Monte Carlos and New Physics

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Lecture 2:
Parton Showers and Matrix-Element Corrections

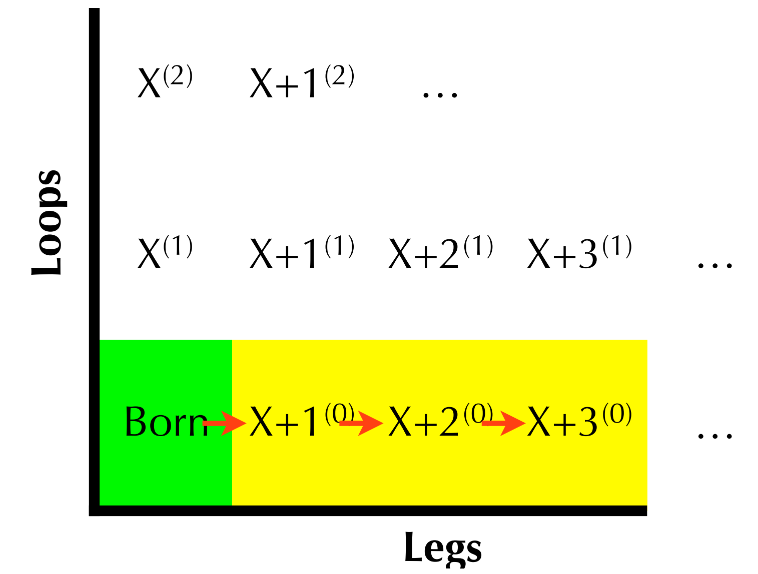
Pre-SUSY - June 2016

Lecture Notes: P. Skands, arXiv:1207.2389

Recap from Yesterday: Loops and Legs

Factorisation of amplitudes (squared) → approximate all-orders fractal

Jet-within-a-jet-within-a-jet-...

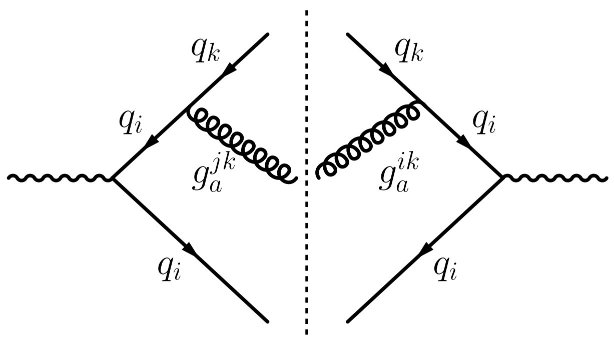


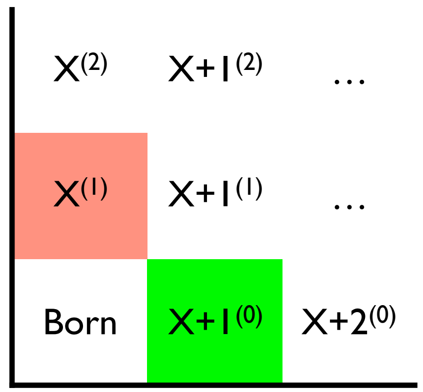
Universality (scaling)

$$\frac{|M_{X+1}|^2}{|M_X|^2} \propto \quad g_s^2\, 2C_F \left[\frac{2s_{ik}}{s_{ij}s_{jk}} + \frac{1}{s_{IK}} \left(\frac{s_{ij}}{s_{jk}} + \frac{s_{jk}}{s_{ij}} \right) \right]$$

Universal **poles** for soft & collinear bremsstrahlung

$$\sigma^{\mathrm{LO}}_{\mathrm{X+1}}(R) = \int_R |M^{(0)}_{X+1}|^2$$





R = some "Infrared Safe" phase space region (Often a cut on $p_\perp > X$ GeV)

The corrections from Quantum Loops are missing

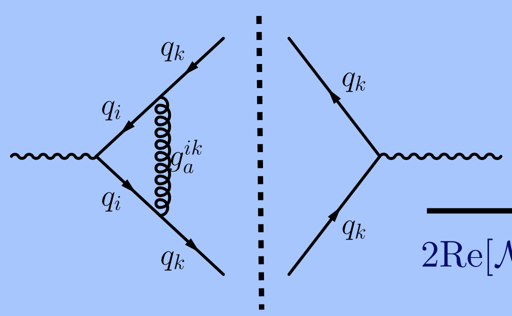
We know from Unitarity (KLN): Real + Virtual = Finite

From Legs to Loops

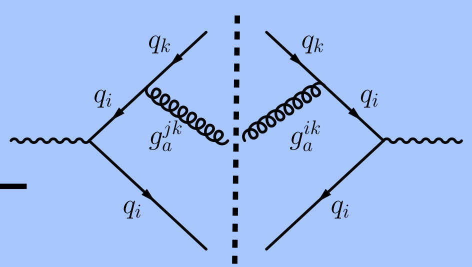
see PS, *Introduction to QCD*, TASI 2012, arXiv:1207.2389

Unitarity: sum(probability) = 1

Kinoshita-Lee-Nauenberg:
(sum over degenerate quantum states = finite: infinities must cancel!)



$$\xrightarrow{2\mathrm{Re}[\mathcal{M}^{(1)}\mathcal{M}^{(0)*}]} \quad \mathrm{Loop} = -\int \mathrm{Tree} + F \quad \xleftarrow{\left|\mathcal{M}^{(0)}_{+1}\right|^2}$$



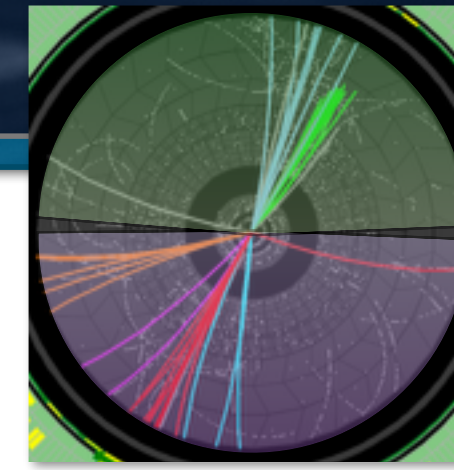
Neglect non-singular piece, F → "Leading-Logarithmic" (LL) Approximation

→ Can also include loops-within-loops-within-loops …
→ Bootstrap for approximate All-Orders Quantum Corrections!

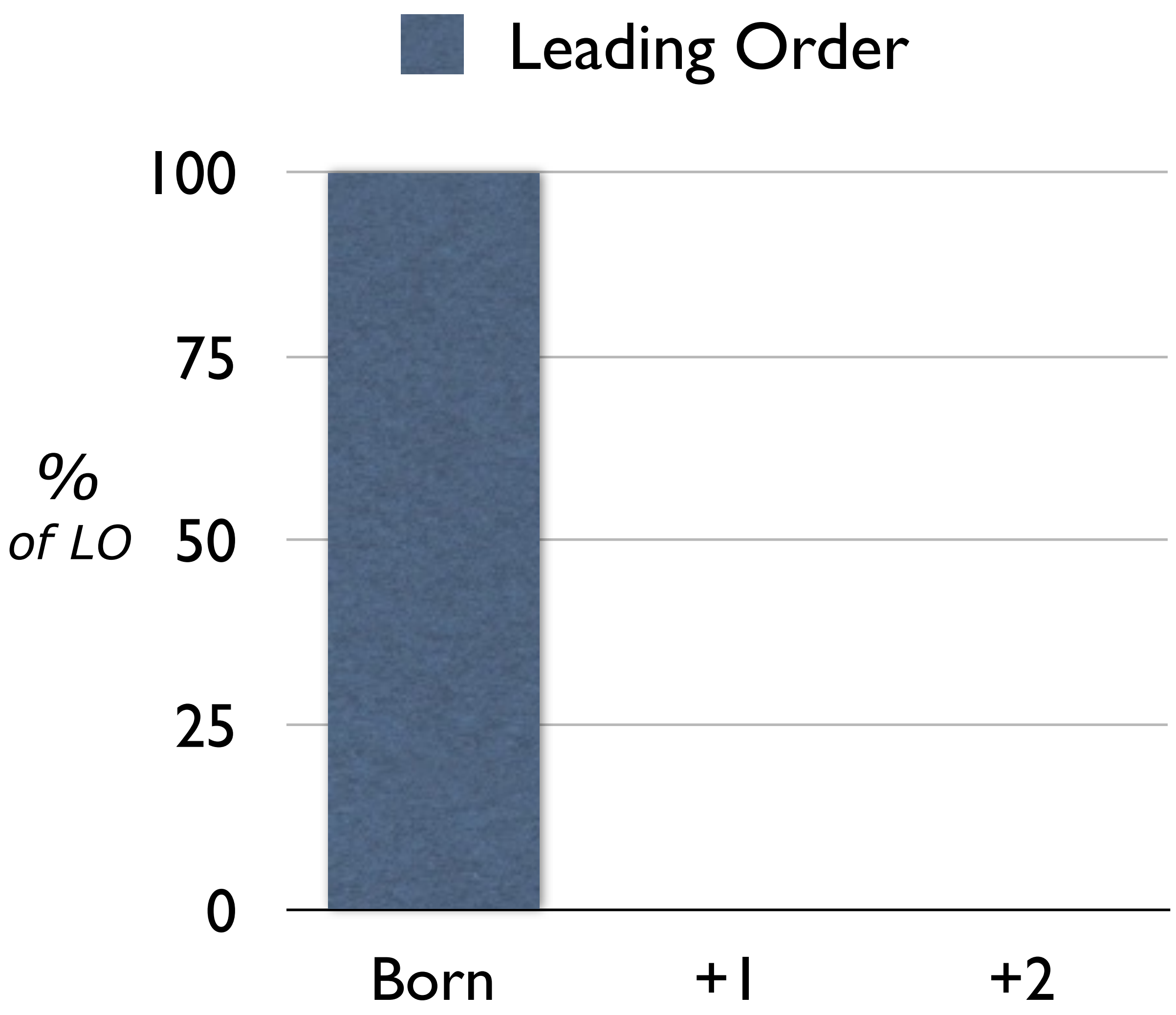
Parton Showers: reformulation of pQCD corrections as gain-loss diff eq.

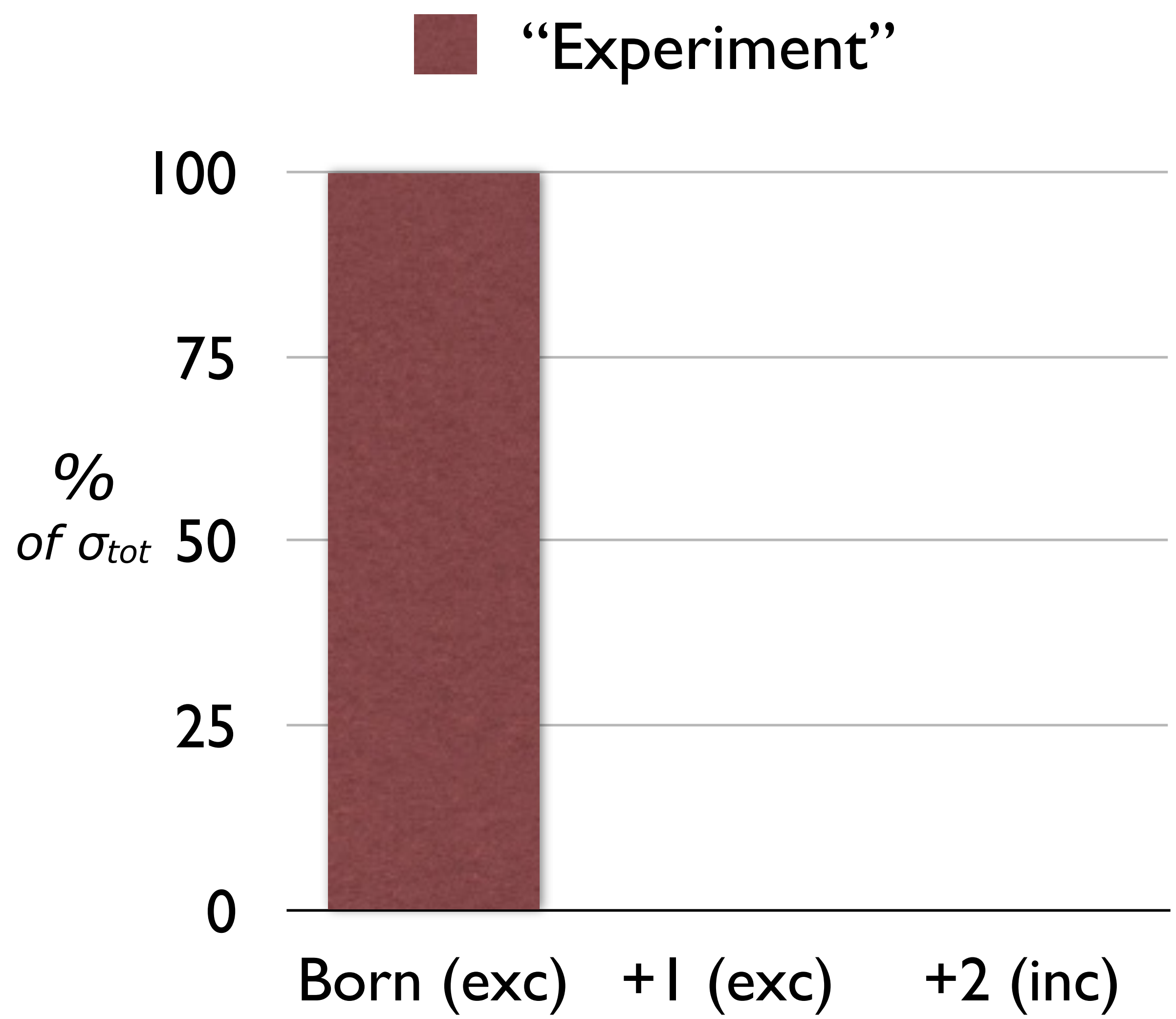
- Iterative (Markov-Chain) evolution algorithm, based on universality and unitarity
- With evolution kernel ~ $\frac{|\mathcal{M}_{n+1}|^2}{|\mathcal{M}_n|^2}$ (or soft/collinear approx thereof)
- Generate explicit fractal structure across all scales (via Monte Carlo Simulation)
- Evolve in some measure of *resolution* ~ hardness, virtuality, 1/time … ~ fractal scale
- + account for scaling violation via quark masses and $g_s^2 \to 4\pi\alpha_s(Q^2)$

Evolution



$$Q \sim Q_{\mathrm{HARD}}$$



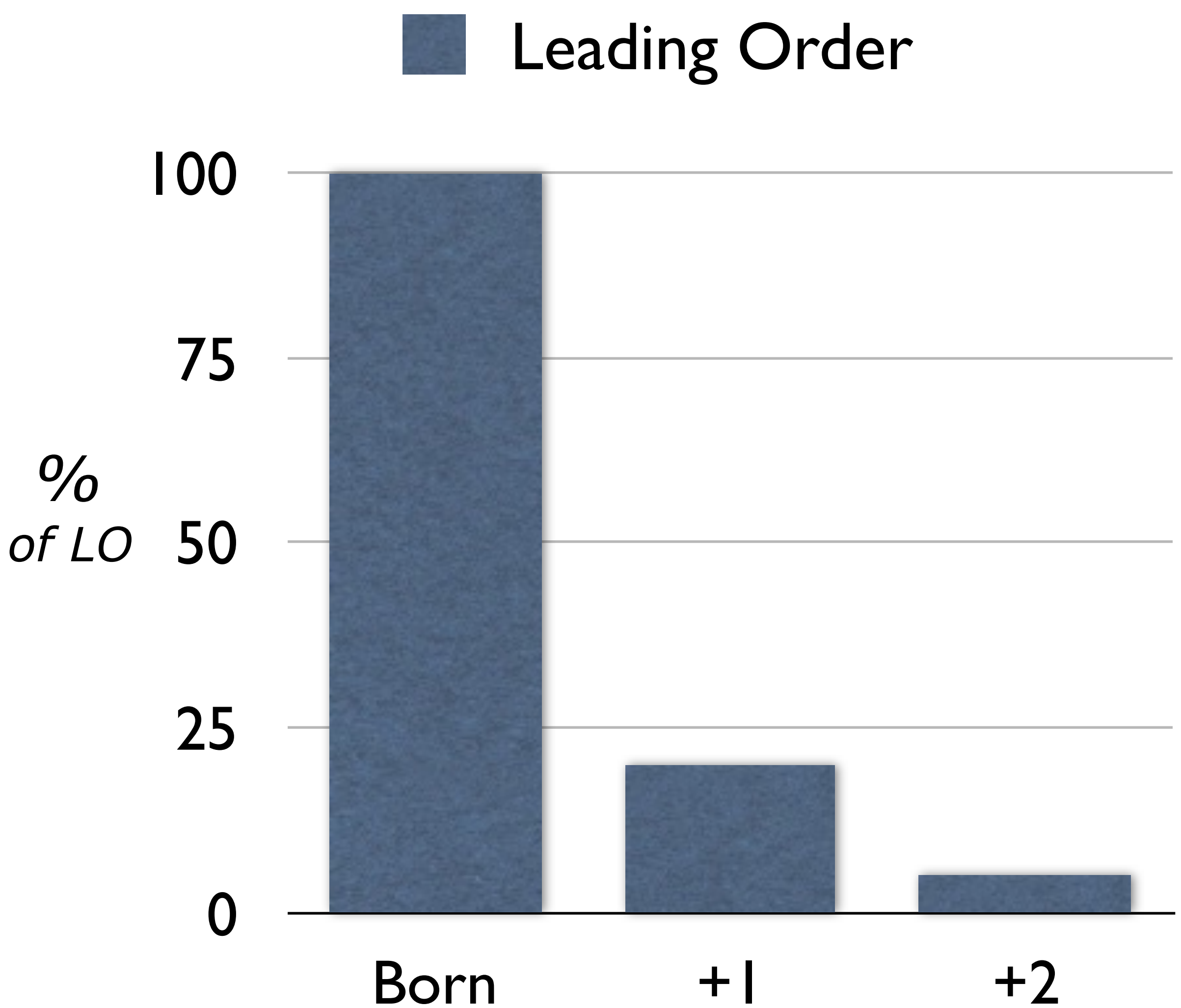


Exclusive = n and only n jets

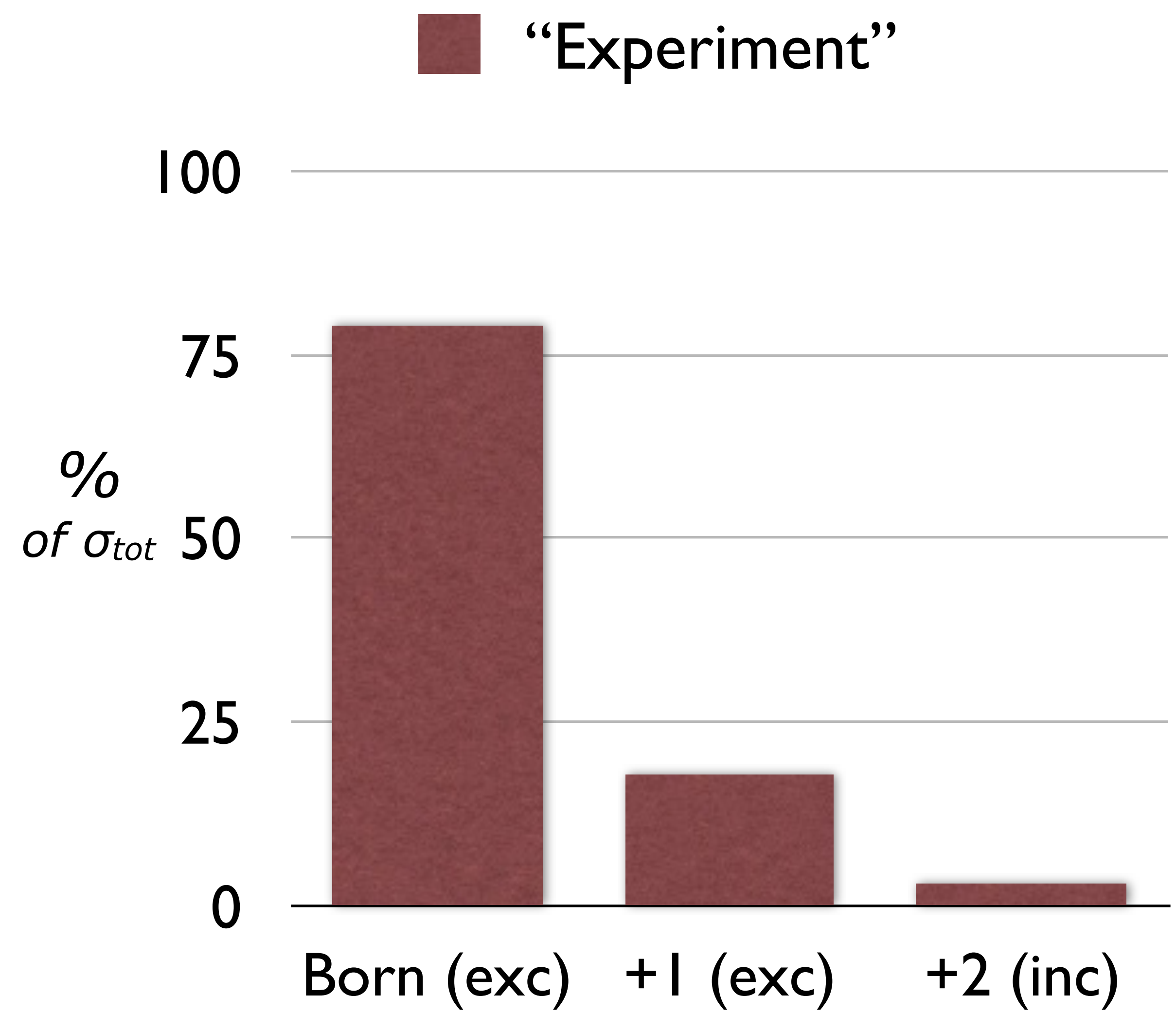
Inclusive = n or more jets

Evolution

$$Q_{\mathrm{HARD}}/Q < \text{“A few”}$$



Separate Matrix-Element Calculations

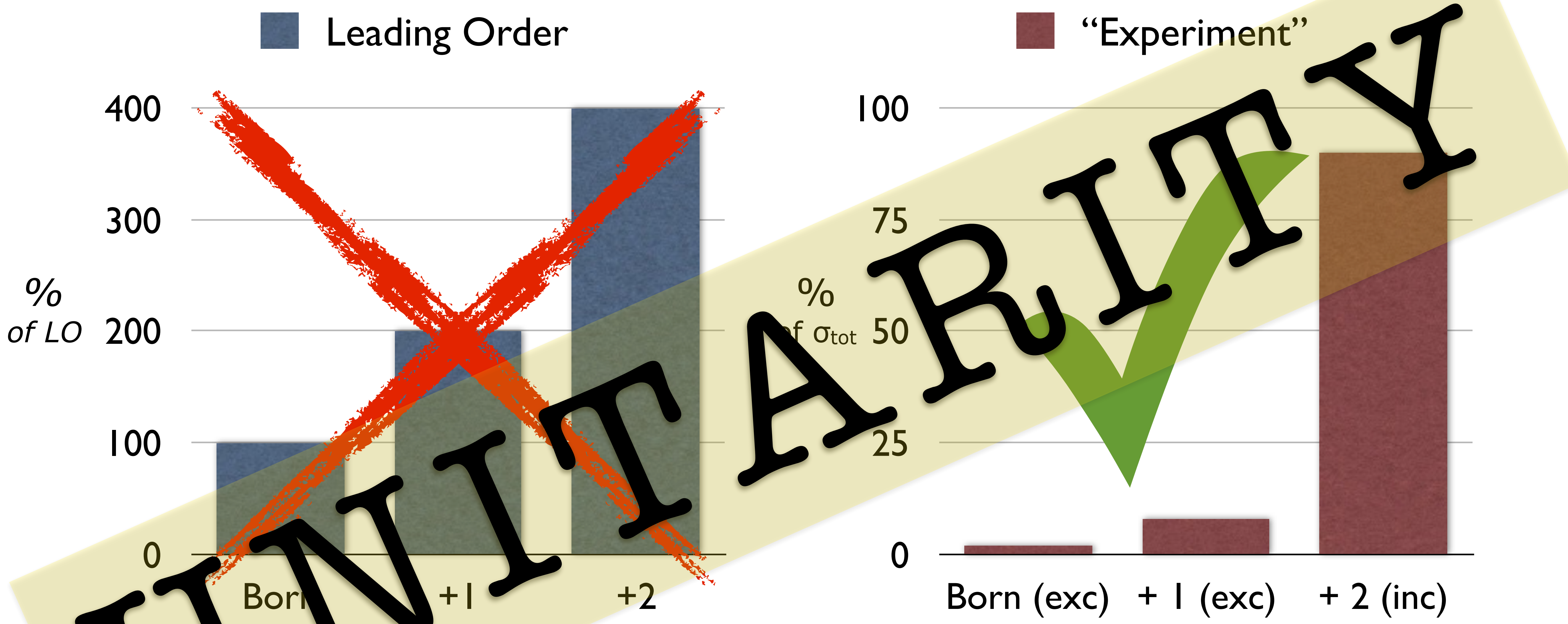


Exclusive = n and only n jets

Inclusive = n or more jets

Evolution

$$Q \ll Q_{\mathrm{HARD}}$$



Cross Section Diverges
cf our 50-GeV example yesterday

Cross Section Remains = Total (IR safe)
Number of Partons Diverges (IR unsafe)

Evolution Equations

What we need is a differential equation

Boundary condition: a few partons defined at a high scale (Q_F)

Then evolves (or "runs") that parton system down to a low scale (the hadronization cutoff ~ 1 GeV)

→ It's an evolution equation in Q_F

Close analogy: nuclear decay

Evolve an unstable nucleus (+ follow chains of decays)

Decay constant

$$\frac{\mathrm{d}P(t)}{\mathrm{d}t} = c_N$$

Probability to remain undecayed in the time interval [t_1,t_2]

$$\Delta(t_1,t_2) = \exp\left(-\int_{t_1}^{t_2} c_N \,\mathrm{d}t\right) = \exp\left(-c_N\,\Delta t\right)$$

$$= 1 - c_N\Delta t + \mathcal{O}(c_N^2)$$

Decay probability per unit time

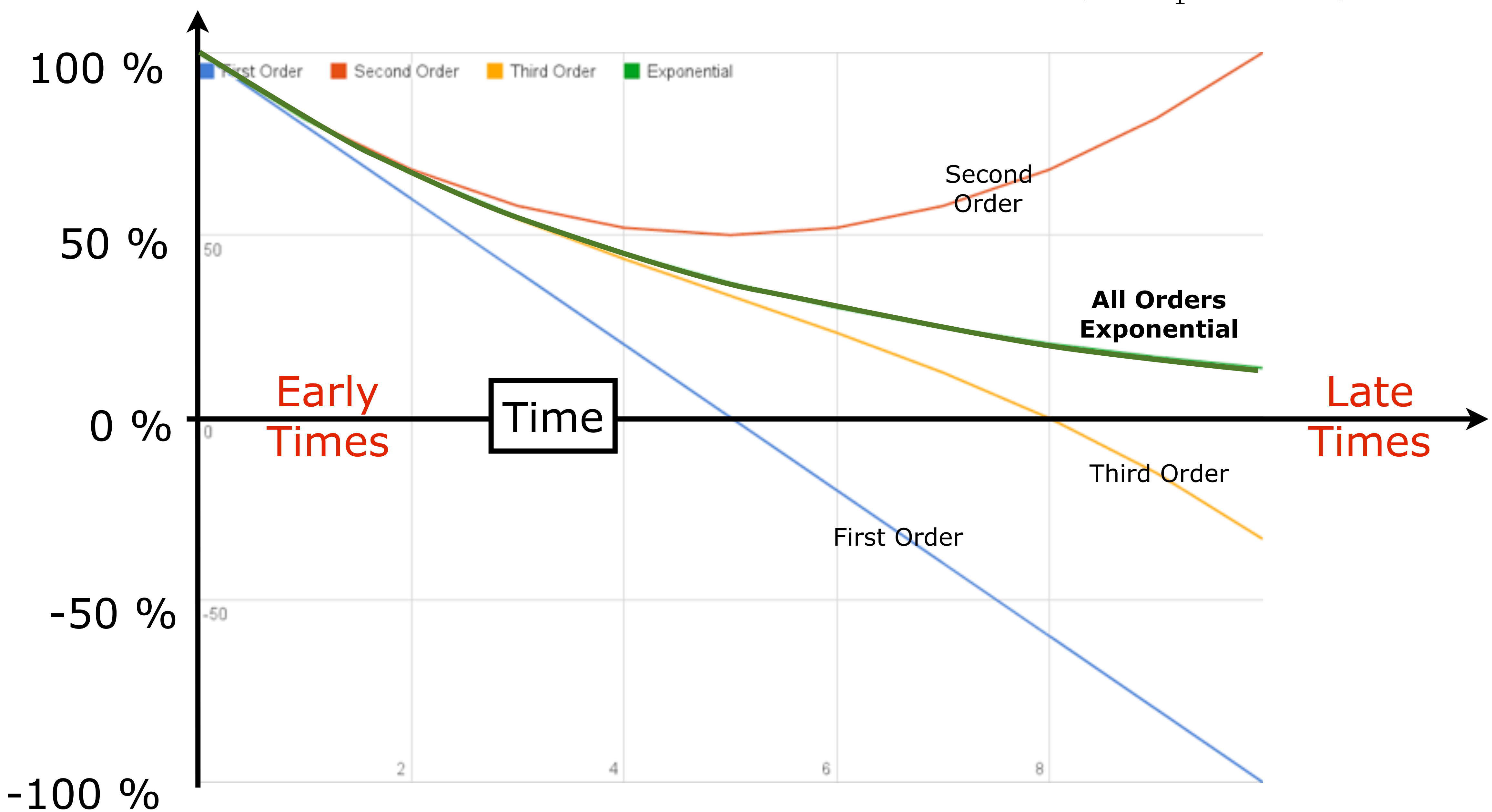
$$\frac{\mathrm{d}P_{\mathrm{res}}(t)}{\mathrm{d}t} = \frac{-\mathrm{d}\Delta}{\mathrm{d}t} = c_N\,\Delta(t_1,t)$$

(requires that the nucleus did not already decay)

$\Delta(t_1,t_2)$: "Sudakov Factor"

Fixed vs Infinite Orders

Nuclei remaining undecayed after time t $= \Delta(t_1, t_2) = \exp\left(- \int_{t_1}^{t_2} \mathrm{d}t \, \frac{\mathrm{d}\mathcal{P}}{\mathrm{d}t} \right)$



The Sudakov Factor

In nuclear decay, the Sudakov factor counts:

How many nuclei remain undecayed after a time t

Probability to remain undecayed in the time interval [t_1,t_2]

$$\Delta(t_1,t_2) = \exp\left(-\int_{t_1}^{t_2} c_N \,\mathrm{d}t\right) \;=\; \exp\left(-c_N\,\Delta t\right)$$

The Sudakov factor for a parton system counts:

The probability that the parton system doesn't evolve (branch) when we run the factorization scale (~1/time) from a high to a low scale

Evolution probability per unit "time"

$$\frac{\mathrm{d}P_{\mathrm{res}}(t)}{\mathrm{d}t} = \frac{-\mathrm{d}\Delta}{\mathrm{d}t} = c_N\,\Delta(t_1,t)$$

(replace *t* by shower evolution scale)

(replace c_N by proper shower evolution kernels)

The Shower Operator

H = Hard process

Born $$\left.\frac{d\sigma_H}{d\mathcal{O}}\right|_{\text{Born}} = \int d\Phi_H \; |M_H^{(0)}|^2 \, \delta(\mathcal{O} - \mathcal{O}(\{p\}_H))$$

{p} : partons

But instead of evaluating O directly on the Born final state, first insert a showering operator

Born + shower $$\left.\frac{d\sigma_H}{d\mathcal{O}}\right|_{\mathcal{S}} = \int d\Phi_H \; |M_H^{(0)}|^2 \, \mathcal{S}(\{p\}_H, \mathcal{O})$$

{p} : partons

S : showering operator

Unitarity: to first order, S does nothing

$$\mathcal{S}(\{p\}_H, \mathcal{O}) = \delta\left(\mathcal{O} - \mathcal{O}(\{p\}_H)\right) \; + \; \mathcal{O}(\alpha_s)$$

The Shower Operator

To ALL Orders

(Markov Chain)

$$S(\{p\}_X, \mathcal{O}) = \Delta(t_{\mathrm{start}}, t_{\mathrm{had}})\delta(\mathcal{O}-\mathcal{O}(\{p\}_X))$$

"Nothing Happens" → "Evaluate Observable"

$$- \int_{t_{\mathrm{start}}}^{t_{\mathrm{had}}} \mathrm{d}t \frac{\mathrm{d}\Delta(t_{\mathrm{start}}, t)}{\mathrm{d}t} S(\{p\}_{X+1}, \mathcal{O})$$

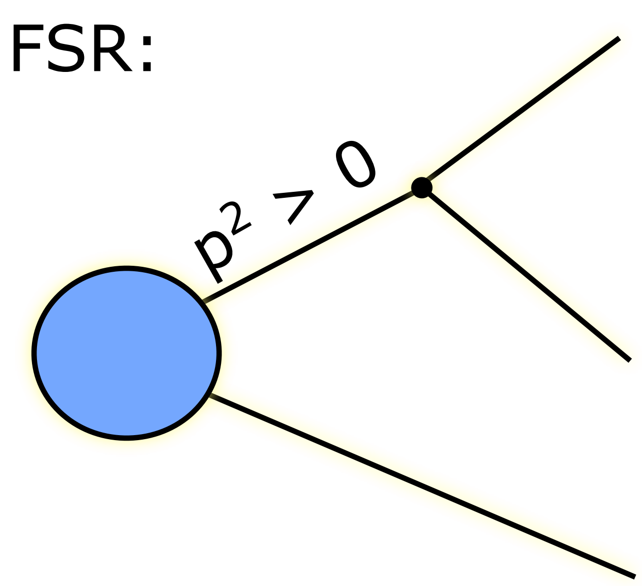
"Something Happens" → "Continue Shower"

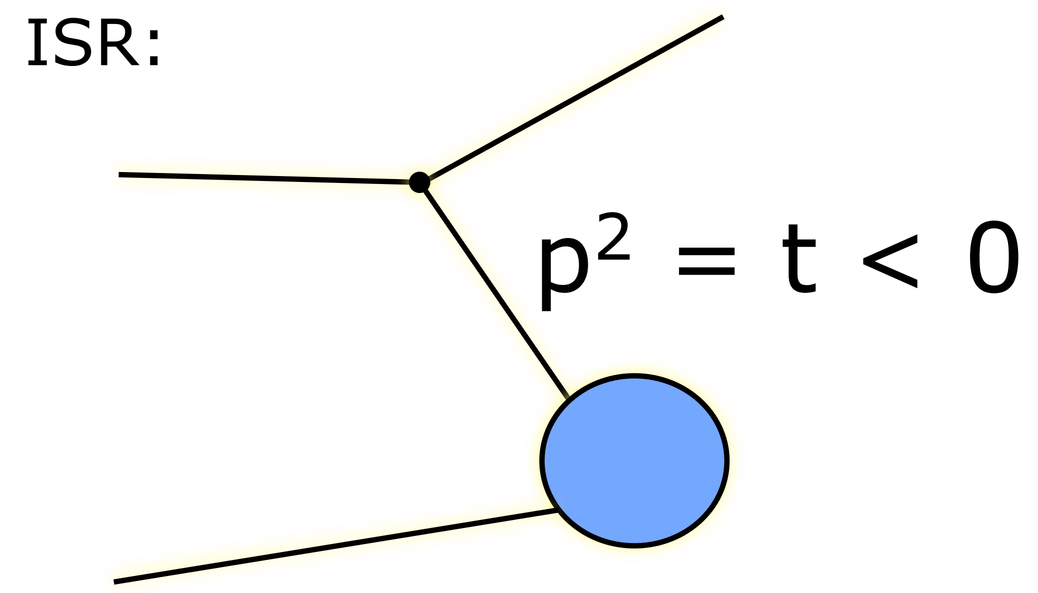
All-orders Probability that nothing happens

$$\Delta(t_1, t_2) = \exp\left(- \int_{t_1}^{t_2} \mathrm{d}t \, \frac{\mathrm{d}\mathcal{P}}{\mathrm{d}t} \right)$$

(Exponentiation)
Analogous to nuclear decay
$N(t) \approx N(0) \exp(-ct)$

Initial-State vs Final-State Evolution





Virtualities are
Timelike: $p^2>0$

Start at $Q^2 = Q_F^2$
"Forwards evolution"

Virtualities are
Spacelike: $p^2<0$

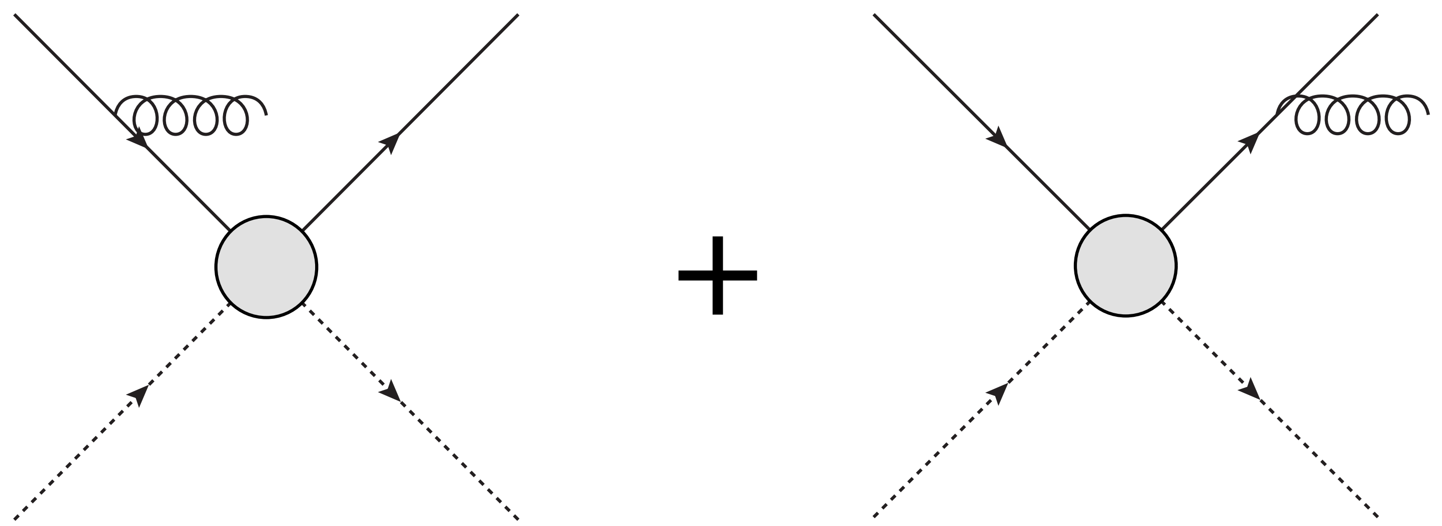
Start at $Q^2 = Q_F^2$
Constrained backwards evolution
towards boundary condition = proton

Separation meaningful for collinear radiation, but not for soft …

Initial-Final Interference

Illustrates **quantum ≠ classical**

Who emitted that gluon?

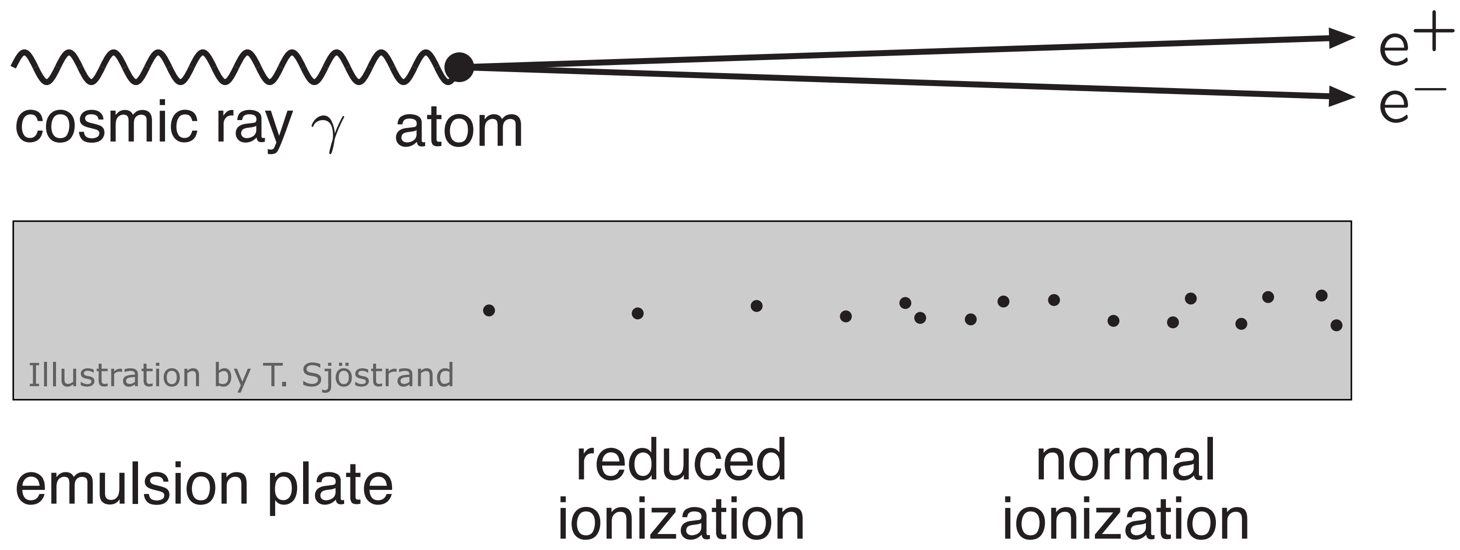


Real QFT = sum over amplitudes, then square → interference (IF coherence)
Respected by dipole/antenna languages (and by angular ordering), but not by conventional DGLAP (→ all PDFs are "wrong")

Separation meaningful for collinear radiation, but not for soft …

Coherence

QED: Chudakov effect (mid-fifties)



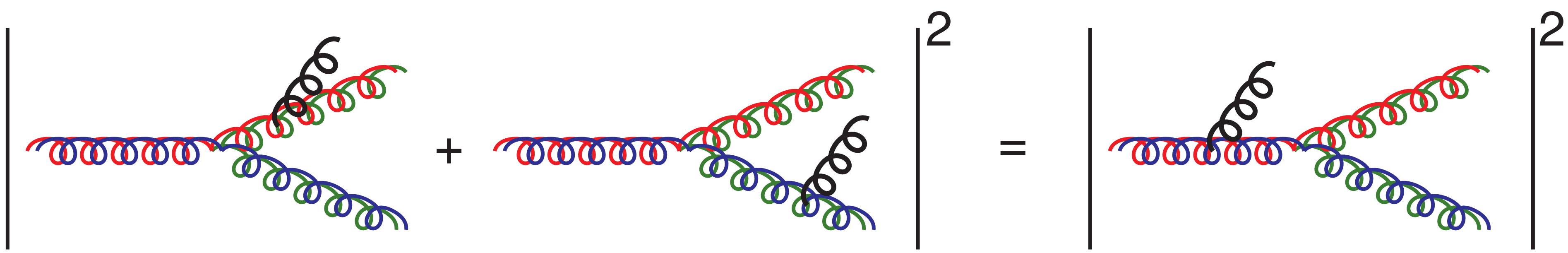
Approximations to Coherence:

Angular Ordering (HERWIG)

Angular Vetos (PYTHIA)

Coherent Dipoles/Antennae (ARIADNE, Catani-Seymour, DIRE, VINCIA)

QCD: colour coherence for **soft** gluon emission



→ an example of an interference effect that can be treated probabilistically

More interference effects can be included by matching to full matrix elements

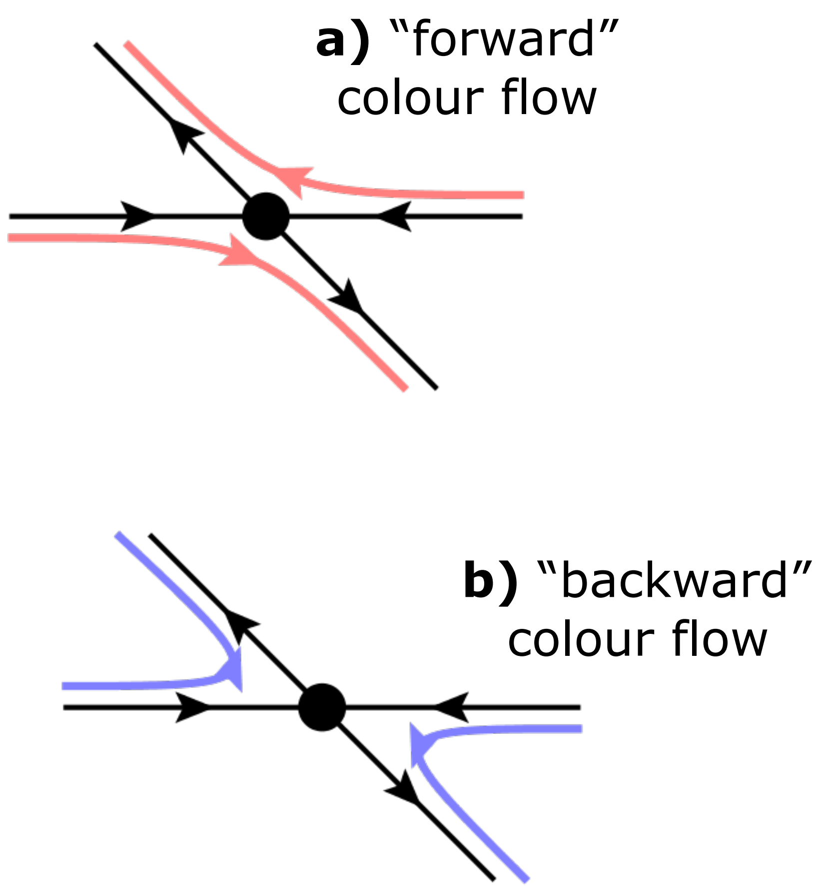
Coherence at Work

Example taken from: Ritzmann, Kosower, PS, PLB718 (2013) 1345

Example: quark-quark scattering in hadron collisions

Consider one specific phase-space point (eg scattering at 45o)

2 possible colour flows: **a** and **b**



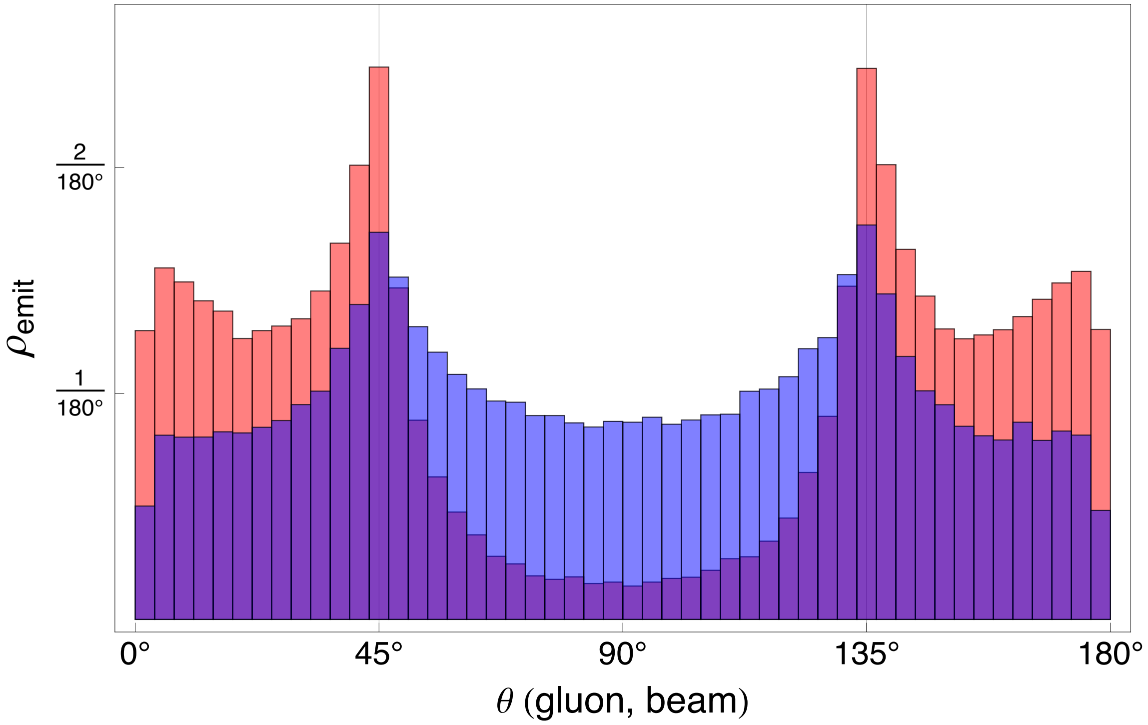


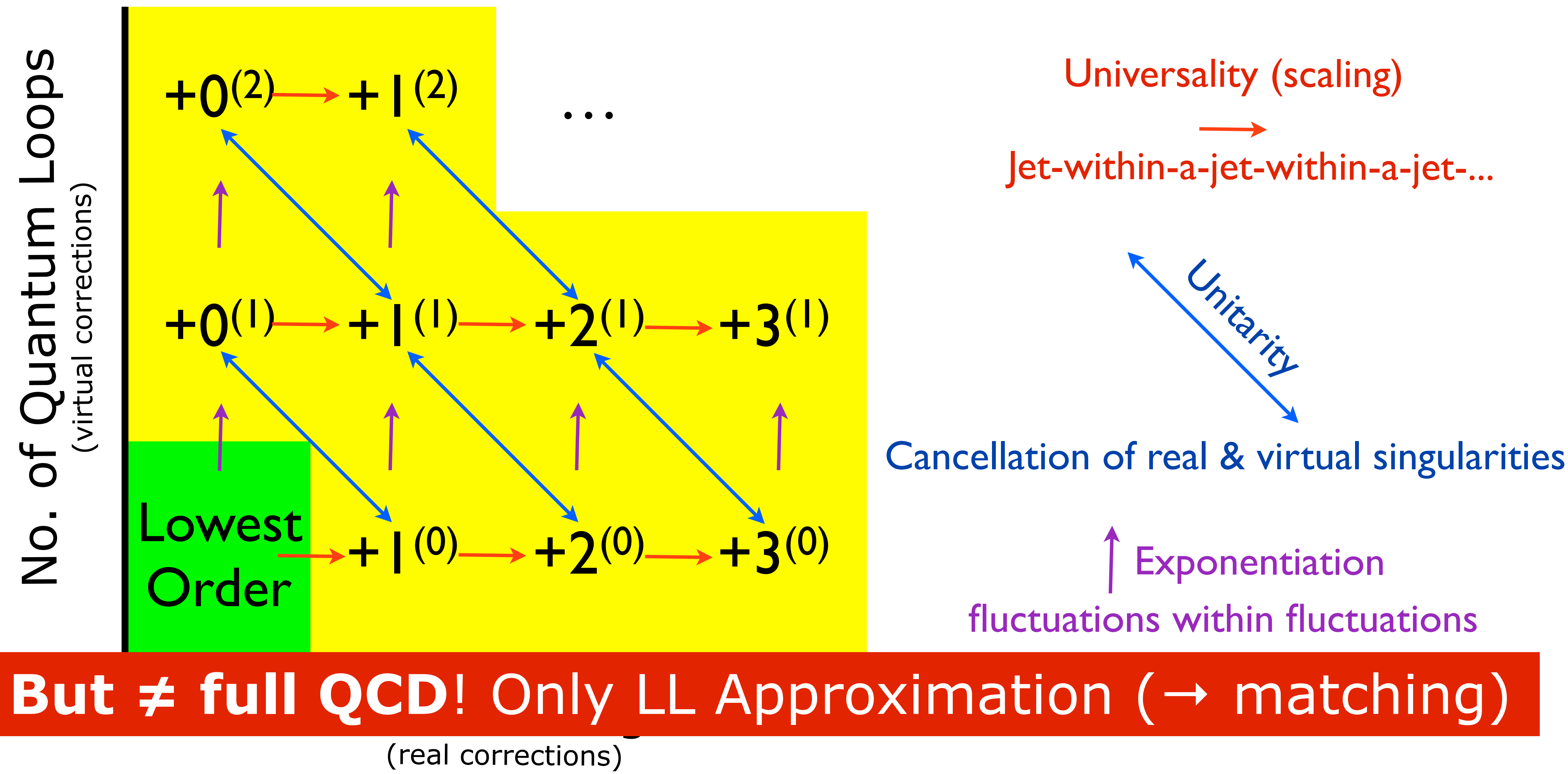
Figure 4: Angular distribution of the first gluon emission in $qq \to qq$ scattering at 45°, for the two different color flows. The light (red) histogram shows the emission density for the forward flow, and the dark (blue) histogram shows the emission density for the backward flow.

Another good recent example is the SM contribution to the Tevatron top-quark forward-backward asymmetry from coherent showers, see: PS, Webber, Winter, JHEP 1207 (2012) 151

Bootstrapped Perturbation Theory

Start from an **arbitrary lowest-order** process (green = QFT amplitude squared)

Parton showers generate the bremsstrahlung terms of the rest of the perturbative series (approximate infinite-order resummation)



But ≠ full QCD! Only LL Approximation (→ matching)

Perturbative Ambiguities

The final states generated by a shower algorithm will depend on

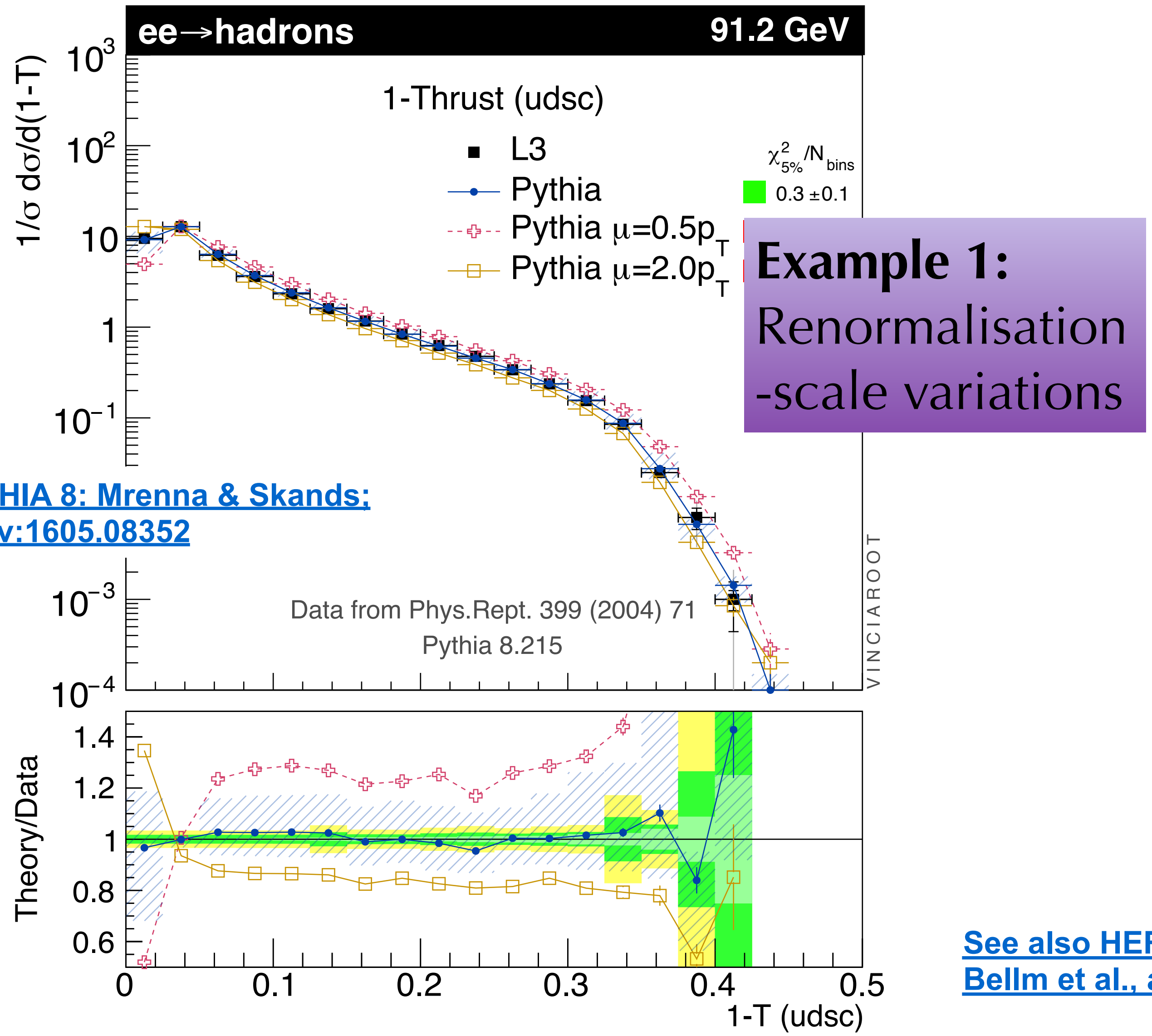
1. The choice of perturbative evolution variable(s) $t^{[i]}$. ← Ordering & Evolution-scale choices
2. The choice of phase-space mapping $\mathrm{d}\Phi_{n+1}^{[i]}/\mathrm{d}\Phi_n$. ← Recoils, kinematics
3. The choice of radiation functions a_i, as a function of the phase-space variables. ← Non-singular terms, Reparametrizations, Subleading Colour
4. The choice of renormalization scale function μ_R.
5. Choices of starting and ending scales. ← Phase-space limits / suppressions for hard radiation and choice of hadronization scale

→ can give additional handles for uncertainty estimates, beyond just μ_R
(+ ambiguities can be reduced by including more pQCD → matching!)

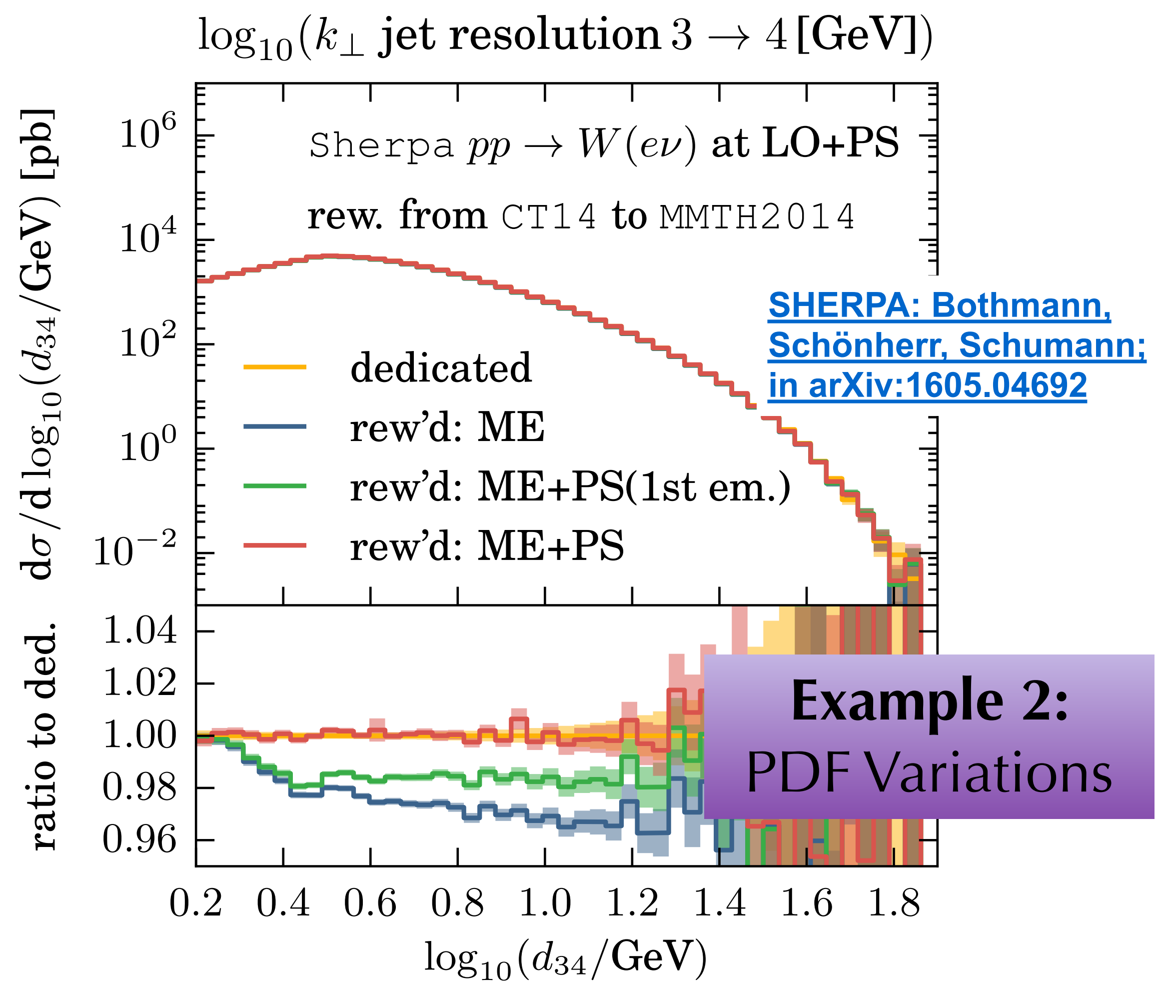
Very recently, HERWIG, SHERPA, PYTHIA, all published papers on automated calculations of shower uncertainties

Weight of event = { 1 , 0.7, 1.2, … }

Originally proposed (for VINCIA) in Giele, Kosower & Skands; arXiv:1102.2126



PYTHIA 8: Mrenna & Skands; arXiv:1605.08352



SHERPA: Bothmann, Schönherr, Schumann; in arXiv:1605.04692

See also HERWIG++ : Bellm et al., arXiv:1605.08256

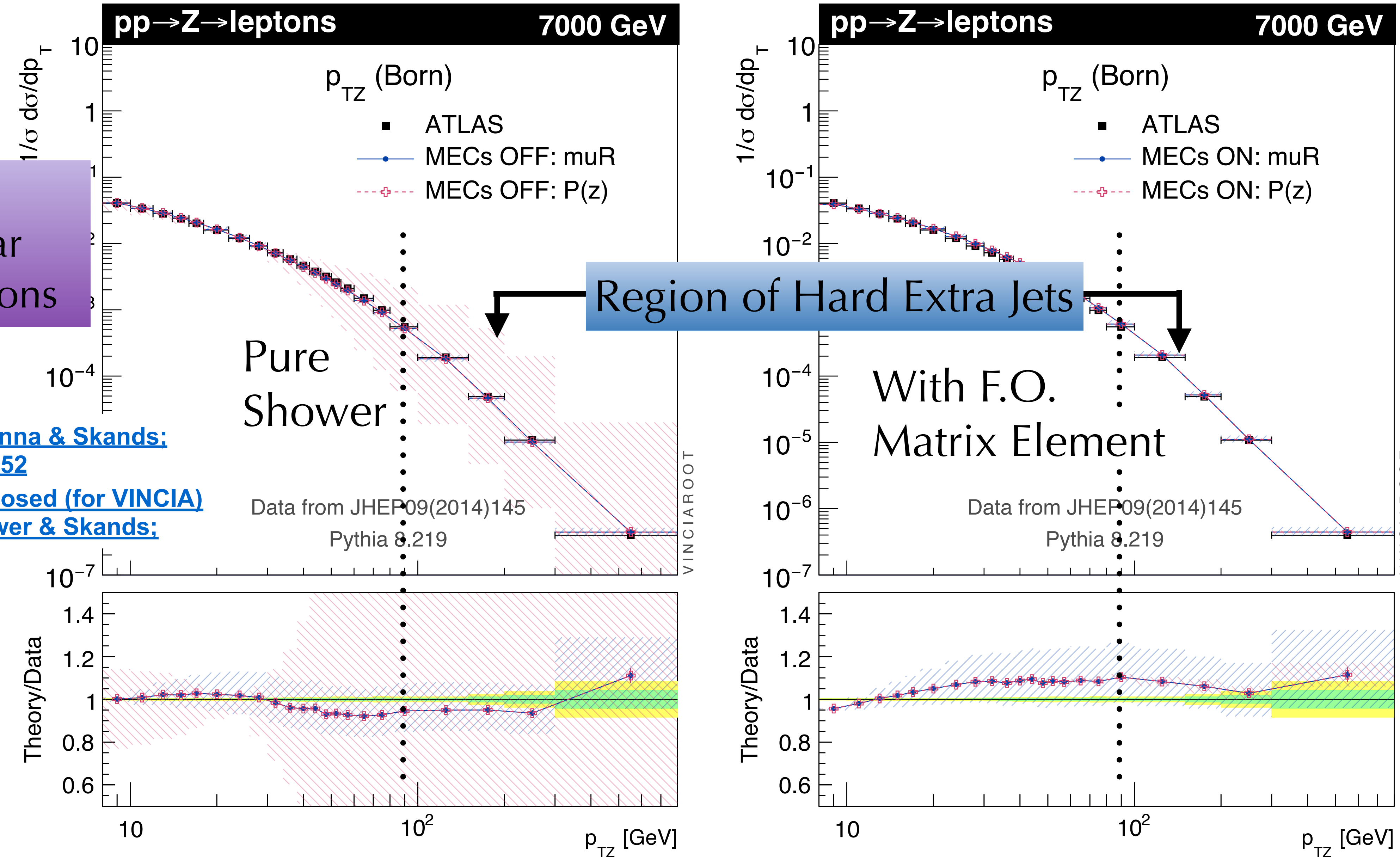
I encourage to start using those, and provide feedback

Hard Jets

Variation of non-singular terms; not controlled by shower

Example pT of Z boson in Drell-Yan production (= zero at LO)

Example 3:
Non-Singular Term Variations



[PYTHIA 8: Mrenna & Skands; arXiv:1605.08352](#)

[Originally proposed (for VINCIA) in Giele, Kosower & Skands;](#)

I encourage to start using those, and provide feedback

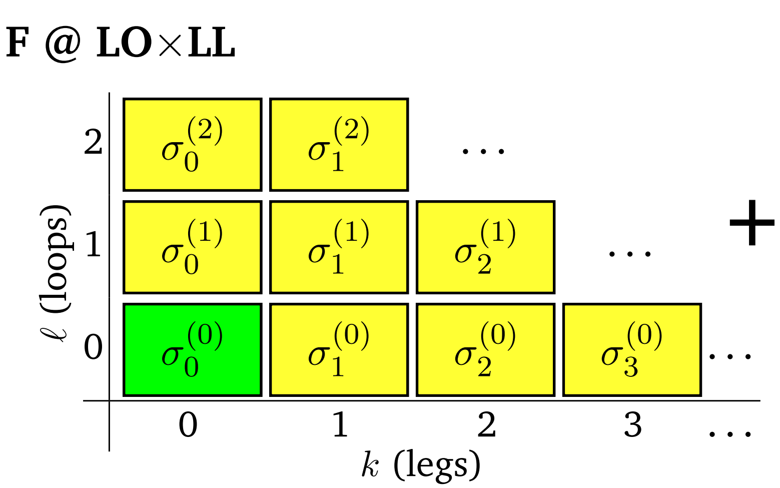
Nice to have all-orders solution

But it is only exact in the singular (soft & collinear) limits

→ gets the bulk of bremsstrahlung corrections right, but fails equally spectacularly: for hard wide-angle radiation: **visible, extra jets**

… which is exactly where fixed-order calculations work!

So combine them!



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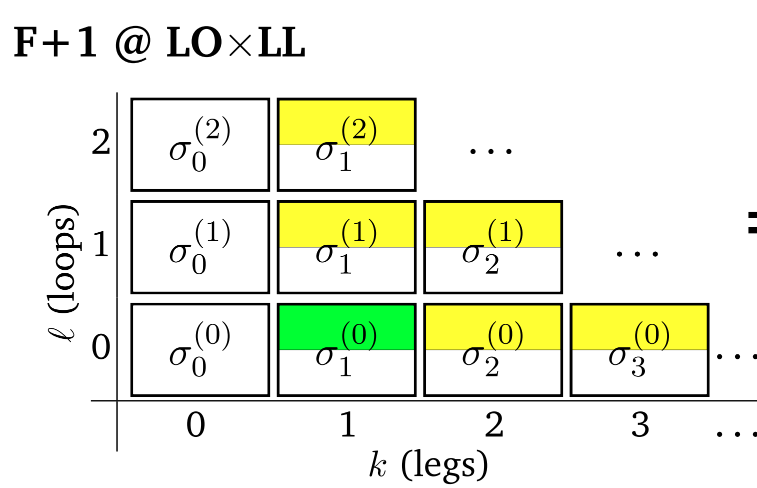

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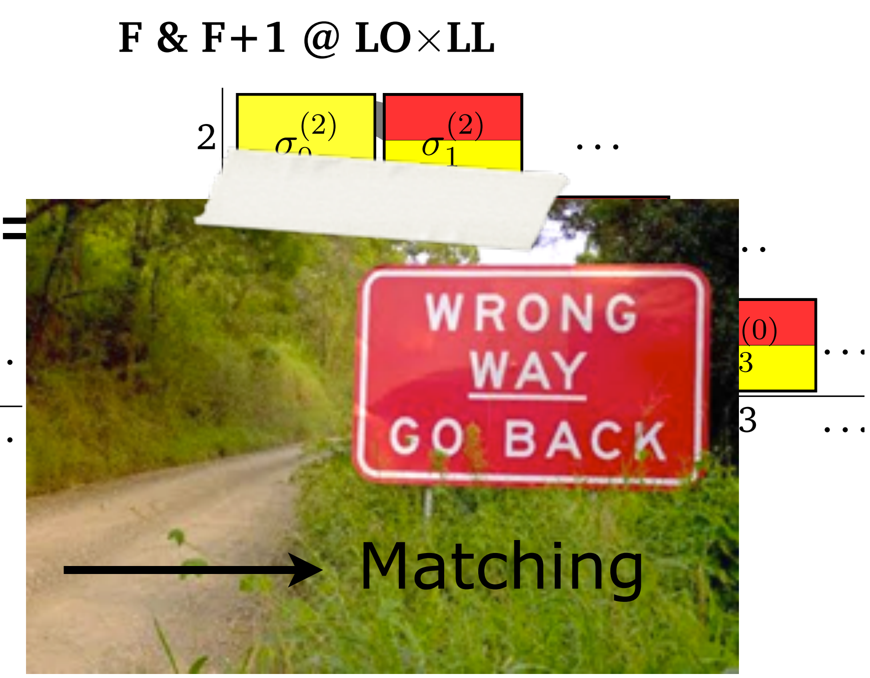
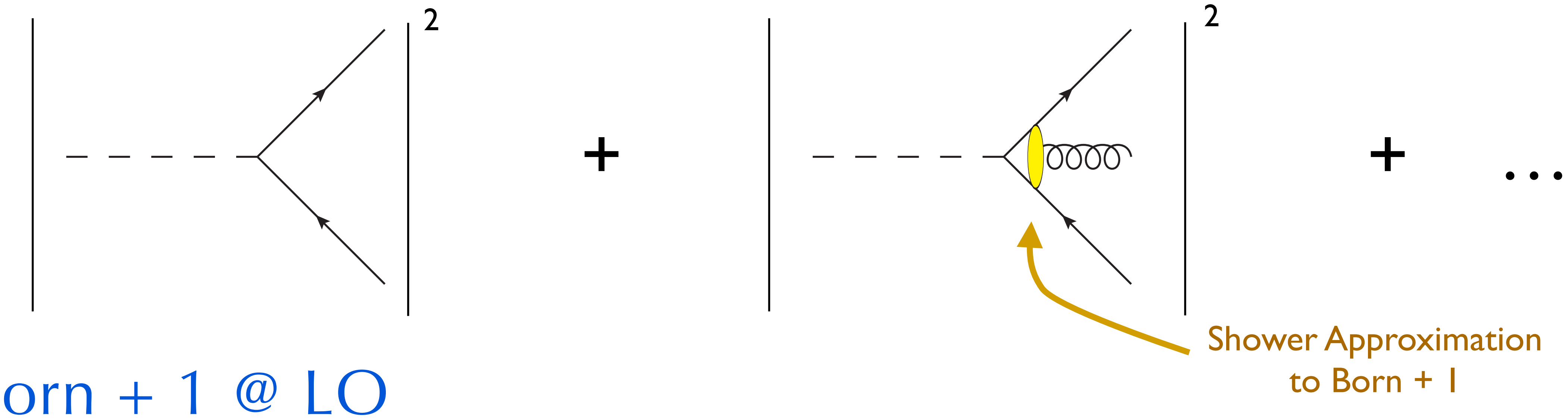


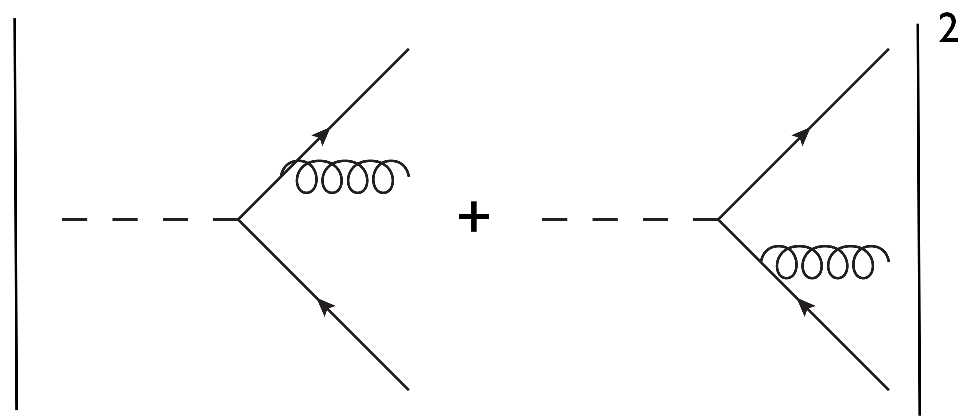
See also: PS, *Introduction to QCD*, TASI 2012, arXiv:1207.2389

Example: $H^0 \to b\bar{b}$

Born + Shower



Born + 1 @ LO



Example: $H^0 \to b\bar{b}$

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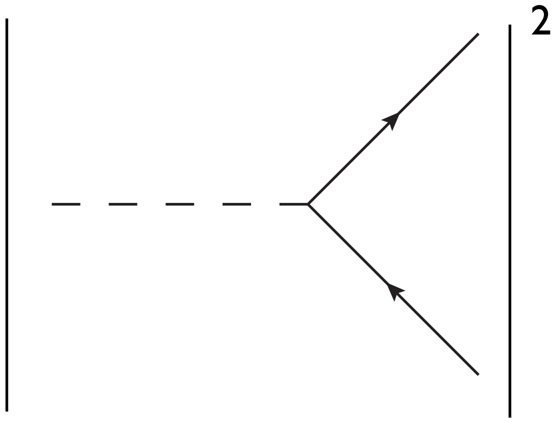
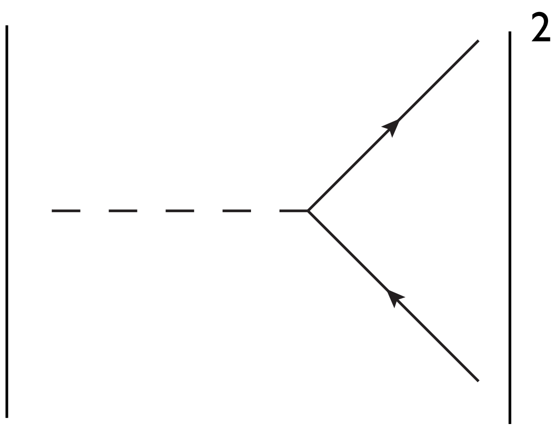


$$\left(1 + g_s^2\, 2C_F \left[\frac{2 s_{ik}}{s_{ij} s_{jk}} + \frac{1}{s_{IK}} \left(\frac{s_{ij}}{s_{jk}} + \frac{s_{jk}}{s_{ij}} \right) \right] + \ldots \right)$$

Born + 1 @ LO



$$\left(g_s^2\, 2C_F \left[\frac{2 s_{ik}}{s_{ij} s_{jk}} + \frac{1}{s_{IK}} \left(\frac{s_{ij}}{s_{jk}} + \frac{s_{jk}}{s_{ij}} + 2 \right) \right] \right)$$

Total Overkill to add these two. All I really need is just that **+2** …

1. Matrix-Element Corrections

Exploit freedom to choose non-singular terms

Bengtsson, Sjöstrand, PLB 185 (1987) 435

- **Modify parton shower** to use process-dependent radiation functions for first emission → absorb real correction

$$\text{Parton Shower} \quad \frac{P(z)}{Q^2} \rightarrow \frac{P'(z)}{Q^2} = \frac{P(z)}{Q^2} \underbrace{\frac{|M_{n+1}|^2}{\sum_i P_i(z)/Q_i^2 |M_n|^2}}_{\text{MEC}}$$

(suppressing α_s and Jacobian factors)

  - Process-dependent MEC → P′ different for each process

- Done in PYTHIA for all SM decays and many BSM ones

Norrbin, Sjöstrand, NPB 603 (2001) 297

  - Based on systematic classification of spin/colour structures
  - Also used to account for mass effects, and for a few 2→2 procs

Difficult to generalise beyond 1st emission

- Parton-shower expansions complicated & can have "dead zones"
- Achieved in VINCIA (by changing from parton showers to "Markovian Antenna Showers") Giele, Kosower, Skands, PRD 84 (2011) 054003
- Only recently done for hadron collisions Fischer et al, arXiv:1605.06142

MECs with Loops: POWHEG

Acronym stands for: **Po**sitive **W**eight **H**ardest **E**mission **G**enerator.

Start at Born level

$$|M_F|^2$$

Generate "shower" emission

$$|M_{F+1}|^2 \overset{LL}{\sim} \sum_{i \in \mathrm{ant}} a_i \; |M_F|^2$$

Correct to Matrix Element

$$a_i \to \frac{|M_{F+1}|^2}{\sum a_i |M_F|^2} a_i$$

Unitarity of Shower

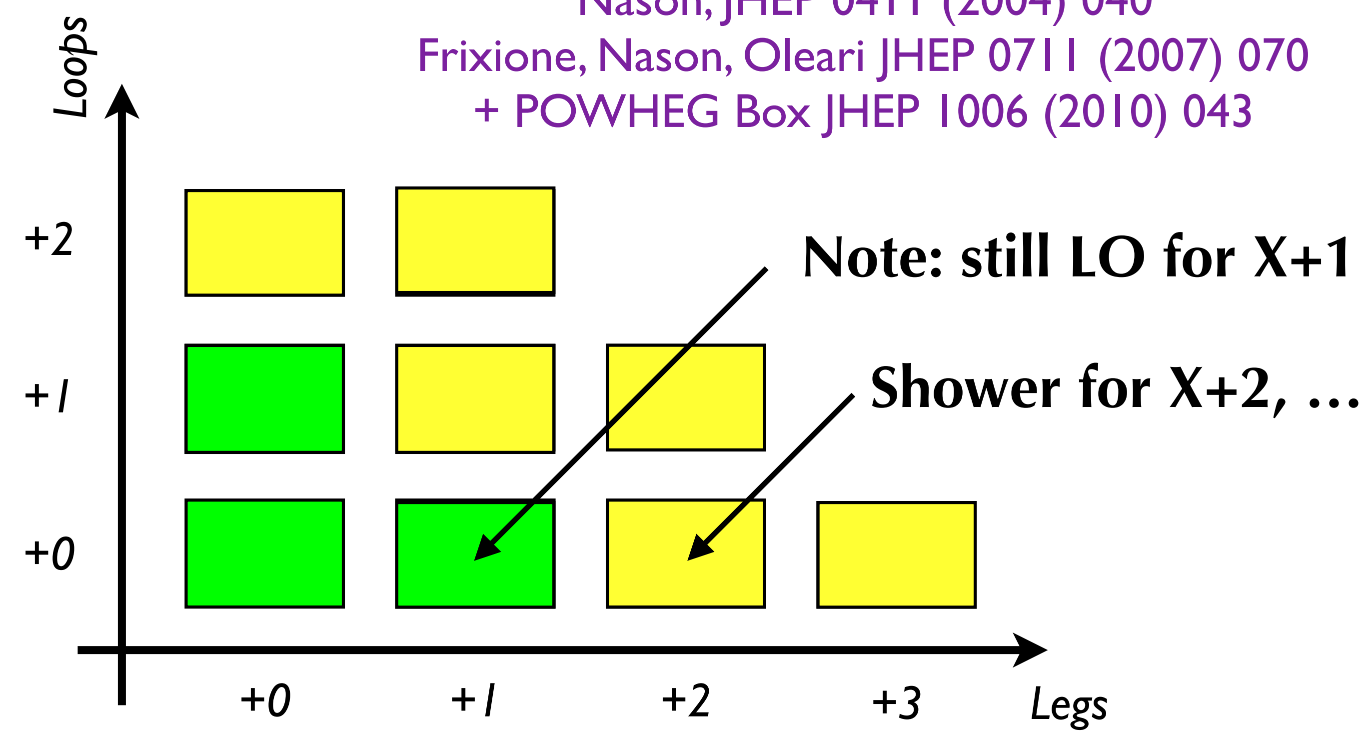
$$\mathrm{Virtual} = - \int \mathrm{Real}$$

Correct to Matrix Element

$$|M_F|^2 \to |M_F|^2 + 2\mathrm{Re}[M_F^1 M_F^0] + \int \mathrm{Real}$$

Repeat:ordinary parton shower

Nason, JHEP 0411 (2004) 040
Frixione, Nason, Oleari JHEP 0711 (2007) 070
+ POWHEG Box JHEP 1006 (2010) 043



Method is widely applied/available, can be used with PYTHIA, HERWIG, SHERPA

Subtlety 1: Connecting with parton shower

Truncated Showers & Vetoed Showers

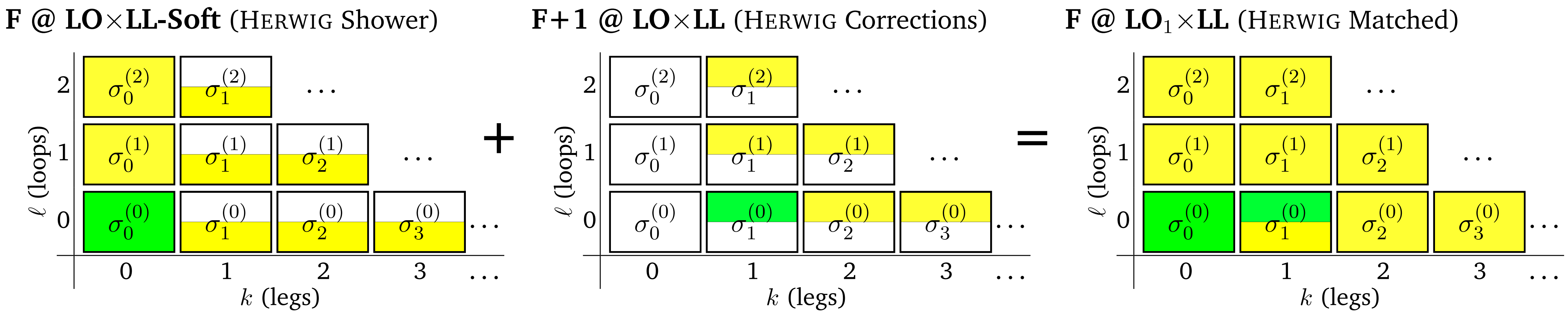
Subtlety 2: Avoiding (over)exponentiation of hard radiation

Controlled by *"hFact parameter"*

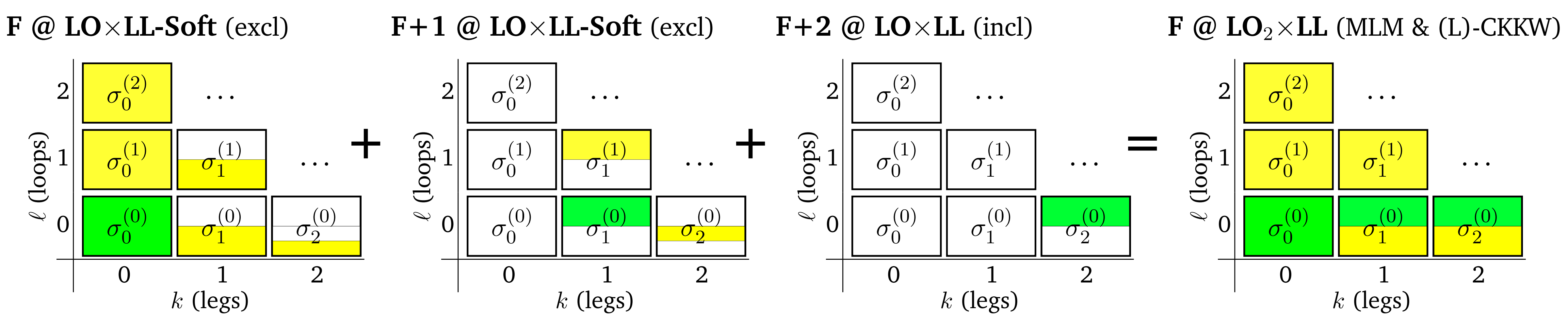
2: Slicing (MLM & CKKW-L)

First emission: "the HERWIG correction"

Use the fact that the angular-ordered HERWIG parton shower has a "dead zone" for hard wide-angle radiation (Seymour, 1995)



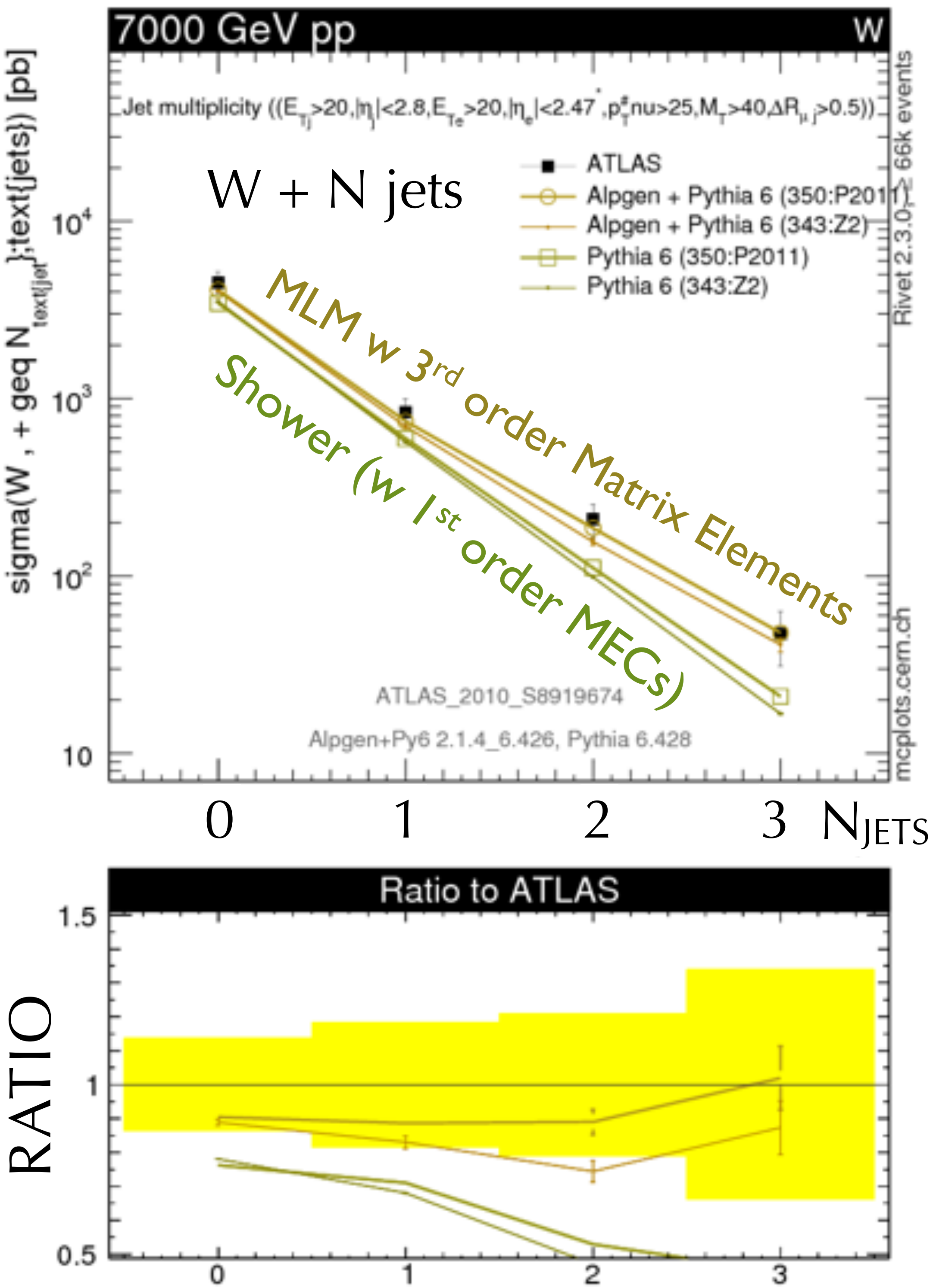
Many emissions: the MLM & CKKW-L prescriptions



(CKKW & Lönnblad, 2001) (Mangano, 2002) (+many more recent; see Alwall et al., EPJC53(2008)473)

The Gain The Cost

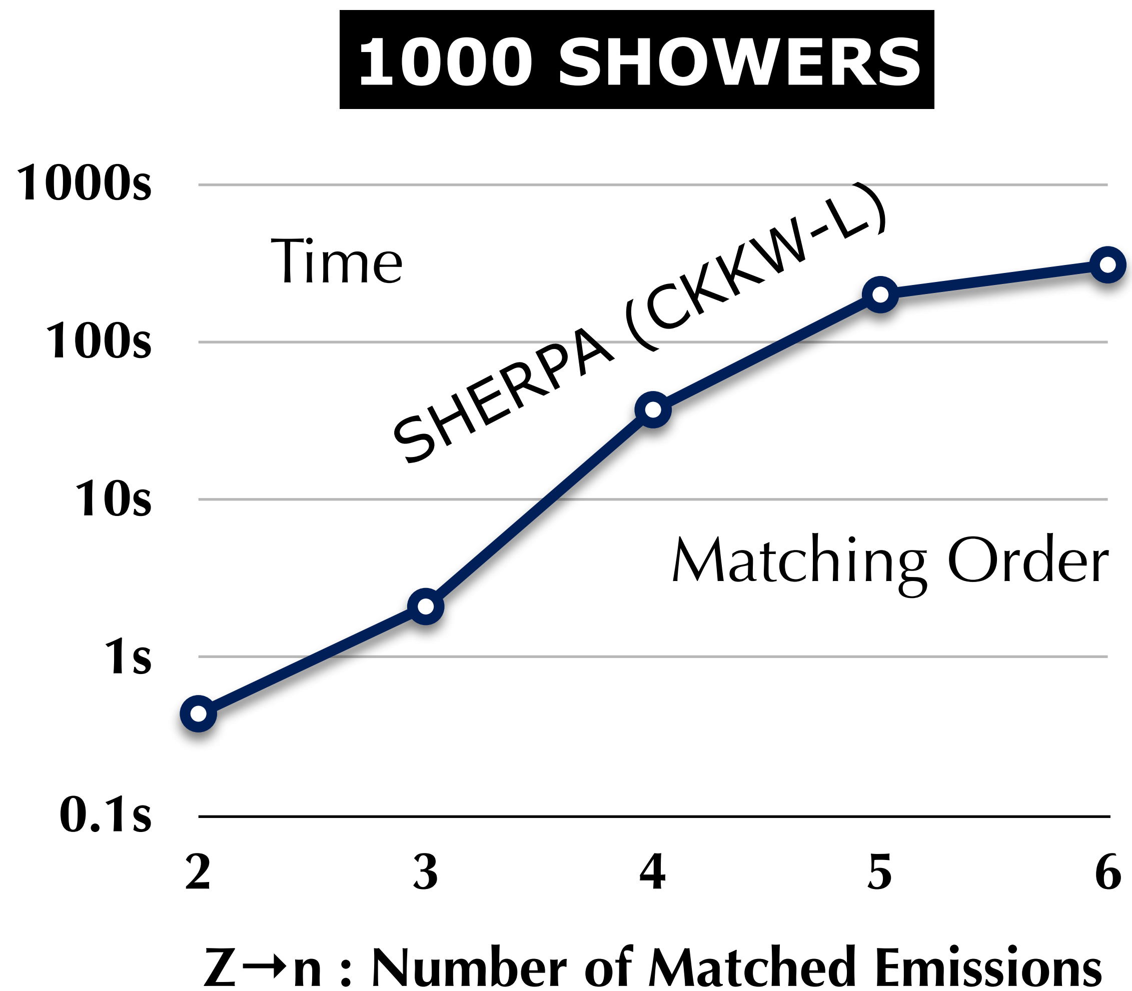
Example: LHC$_7$: W + 20-GeV Jets



Plot from mcplots.cern.ch; see arXiv:1306.3436

Example: $e^+e^- \to Z \to$ Jets

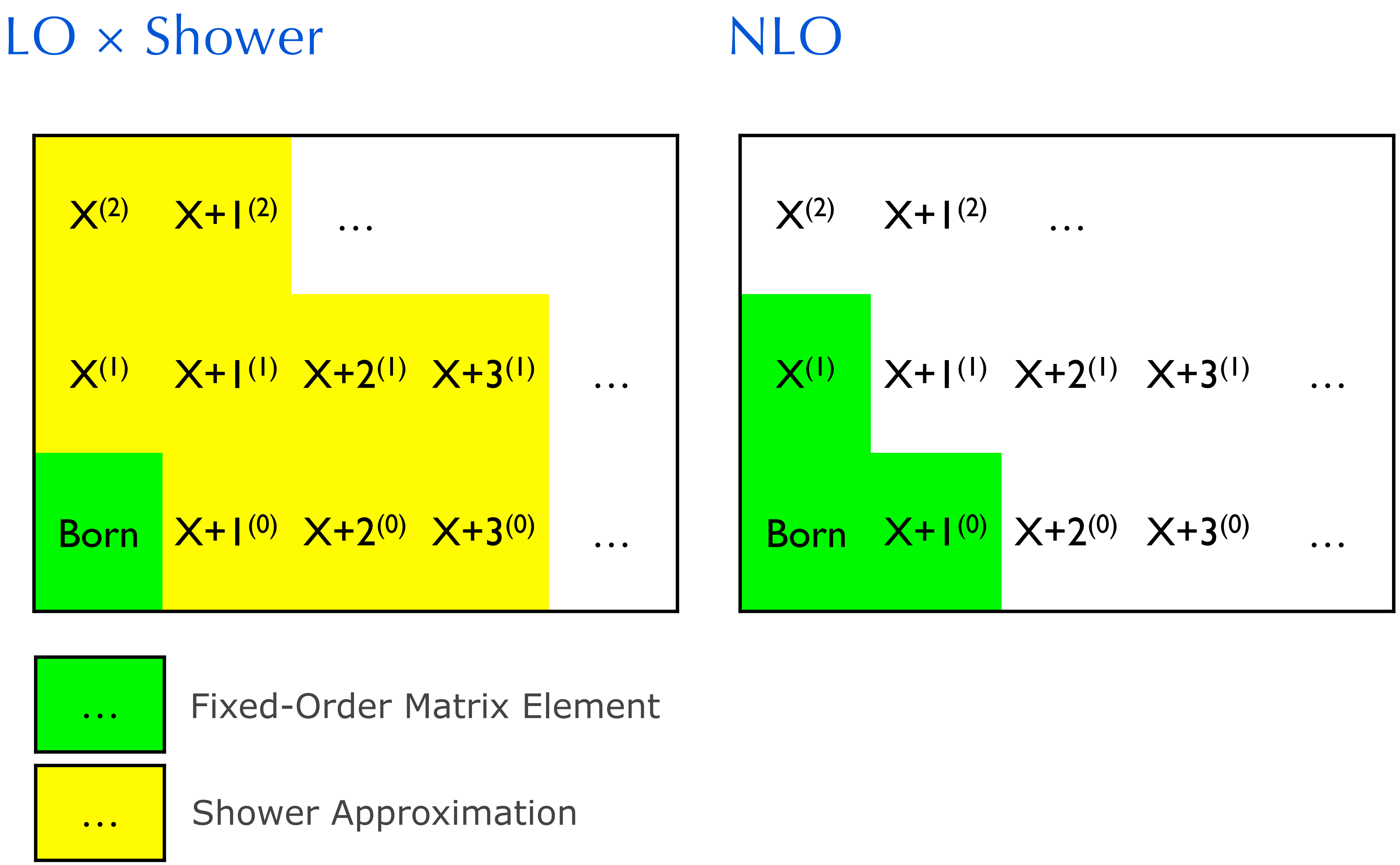
2. Time to generate 1000 events
(Z → partons, fully showered & matched. No hadronization.)



See e.g. Lopez-Villarejo & Skands, arXiv:1109.3608

3: Subtraction

Examples: MC@NLO, aMC@NLO



Matching 3: Subtraction

Examples: MC@NLO, aMC@NLO

LO × Shower

$X^{(2)}$	$X+1^{(2)}$	…		
$X^{(1)}$	$X+1^{(1)}$	$X+2^{(1)}$	$X+3^{(1)}$	…
Born	$X+1^{(0)}$	$X+2^{(0)}$	$X+3^{(0)}$	…

… Fixed-Order Matrix Element

… Shower Approximation

NLO - Shower$_{NLO}$

$X^{(2)}$	$X+1^{(2)}$	…		
$X^{(1)}$	$X+1^{(1)}$	$X+2^{(1)}$	$X+3^{(1)}$	…
Born	$X+1^{(0)}$	$X+2^{(0)}$	$X+3^{(0)}$	…

Expand shower approximation to NLO analytically, then subtract:

… Fixed-Order ME minus Shower Approximation (NOTE: can be < 0!)

Matching 3: Subtraction

Examples: MC@NLO, aMC@NLO

LO × Shower

$X^{(2)}$	$X+1^{(2)}$	…		
$X^{(1)}$	$X+1^{(1)}$	$X+2^{(1)}$	$X+3^{(1)}$	…
Born	$X+1^{(0)}$	$X+2^{(0)}$	$X+3^{(0)}$	…

… Fixed-Order Matrix Element

… Shower Approximation

(NLO - Shower$_{NLO}$) × Shower

$X^{(1)}$	$X^{(1)}$	…		
$X^{(1)}$	$X^{(1)}$	$X^{(1)}$	$X^{(1)}$	…
Born	$X+1^{(0)}$	$X^{(1)}$	$X^{(1)}$	…

… Fixed-Order ME minus Shower Approximation (NOTE: can be < 0!)

… Subleading corrections generated by shower off subtracted ME

Matching 3: Subtraction

Examples: MC@NLO, aMC@NLO

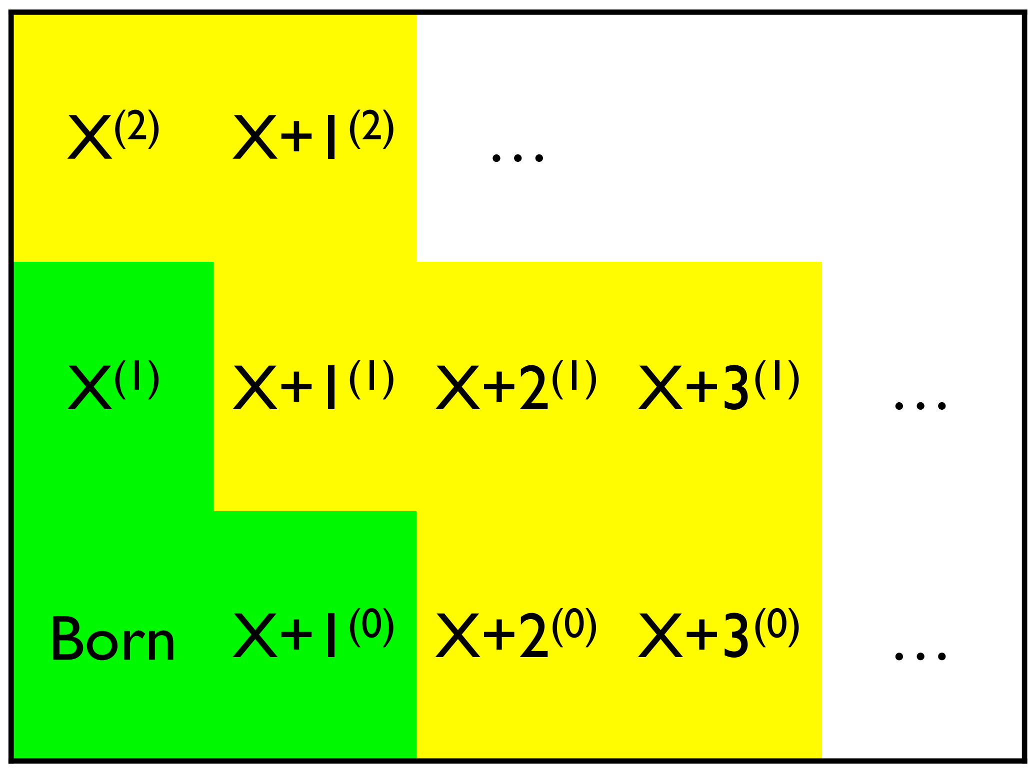
Combine → MC@NLO

Frixione, Webber, JHEP 0206 (2002) 029

Consistent NLO + parton shower (though correction events can have w<0)

Recently, has been fully automated in aMC@NLO

Frederix, Frixione, Hirschi, Maltoni, Pittau, Torrielli, JHEP 1202 (2012) 048



NB: w < 0 are a problem because they kill efficiency:
Extreme example: 1000 positive-weight - 999 negative-weight events → statistical precision of 1 event, for 2000 generated (for comparison, normal MC@NLO has ~ 10% neg-weights)

POWHEG vs MC@NLO

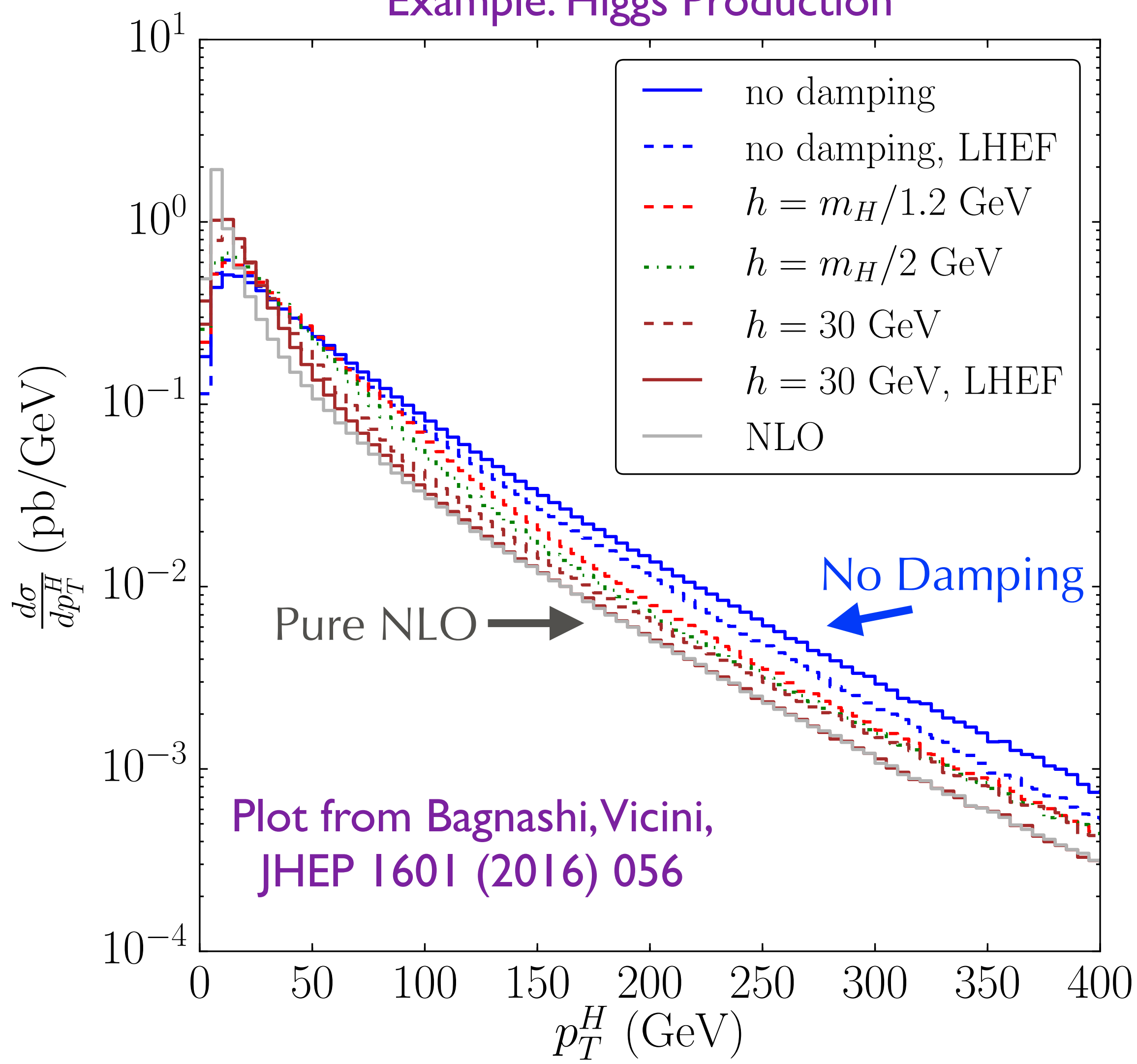
Both methods include the complete first-order (NLO) matrix elements.

Difference is in whether **only** the shower kernels are exponentiated (MC@NLO) or whether part of the matrix-element corrections are too (POWHEG)

In POWHEG, how much of the MEC you exponentiate can be controlled by the "hFact" parameter

Variations basically span range between MC@NLO-like case, and original (hFact=1) POWHEG case (~ PYTHIA-style MECs)

Example: Higgs Production



Plot from Bagnashi, Vicini, JHEP 1601 (2016) 056

$$D_h = \frac{h^2}{h^2 + (p_\perp^H)^2}$$

$R^s = D_h \; R_{\rm div}$ exponentiated

$R^f = (1 - D_h) \; R_{\rm div}$ not exponentiated

(Multi-Leg Merging at NLO)

Currently, much activity on how to combine several NLO matrix elements for the same process: NLO for X, X+1, X+2, …

- Unitarity is a common main ingredient for all of them
- Most also employ **slicing** (separating phase space into regions defined by one particular underlying process)

Methods

- UNLOPS, generalising CKKW-L/UMEPS: Lonnblad, Prestel, arXiv:1211.7278
- MiNLO, based on POWHEG: Hamilton, Nason, Zanderighi (+more) arXiv:1206.3572, arXiv:1512.02663
- FxFx, based on MC@NLO: Frederix & Frixione, arXiv:1209.6215
- (VINCIA, based on NLO MECs): Hartgring, Laenen, Skands, arXiv:1303.4974

Most (all?) of these will also allow for reaching NNLO accuracy on the total inclusive cross section

- Will soon define the state-of-the-art for SM processes
- For BSM, the state-of-the-art is generally one order less than SM

Summary

This Lecture:

- From Unitarity to Evolution Equations
- Parton Showers; the Sudakov no-emission probability
- Interference and Coherence
- Colour Flow
- Ambiguities in Parton Showers ↔ Uncertainties

- Matching & Merging
  - Matrix-Element Corrections: PYTHIA, POWHEG, VINCIA
  - Slicing: CKKW-L (SHERPA + others), MLM (ALPGEN + others)
  - Subtraction (MC@NLO, aMC@NLO + others)
  - State-of-the-art: **Multi-Leg NLO** (UNLOPS, MiNLO, FxFx)

Last Lecture (Friday)

- Lecture 3: Hadronisation + BSM Signals and Backgrounds

Extra Slides

Simple Monte Carlo Example: Number of AEPSHEP students who will get hit by a car this week

Complicated Function:

Time-dependent

Traffic density during day, week-days vs week-ends

(i.e., non-trivial time evolution of system)

No two students are the same

Need to compute probability for each and sum

(simulates having several distinct types of “evolvers”)

Multiple outcomes:

Hit → keep walking, or go to hospital?

Multiple hits = Product of single hits, or more complicated?

Monte Carlo Approach

Approximate Traffic

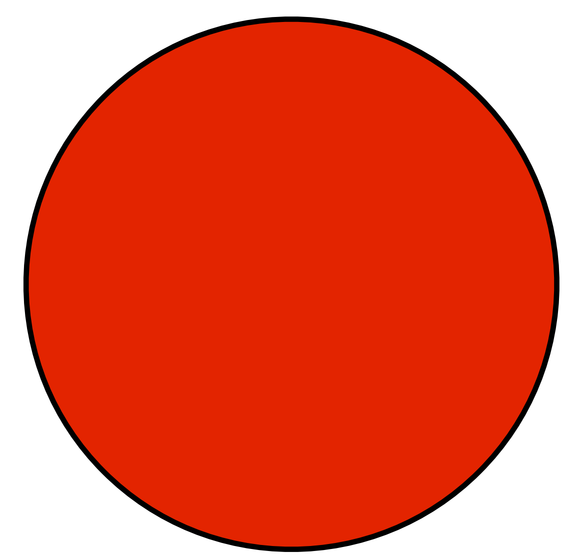
Simple overestimate:

highest recorded density
of most careless drivers,
driving at highest recorded speed
…

Approximate Student

by most completely reckless and accident-prone student
(wandering the streets lost in thought after these lectures …)

This extreme guess will be the equivalent of our simple overestimate from yesterday:



Hit Generator

Off we go…

Throw random accidents according to:

$$R = \int_{t_0}^{t_e} \mathrm{d}t \int_x \mathrm{d}x \sum_{i=1}^{n_{\mathrm{stud}}} \alpha_i(x,t)\, \rho_i(x,t)\, \rho_c(x,t)$$

Sum over students

Student-Car hit rate

Density of Student i

Density of Cars

t_e : time of accident

Too Difficult

↓

Simple Overestimate

$$R = (t_e - t_0)\Delta x \quad \alpha_{\max}\, n_{\mathrm{stud}}\, \rho_{c\max}$$

Hit rate for most accident-prone student

Rush-hour density of cars

(Also generate trial x_e, e.g., uniformly in circle around Puri)

(Also generate trial *i;* a random student gets hit)

Hit Generator

Accept trial hit (i,x,t) with probability

$$\text{Prob(accept)} = \frac{\alpha_i(x,t)\ \rho_i(x,t)\ \rho_c(x,t)}{\alpha_{\mathrm{max}}\ n_{\mathrm{stud}}\ \rho_{\mathrm{cmax}}}$$

Using the following:

ρ_c : actual density of cars at location x at time t

ρ_i : actual density of student i at location x at time t

α_i : The actual "hit rate" (OK, not really known, but can make one up)

→ True number = number of accepted hits

(note: we didn't really treat multiple hits … → Markov Chain)

Summary: How we do Monte Carlo

Take your system

Generate a "trial" (event/decay/interaction/...)

Not easy to generate random numbers distributed according to exactly the right distribution?

May have complicated dynamics, interactions ...

→ use a simpler "trial" distribution

Flat with some stratification

Or importance sample with simple overestimating function (for which you can generate random #s)

Summary: How we do Monte Carlo

Take your system

Generate a "trial" (event/decay/interaction/…)

Accept trial with probability f(x)/g(x)

f(x) contains all the complicated dynamics

g(x) is the simple trial function

If accept: replace with new system state

If reject: keep previous system state

no dependence on g in final result - only affects convergence rate

And keep going: generate next trial …

Summary: How we do Monte Carlo

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Generate a "trial" (event/decay/in

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Sounds deceptively simple, but …

with it, you can integrate

arbitrarily complicated functions *(in particular chains of nested functions)*, over arbitrarily complicated regions, in arbitrarily many dimensions …

And keep going: generate next trial …