathcal{M}_{F+1}(\ldots,i,j,k\ldots)|^2 \stackrel{j_g\to 0}{\to} g_s^2\mathcal{C}\,\frac{(p_i\cdot p_k)}{(p_i\cdot p_j)(p_j\cdot p_k)}|\mathcal{M}_F(\ldots,i,k,\ldots)|^2$$

→ can build different types of parton showers
(and, in general, different kinds of resummations)

TYPES OF PARTON SHOWERS



	Coll(I)	Soft(IK)
Parton Shower (DGLAP)	a_I	$a_I + a_K$
*Coherent Parton Shower (*HERWIG *[12,40],* PYTHIA6 *[11])*	$\Theta_I a_I$	$\Theta_I a_I + \Theta_K a_K$
*Global Dipole-Antenna (*ARIADNE *[17], GGG [36], WK [32],* VINCIA*)*	$a_{IK} + a_{HI}$	a_{IK}
Sector Dipole-Antenna (LP [41], VINCIA*)*	$\Theta_{IK} a_{IK} + \Theta_{HI} a_{HI}$	a_{IK}
Partitioned-Dipole Shower (SK [23], NS [42], DTW [24], PYTHIA8 *[38],* SHERPA*,* DIRE*)*	$a_{I,K} + a_{I,H}$	$a_{I,K} + a_{K,I}$

Starting from collinear (parton) limit:

DGLAP evolution, collinear factorisation (MSbar PDFs)

"Conventional Parton Showers" : earliest shower models

Modified for correct soft limits: **angular ordering*** (or vetos), **(CS) Dipole showers**

Herwig · Pythia · Herwig CS, Sherpa CS, Dire

*angular ordering → coherence only in an averaged sense; discussed later

Starting from soft (dipole) limit:

related to HEJ

DLA (only double-pole piece), eikonal approximations

Extended to include DGLAP collinear limits: **(Lund) Dipole / Antenna showers**

Ariadne · Vincia

EXAMPLE: DGLAP KERNELS

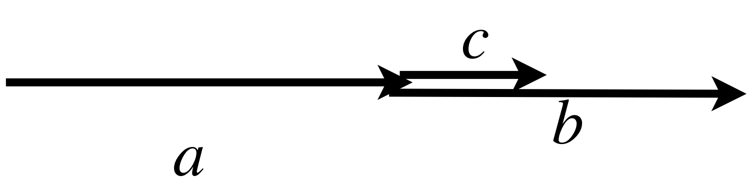
DGLAP: from *collinear limit* of MEs $(p_b+p_c)^2 \to 0$

+ evolution equation from invariance with respect to Q_F → RGE

DGLAP

(E.g., PYTHIA)

$$\mathrm{d}\mathcal{P}_a = \sum_{b,c} \frac{\alpha_{abc}}{2\pi} P_{a\to bc}(z)\, \mathrm{d}t\, \mathrm{d}z \ .$$



$p_b = z\, p_a$

$p_c = (1\text{-}z)\, p_a$

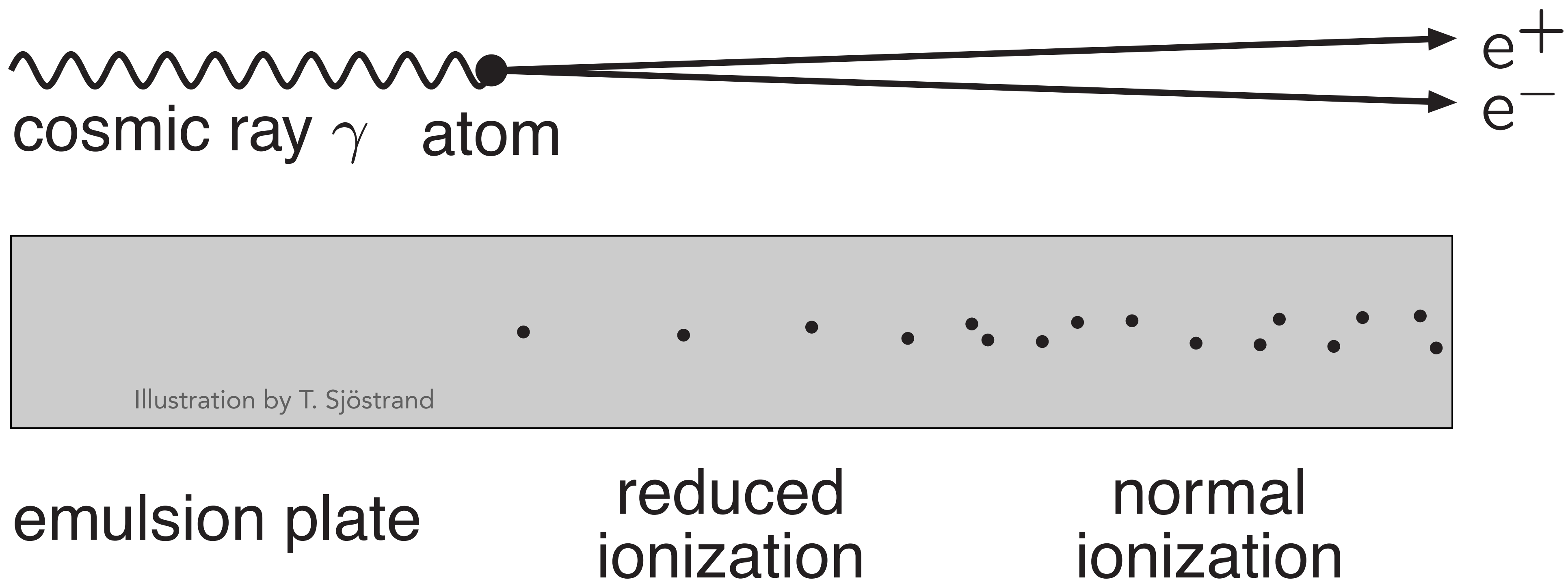
$$\begin{aligned}
P_{\mathrm{q}\to\mathrm{qg}}(z) &= C_F \frac{1+z^2}{1-z} \ , \\
P_{\mathrm{g}\to\mathrm{gg}}(z) &= N_C \frac{(1-z(1-z))^2}{z(1-z)} \ , \\
P_{\mathrm{g}\to\mathrm{q\bar{q}}}(z) &= T_R\, (z^2+(1-z)^2) \ , \\
P_{\mathrm{q}\to\mathrm{q}\gamma}(z) &= e_{\mathrm{q}}^2 \frac{1+z^2}{1-z} \ , \\
P_{\ell\to\ell\gamma}(z) &= e_{\ell}^2 \frac{1+z^2}{1-z} \ ,
\end{aligned}$$

$$\mathrm{d}t = \frac{\mathrm{d}Q^2}{Q^2} = \mathrm{d}\ln Q^2$$

… with Q^2 some measure of "hardness"
= event/jet resolution
measuring parton virtualities / formation time / …

NB: dipoles, antennae, also have DGLAP kernels as their collinear limits

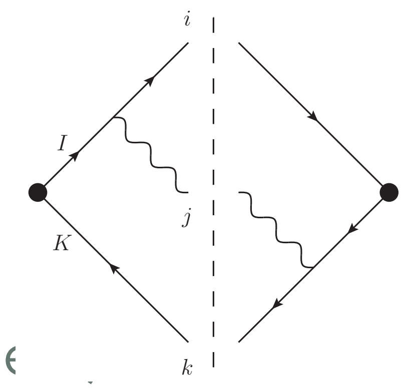
QED: Chudakov effect (mid-fifties)



Physics: (applies to any gauge theory)

Interference between emissions from colour-connected partons (e.g. i and k) → coherent **dipole** patterns

(More complicated multipole effects beyond leading colour; ignored he



DGLAP kernels, though incoherent a priori, can reproduce this pattern (at least in an azimuthally averaged sense) by *angular ordering*

Start from the M.E. factorisation formula in the ***soft limit***

$$\frac{E_j^2\,(p_i\cdot p_k)}{(p_i\cdot p_j)(p_j\cdot p_k)} = \frac{1-\cos\theta_{ik}}{(1-\cos\theta_{ij})(1-\cos\theta_{jk})} = \frac{1-\cos\theta_{ik}}{(1-\cos\theta_{ij})(1-\cos\theta_{jk})} \pm \frac{1}{2(1-\cos\theta_{ij})} \mp \frac{1}{2(1-\cos\theta_{jk})}$$

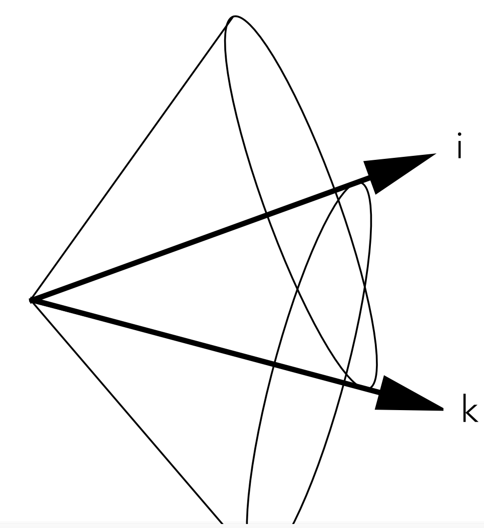
Soft Eikonal Factor — (write out 4-products) — Add and subtract $1/(1-\cos\theta_{ij})$ and $1/(1-\cos\theta_{jk})$ to isolate ij and jk collinear pieces

$$\int_0^{2\pi}\frac{\mathrm{d}\varphi_{ij}}{4\pi}\left(\frac{1-\cos\theta_{ik}}{(1-\cos\theta_{ij})(1-\cos\theta_{jk})}+\frac{1}{1-\cos\theta_{ij}}-\frac{1}{1-\cos\theta_{jk}}\right) = \frac{1}{2(1-\cos\theta_{ij})}\left(1+\frac{\cos\theta_{ij}-\cos\theta_{ik}}{|\cos\theta_{ij}-\cos\theta_{ik}|}\right)$$

Take the ij piece and integrate over azimuthal angle $\mathrm{d}\varphi_{ij}$ (using explicit momentum representations)

⇛ Soft radiation averaged over φ_{ij} : $\rightarrow \dfrac{1}{1-\cos\theta_{ij}}$ if $\theta_{ij} < \theta_{ik}$; **otherwise 0**

↑ kill radiation outside ik opening angle



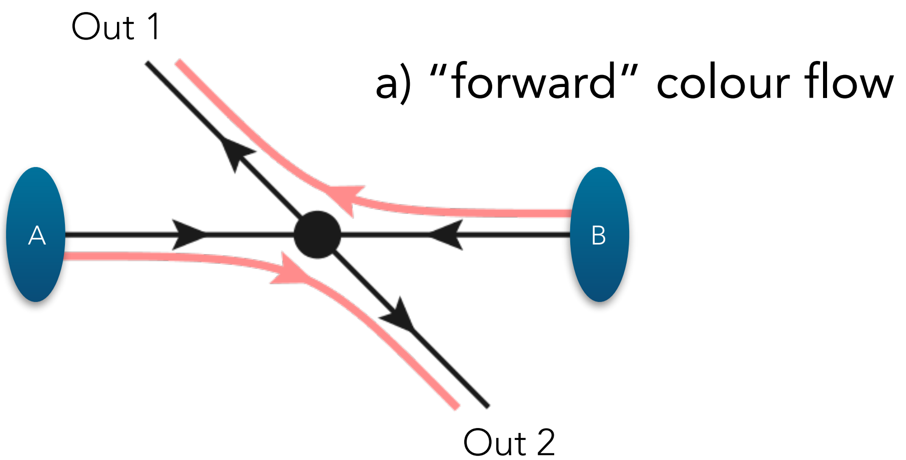
Note: Dipole & antenna showers include this effect point by point in φ (without averaging)

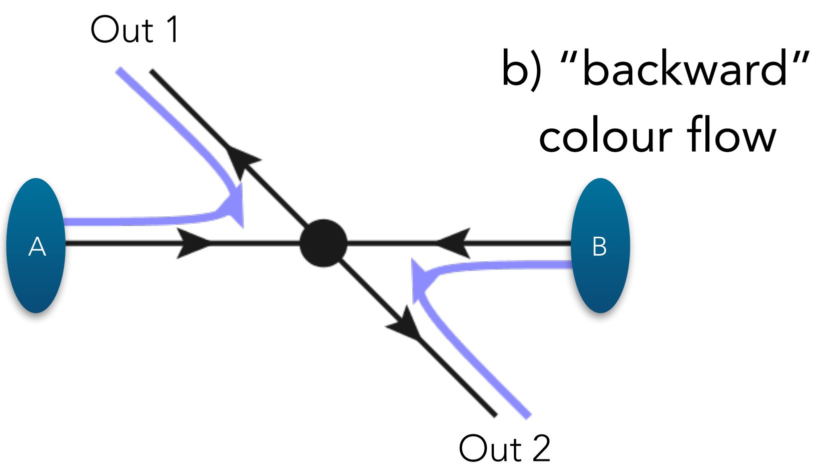
Example taken from: Ritzmann, Kosower, PS, PLB718 (2013) 1345

Example: quark-quark scattering in hadron collisions

Consider, for instance, scattering at 45°

2 possible colour flows :





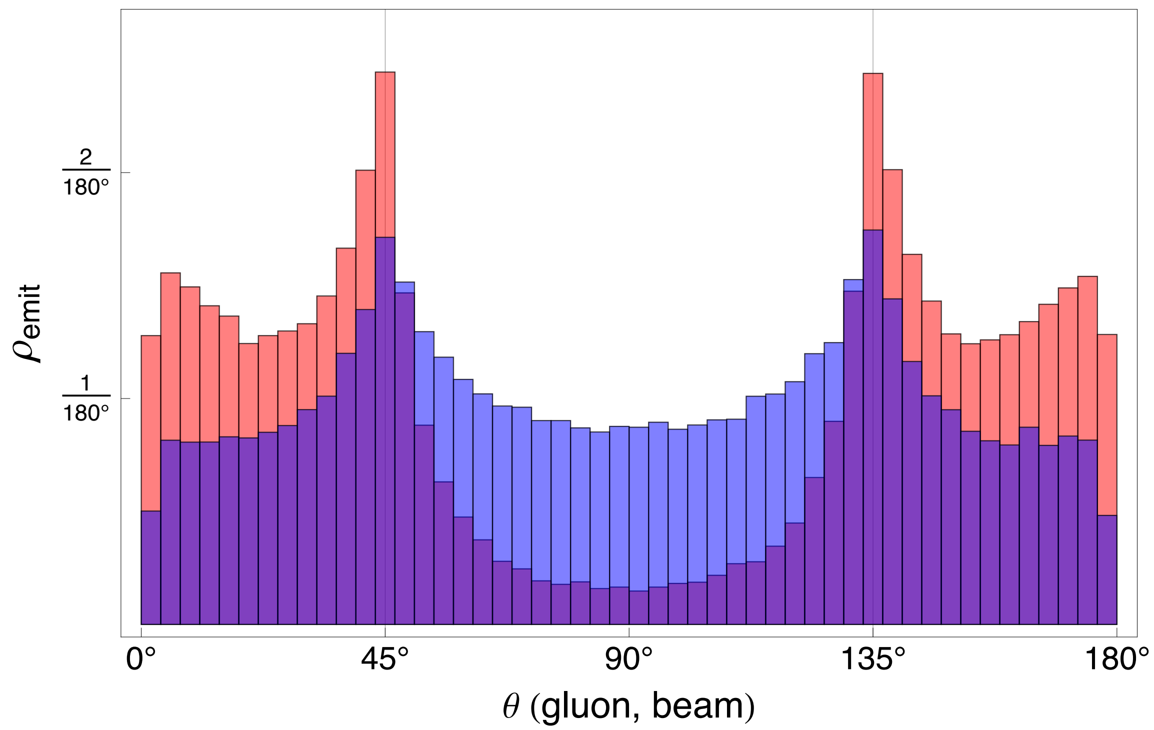
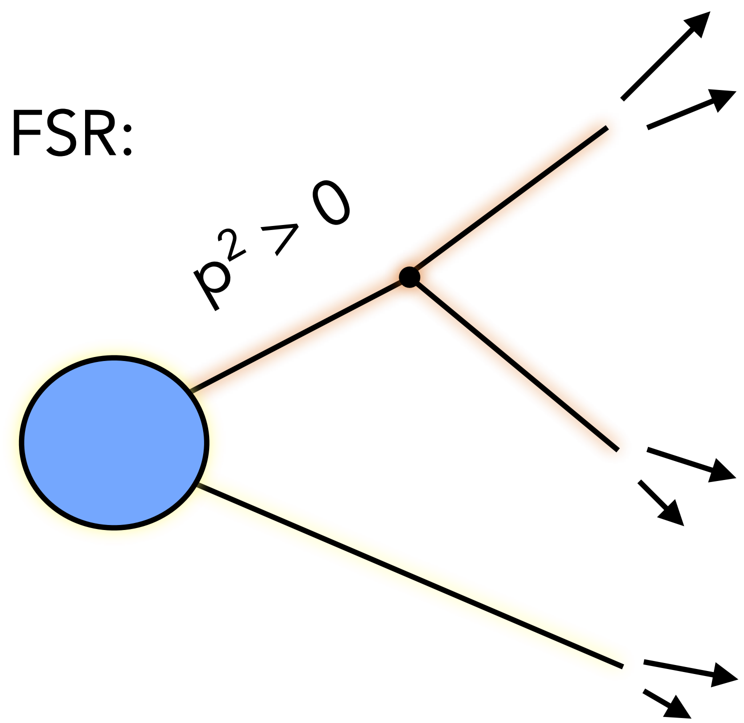
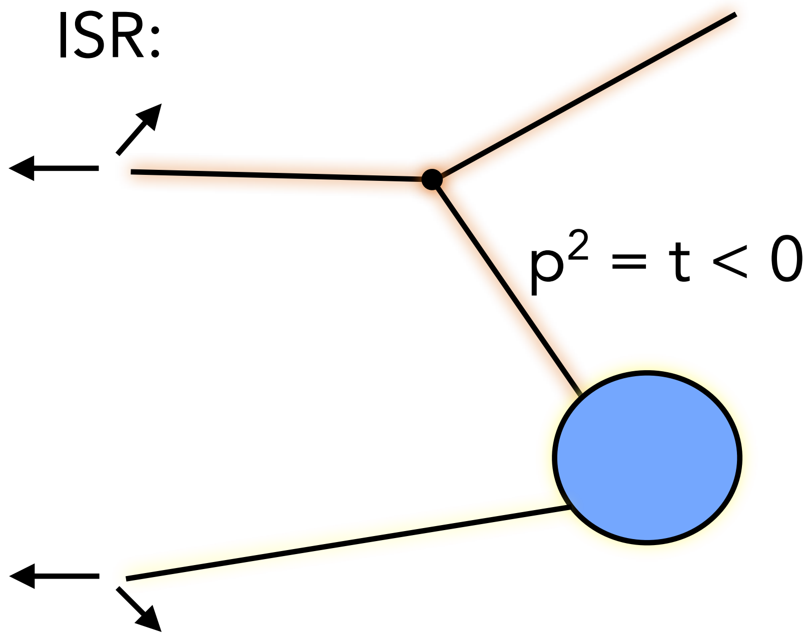


Figure 4: Angular distribution of the first gluon emission in $qq \to qq$ scattering at 45°, for the two different color flows. The light (red) histogram shows the emission density for the forward flow, and the dark (blue) histogram shows the emission density for the backward flow.

Another nice physics example is the SM contribution to the Tevatron top-quark forward-backward asymmetry from coherent showers, see: PS, Webber, Winter, JHEP 1207 (2012) 151

INITIAL-STATE VS FINAL-STATE EVOLUTION





Virtualities are

Timelike: $p^2>0$

Start at $Q^2 = Q_F^2$

"Forwards evolution"

Virtualities are

Spacelike: $p^2<0$

Start at $Q^2 = Q_F^2$

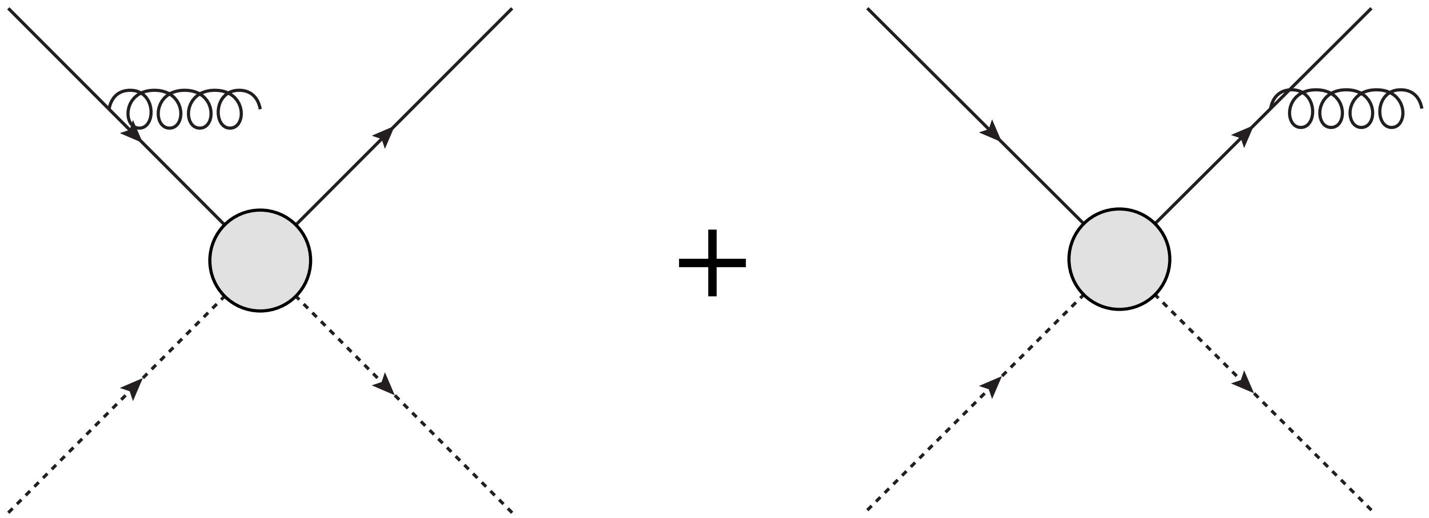
Constrained backwards evolution towards boundary condition = proton

Separation meaningful for collinear radiation, but not for soft …

INITIAL-FINAL INTERFERENCE

A tricky aspect for many parton showers. Illustrates that quantum ≠ classical !

Who emitted that gluon?



Real QFT = sum over amplitudes, then square → interference (IF coherence)
Respected by dipole/antenna languages (and by angular ordering, azimuthally averaged), but not by conventional DGLAP (→ all PDFs are "wrong")

Separation meaningful for collinear radiation, but not for soft …

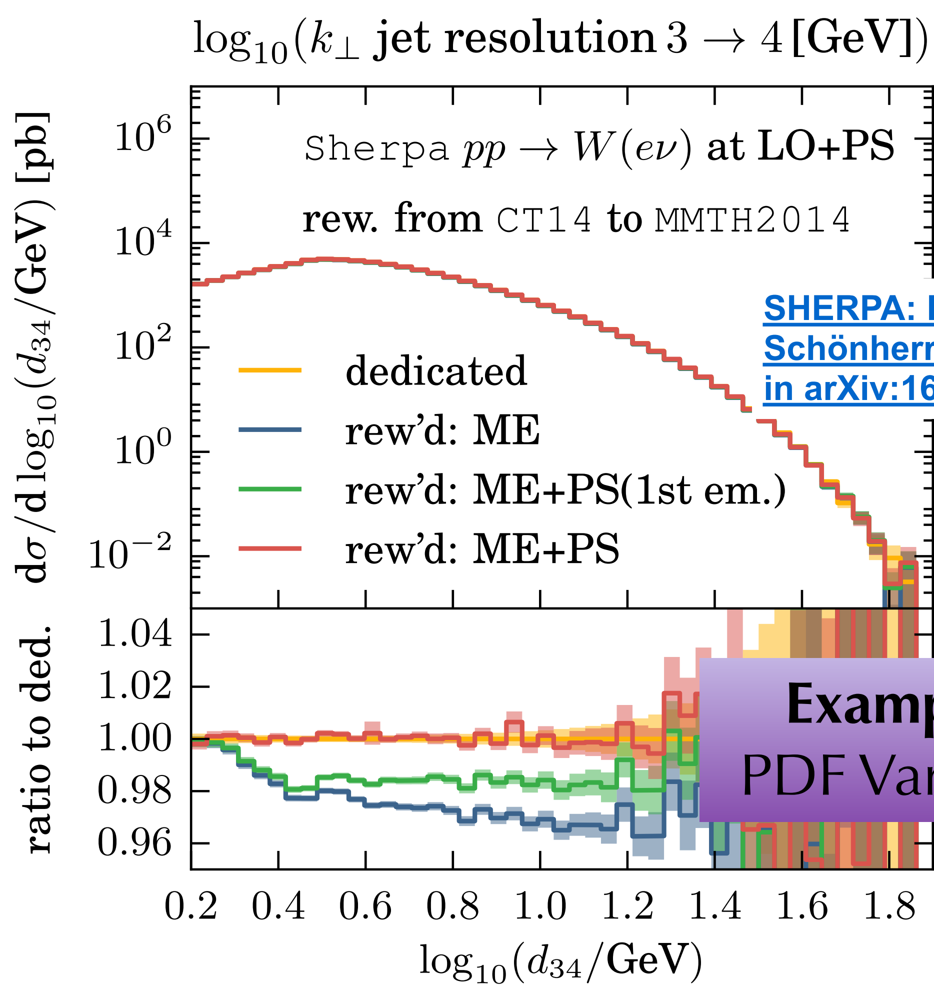
The final states generated by a shower algorithm will depend on

1. The choice of perturbative evolution variable(s) $t^{[i]}$. ← Ordering & Evolution-scale choices
2. The choice of phase-space mapping $\mathrm{d}\Phi_{n+1}^{[i]}/\mathrm{d}\Phi_n$. ← Recoils, kinematics
3. The choice of radiation functions a_i, as a function of the phase-space variables. ← Non-singular terms, Reparametrizations, Subleading Colour
4. The choice of renormalization scale function μ_R.
5. Choices of starting and ending scales. ← Phase-space limits / suppressions for hard radiation and choice of hadronization scale

→ gives us additional handles for uncertainty estimates, beyond just μ_R
(+ ambiguities can be reduced by including more pQCD → matching!)

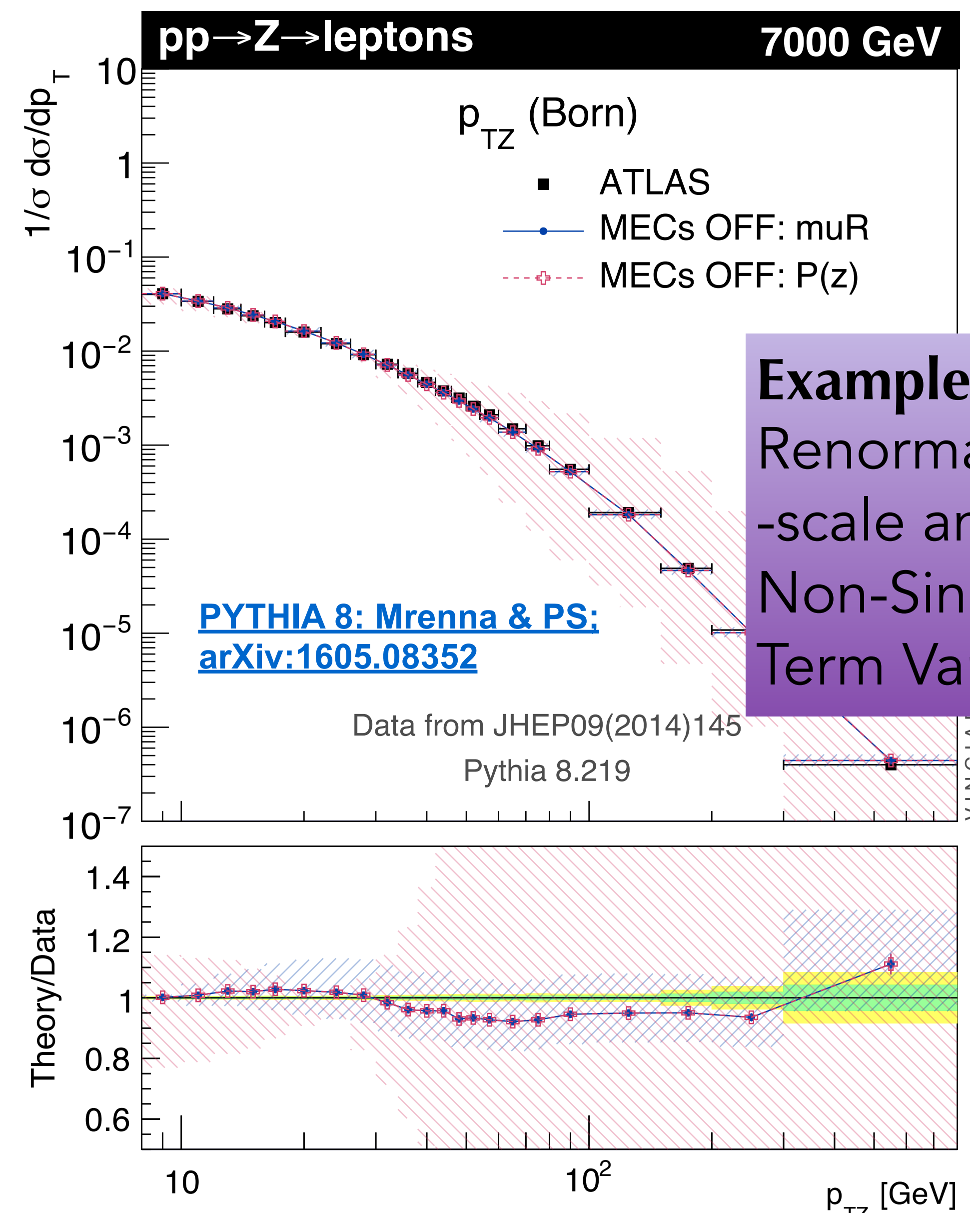
Recently, HERWIG, PYTHIA & SHERPA all published papers on automated calculations of shower uncertainties (based on tricks with the Sudakov algorithm)

Weight of event = { 1 , 0.7, 1.2, … }



SHERPA: Bothmann, Schönherr, Schumann; in arXiv:1605.04692

Example 1: PDF Variations



PYTHIA 8: Mrenna & PS; arXiv:1605.08352

Example 2: Renormalisation -scale and Non-Singular Term Variations

See also HERWIG++ : Bellm et al., arXiv:1605.08256

VINCIA: Giele, Kosower PS; arXiv:1102.2126

Encouraged to start using those, and provide feedback

Fixed Order Paradigm: consider a single physical process

Explicit solutions, process-by-process (to some extent automated)

- Standard-Model: typically NLO or NNLO
- Beyond-SM: typically LO or NLO

Accurate for hard process, to given perturbative order

Limited generality

Event Generators (Showers): consider all physical processes

Universal solutions, applicable to any/all processes

- Process-dependence = subleading correction (→ matrix-element corrections)

Maximum generality

- Common property of all processes is, e.g., limits in which they factorise!

Accurate in strongly ordered (soft/collinear) limits (=bulk of radiation)

MCnet

From $\overline{\text{MS}}$ to MC

PDG: 0.119
ME : 0.127
PS: 0.138

$\alpha_s(M_Z)$

CMW Nucl Phys B 349 (1991) 635 : Drell-Yan and DIS processes

$$P(\alpha_s, z) = \frac{\alpha_s}{2\pi} \underbrace{C_F}_{A^{(1)}} \frac{1+z^2}{1-z} + \left(\frac{\alpha_s}{\pi}\right)^2 \frac{A^{(2)}}{1-z}$$

Eg Analytic resummation (in Mellin space): General Structure

$$\propto \exp\left[\int_0^1 dz \frac{z^{N-1}-1}{1-z}\left[\int \frac{dp_\perp^2}{p_\perp^2}\left(A(\alpha_s) + \underbrace{B(\alpha_s)}_{\text{for DIS}}\right)\right]\right]$$

$$B^{(1)} = -3C_F/2$$

$$A(\alpha_s) = A^{(1)}\frac{\alpha_s}{\pi} + A^{(2)}\left(\frac{\alpha_s}{\pi}\right)^2 + \ldots$$

$$A^{(2)} = \frac{1}{2}C_F\left(C_A\left(\frac{67}{18} - \frac{1}{6}\pi^2\right) - \frac{5}{9}N_F\right) = \frac{1}{2}C_F K_{\mathrm{CMW}}$$

Replace

(for z→1: soft gluon limit):

$$P_i(\alpha_s, z) = \frac{C_i \frac{\alpha_s}{\pi}\left(1 + K_{\mathrm{CMW}}\frac{\alpha_s}{2\pi}\right)}{1-z}$$

PDG: 0.119
ME : 0.127
PS: 0.138

$\alpha_s(M_Z)$

From $\overline{\mathrm{MS}}$ to MC

CMW Nucl Phys B 349 (1991) 635 : Drell-Yan and DIS processes

$$P(\alpha_s, z) = \frac{\alpha_s}{2\pi} \overbrace{C_F}^{A^{(1)}} \frac{1+z^2}{1-z} + \left(\frac{\alpha_s}{\pi}\right)^2 \frac{A^{(2)}}{1-z}$$

Replace
(for z→1: soft gluon limit):

$$P_i(\alpha_s, z) = \frac{C_i \frac{\alpha_s}{\pi}\left(1 + K_{\mathrm{CMW}} \frac{\alpha_s}{2\pi}\right)}{1-z}$$

$$\alpha_s^{(\mathrm{MC})} = \alpha_s^{(\overline{\mathrm{MS}})} \left(1 + K_{\mathrm{CMW}} \frac{\alpha_s^{(\overline{\mathrm{MS}})}}{2\pi}\right)$$

$$\Lambda_{\mathrm{MC}} = \Lambda_{\overline{\mathrm{MS}}} \exp\left(\frac{K_{\mathrm{CMW}}}{4\pi\beta_0}\right) \sim 1.57\Lambda_{\overline{\mathrm{MS}}}$$

(for nF=5)

Note also: used $mu^2 = p_T^2 = (1\text{-}z)Q^2$
Amati, Bassetto, Ciafaloni, Marchesini, Veneziano, 1980

Main Point:
Doing an uncompensated scale variation actually ruins this result

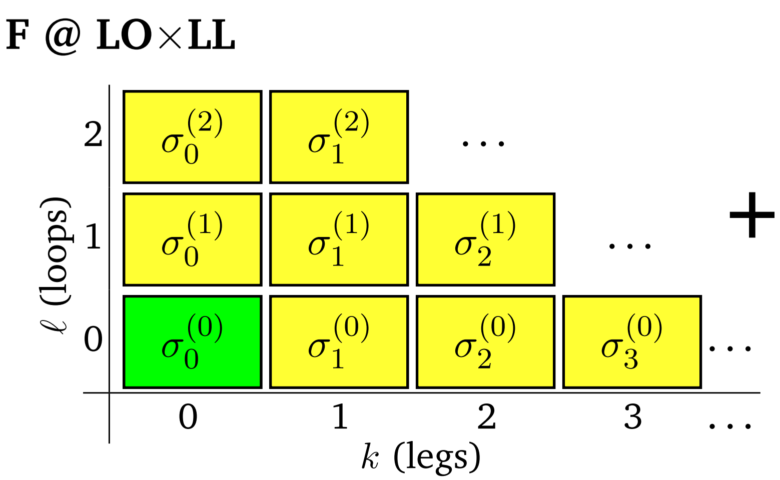
Nice to have all-orders solution

But it is only exact in the singular (soft & collinear) limits

→ gets the bulk of bremsstrahlung corrections right, but fails equally spectacularly: for hard wide-angle radiation: **visible, extra jets**

… which is exactly where fixed-order calculations work!

So combine them!



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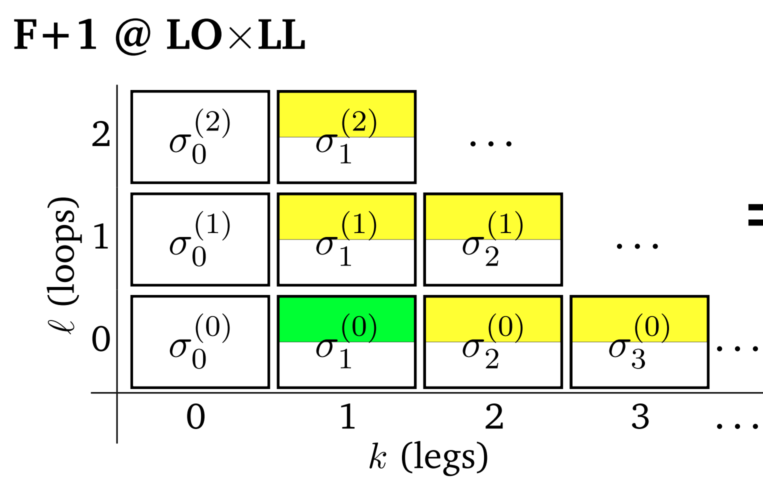

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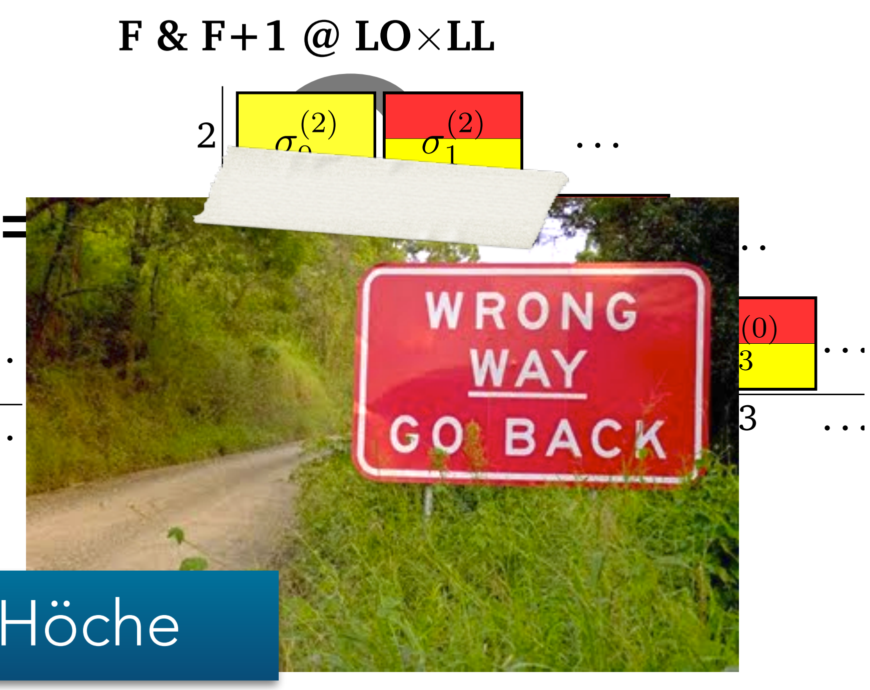
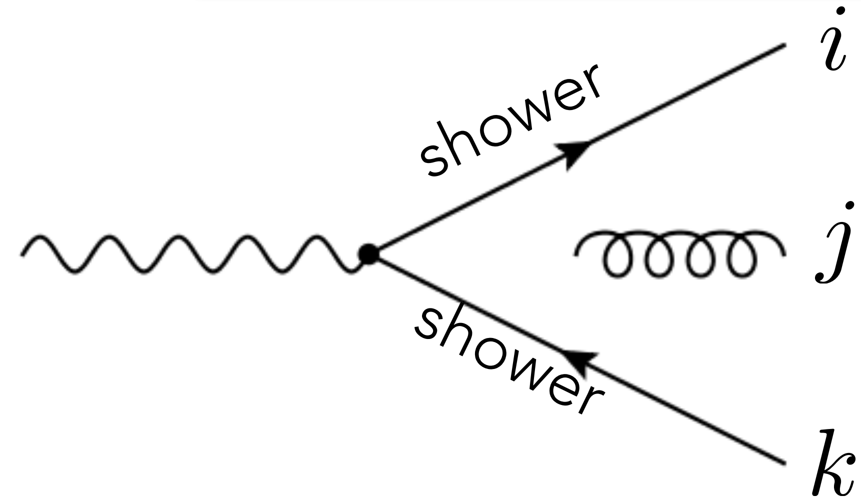


→ Matching Lectures by Stefan Höche

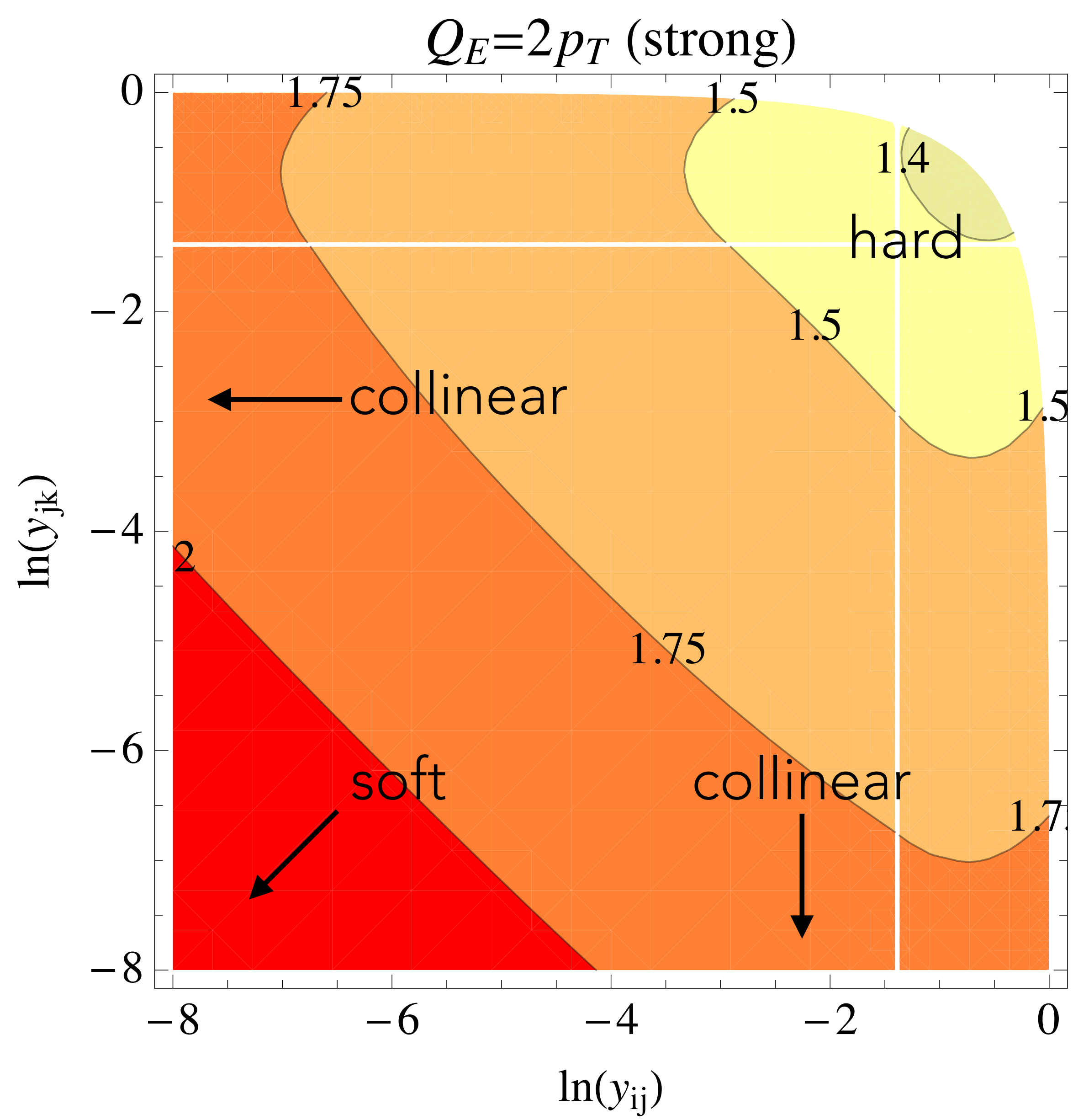
Z → 3 Jets

Size of NLO "K" factor over phase space

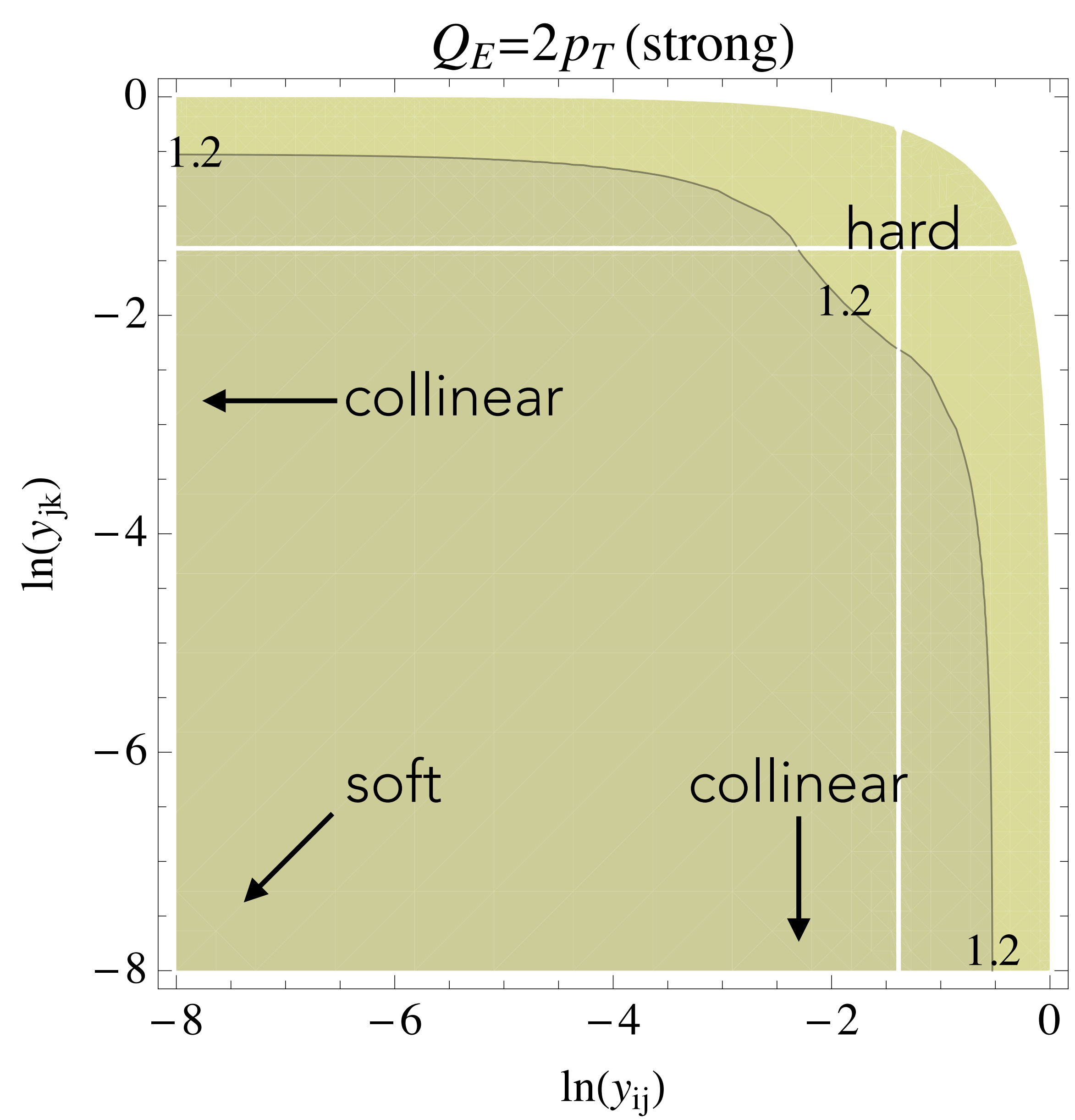


$$y_{ij} = \frac{s_{ij}}{M_Z^2}$$

$$y_{jk} = \frac{s_{jk}}{M_Z^2}$$



(a) $\mu_{\rm PS} = \sqrt{s}$



(b) $\mu_{\rm PS} = p_\perp$

Hartgring, Laenen, Skands, arXiv:1303.4974

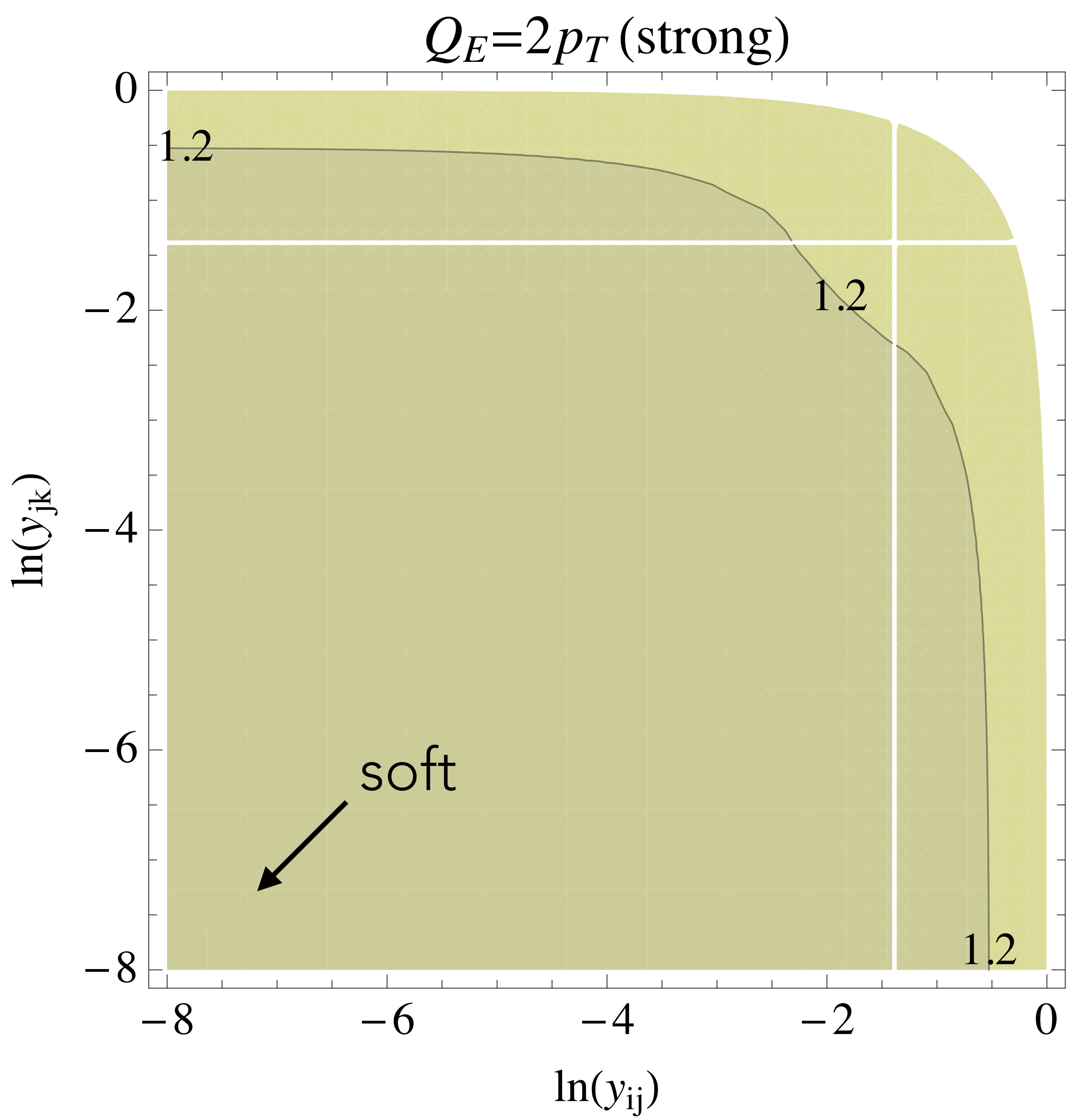
Z → 3 Jets   Size of NLO "K" factor over phase space

The "CMW" factor : Constant shift by

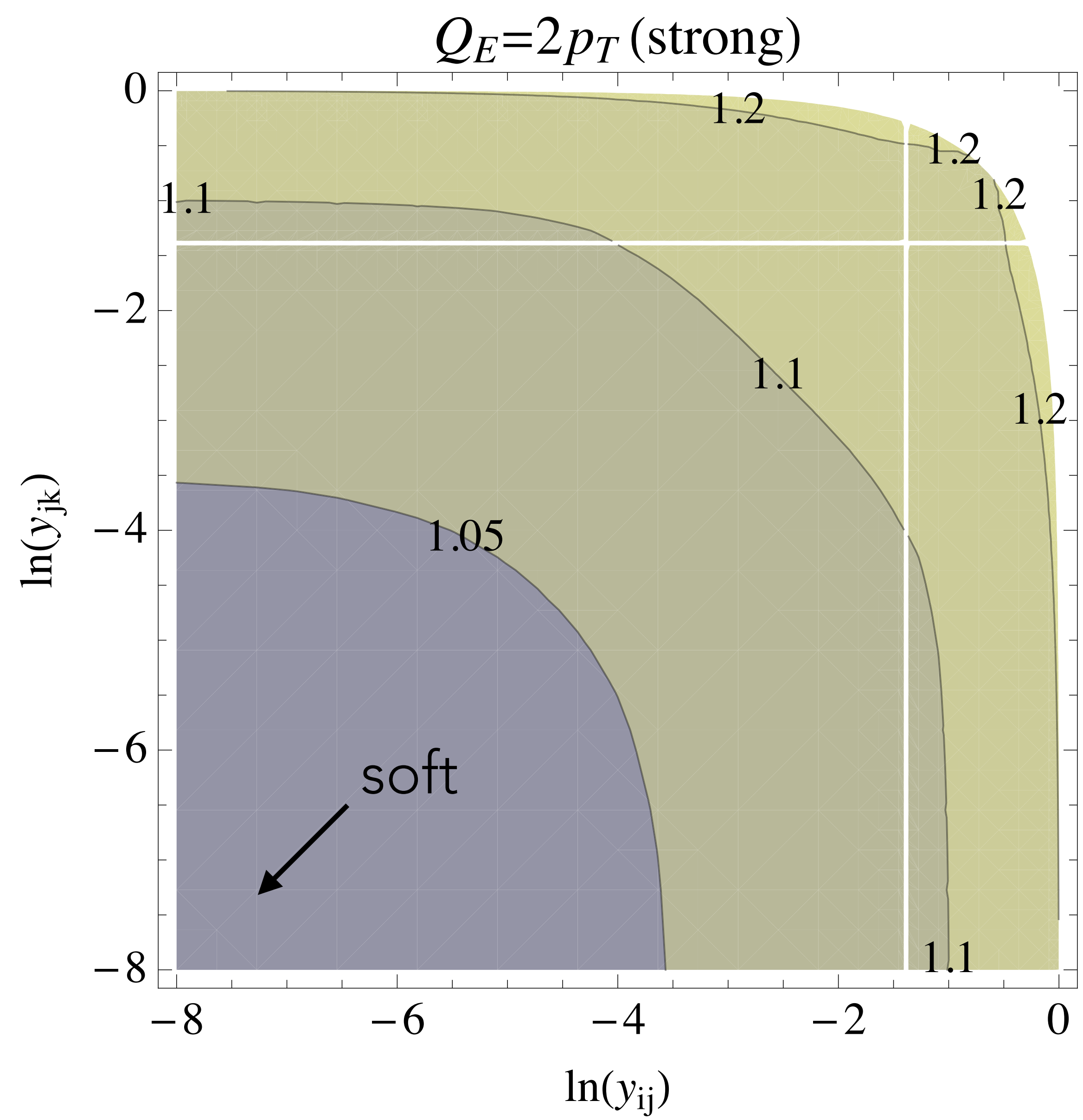
$$k_{\mathrm{CMW}} = \exp\left(\frac{67 - 3\pi^2 - 10n_F/3}{2(33 - 2n_F)}\right) = \begin{cases} 1.513 & n_F = 6 \\ 1.569 & n_F = 5 \\ 1.618 & n_F = 4 \\ 1.661 & n_F = 3 \end{cases}$$

Catani, Marchesini, Webber, NPB349

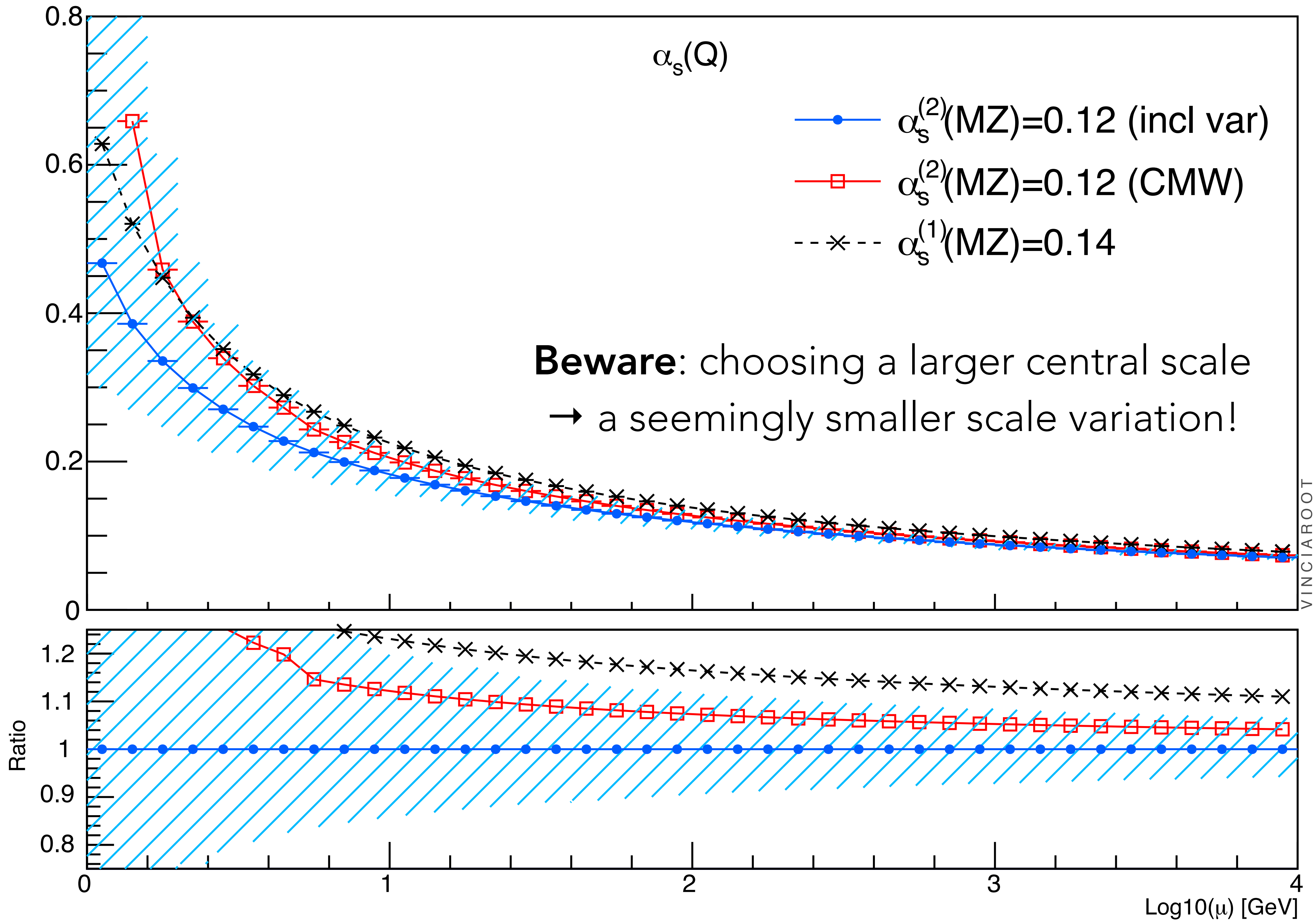
$$\frac{\alpha_s}{2\pi}\frac{\beta_0}{2}\ln\left(k^2_{\mathrm{CMW}}\right) \sim 0.07$$



(b) $\mu_{\mathrm{PS}} = p_\perp$



$\mu_{\mathrm{PS}} = p_\perp$, with CMW



2 Loop: $\alpha_s(M_Z)$=0.12 Λ_3 = 0.37 Λ_4 = 0.32 Λ_5 = 0.23

1 Loop: $\alpha_s(M_Z)$=0.14 Λ_3 = 0.37 Λ_4 = 0.33 Λ_5 = 0.26

(In all cases, 5-flavor running is still used above mt)

DGLAP for Parton Density

$$\frac{\mathrm{d}f_b(x,t)}{\mathrm{d}t} = \sum_{a,c} \int \frac{\mathrm{d}x'}{x'} f_a(x',t) \frac{\alpha_{abc}}{2\pi} P_{a\to bc}\left(\frac{x}{x'}\right)$$

→ Sudakov for ISR

$$\begin{aligned} \Delta(x, t_{\max}, t) &= \exp\left\{ - \int_t^{t_{\max}} \mathrm{d}t' \sum_{a,c} \int \frac{\mathrm{d}x'}{x'} \frac{f_a(x',t')}{f_b(x,t')} \frac{\alpha_{abc}(t')}{2\pi} P_{a\to bc}\left(\frac{x}{x'}\right) \right\} \\ &= \exp\left\{ - \int_t^{t_{\max}} \mathrm{d}t' \sum_{a,c} \int \mathrm{d}z \frac{\alpha_{abc}(t')}{2\pi} P_{a\to bc}(z) \frac{x' f_a(x',t')}{x f_b(x,t')} \right\} , \end{aligned}$$

THE SHOWER OPERATOR

H = Hard process

Born $\left.\frac{d\sigma_H}{d\mathcal{O}}\right|_{\text{Born}} = \int d\Phi_H \; |M_H^{(0)}|^2 \, \delta(\mathcal{O} - \mathcal{O}(\{p\}_H))$

{p} : partons

But instead of evaluating O directly on the Born final state, first insert a showering operator

Born + shower $\left.\frac{d\sigma_H}{d\mathcal{O}}\right|_{\mathcal{S}} = \int d\Phi_H \; |M_H^{(0)}|^2 \, \mathcal{S}(\{p\}_H, \mathcal{O})$

{p} : partons

S : showering operator

Unitarity: to first order, S does nothing

$$\mathcal{S}(\{p\}_H, \mathcal{O}) = \delta\left(\mathcal{O} - \mathcal{O}(\{p\}_H)\right) \; + \; \mathcal{O}(\alpha_s)$$

To ALL Orders

(Markov Chain)

$$S(\{p\}_X, \mathcal{O}) = \Delta(t_{\mathrm{start}}, t_{\mathrm{had}})\delta(\mathcal{O}-\mathcal{O}(\{p\}_X))$$

"Nothing Happens" → "Evaluate Observable"

$$- \int_{t_{\mathrm{start}}}^{t_{\mathrm{had}}} \mathrm{d}t \frac{\mathrm{d}\Delta(t_{\mathrm{start}}, t)}{\mathrm{d}t} S(\{p\}_{X+1}, \mathcal{O})$$

"Something Happens" → "Continue Shower"

All-orders Probability that nothing happens

$$\Delta(t_1, t_2) = \exp\left(-\int_{t_1}^{t_2} \mathrm{d}t \, \frac{\mathrm{d}\mathcal{P}}{\mathrm{d}t}\right)$$

(Exponentiation)

Analogous to nuclear decay

N(t) ≈ N(0) exp(-ct)