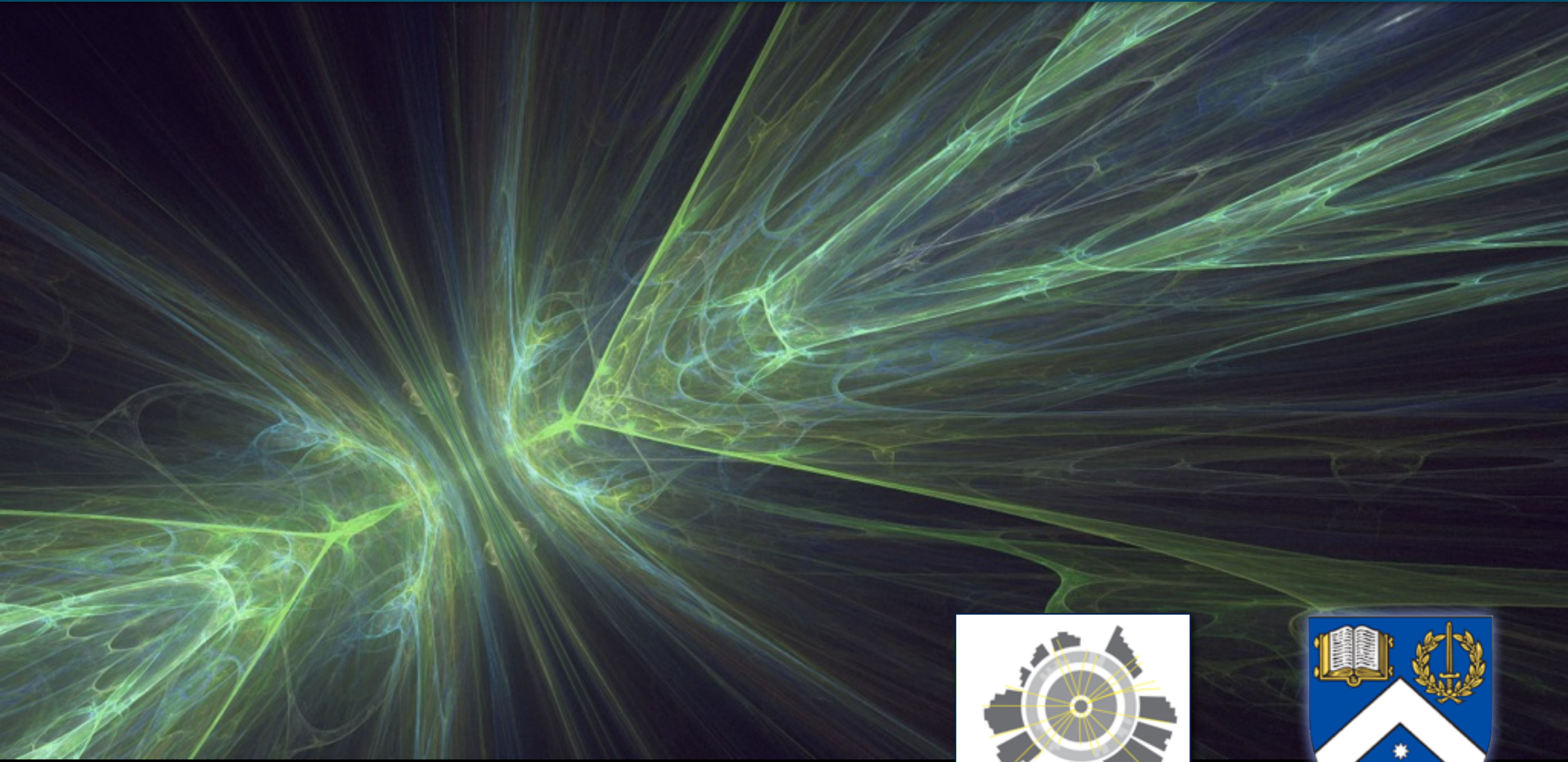
Emergent Phenomena in QCD & at the LHC

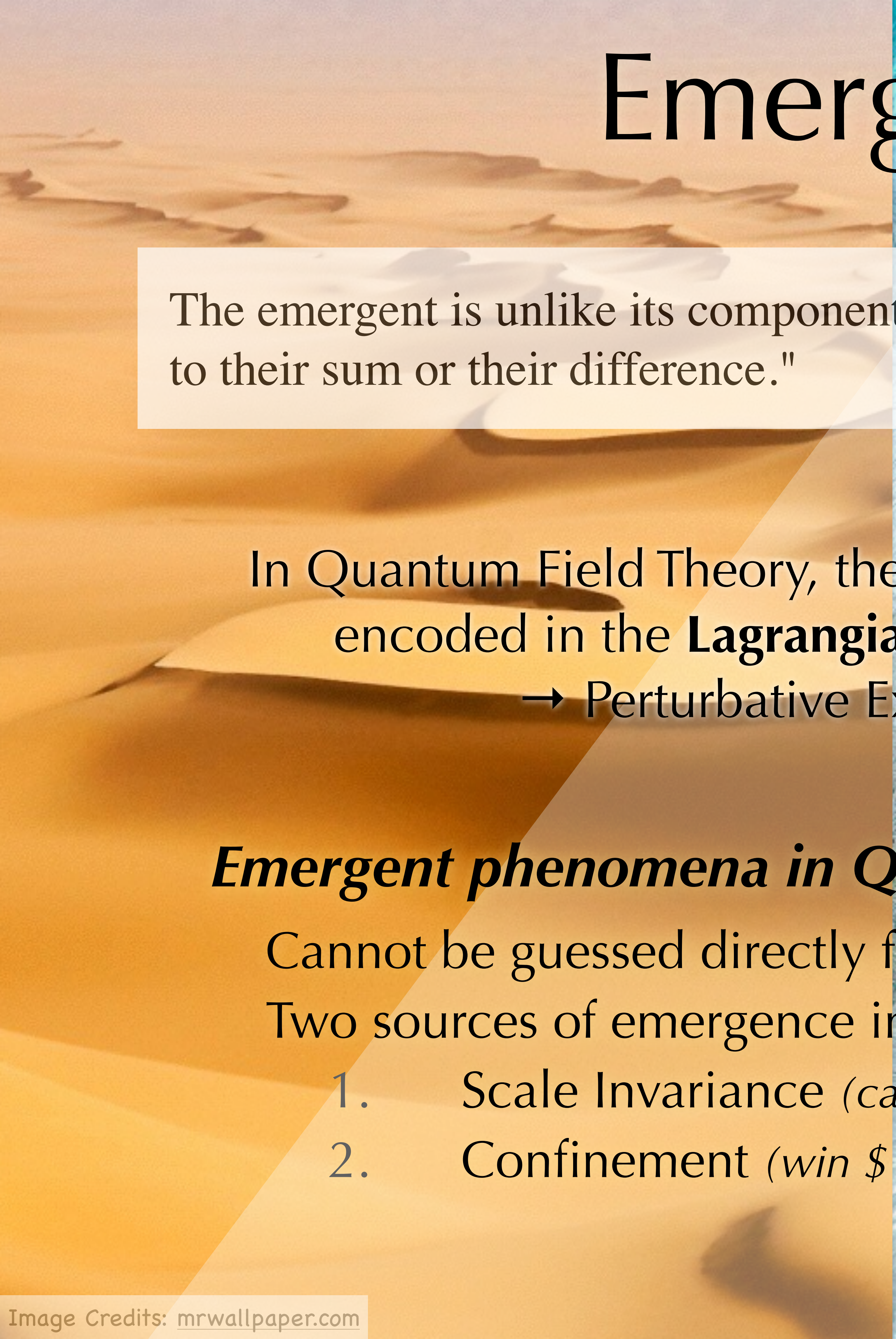
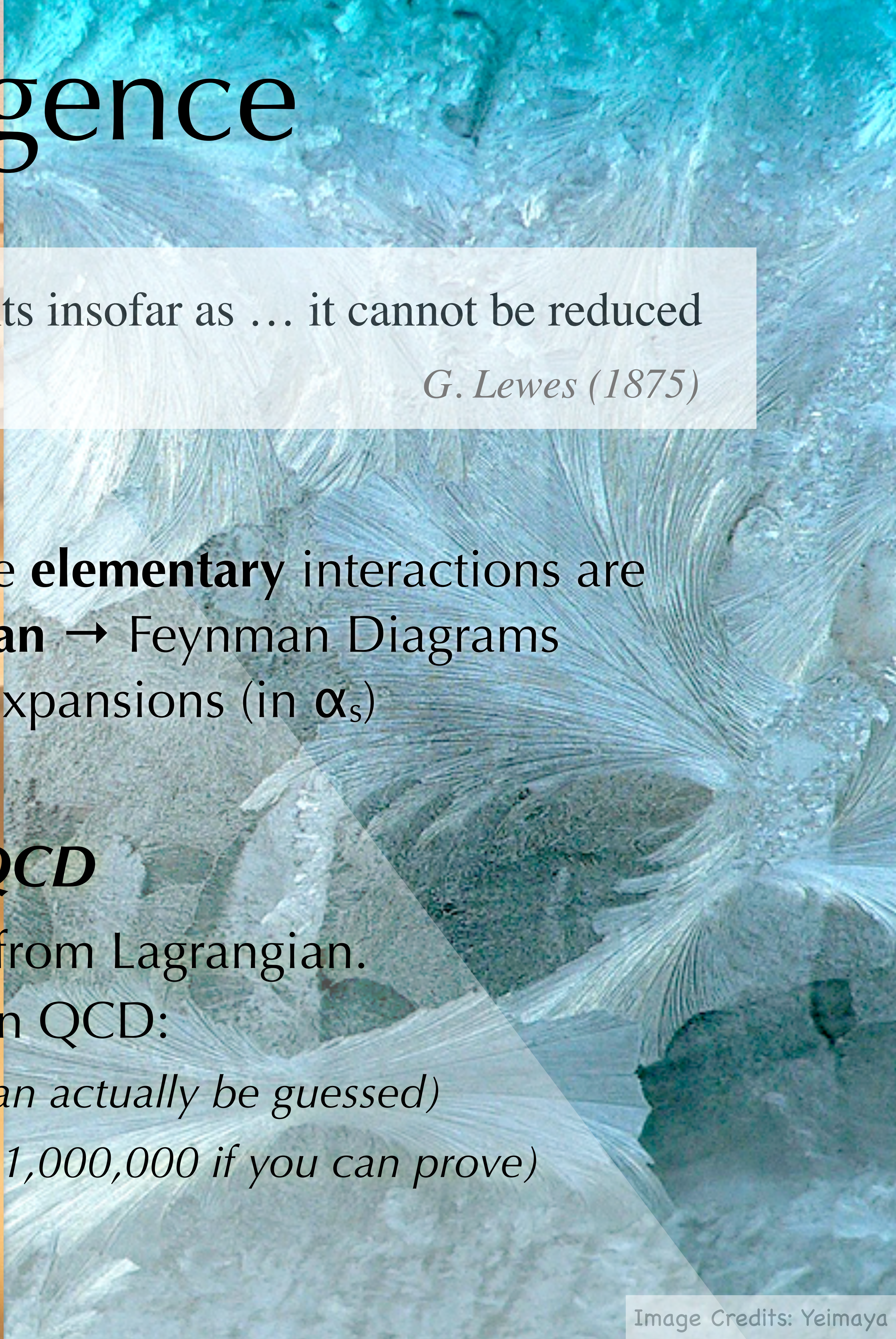
Peter Skands (Monash University)



Physics Colloquium, University of Sydney
11 May 2015

CoEPP
ARC Centre of Excellence for Particle Physics at the Terascale

Emergence

> The emergent is unlike its components insofar as … it cannot be reduced to their sum or their difference."
>
> *G. Lewes (1875)*

In Quantum Field Theory, the **elementary** interactions are encoded in the **Lagrangian** → Feynman Diagrams → Perturbative Expansions (in α_s)

Emergent phenomena in QCD

Cannot be guessed directly from Lagrangian.

Two sources of emergence in QCD:

1. Scale Invariance *(can actually be guessed)*
2. Confinement *(win $1,000,000 if you can prove)*

Image Credits: mrwallpaper.com

Image Credits: Yeimaya

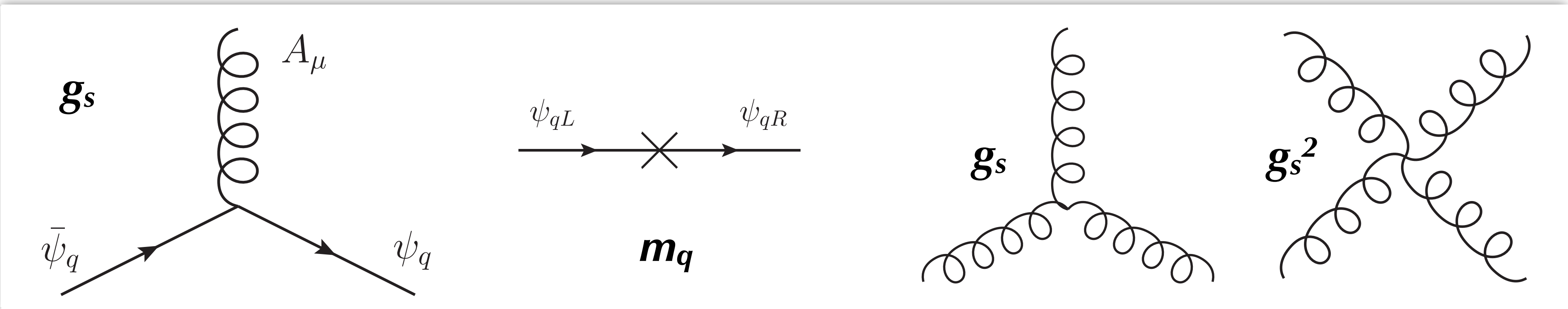
The Constituents of QCD

The **elementary** interactions are encoded in the **Lagrangian**
QFT → Feynman Diagrams → Perturbative Expansions (in α_s)

$g_s^2 = 4\pi\alpha_s$

THE BASIC ELEMENTS OF QCD: QUARKS AND GLUONS

$$\psi_q^j = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}$$



$$\mathcal{L} = \bar{\psi}_q^i (i\gamma^\mu)(D_\mu)_{ij}\psi_q^j - m_q \bar{\psi}_q^i \psi_{qi} - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu}$$

$$D_{\mu ij} = \delta_{ij}\partial_\mu - i g_s T_{ij}^a A_\mu^a$$

Gauge Covariant Derivative: makes *L* invariant under $SU(3)_C$ rotations of ψ_q

m_q: Quark Mass Terms
(Higgs + QCD condensates)

Gluon-Field Kinetic Terms
and Self-Interactions

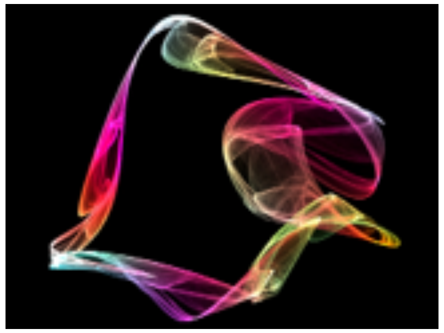
$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g_s f^{abc} A_\mu^b A_\nu^c$$

Beyond Fixed Order

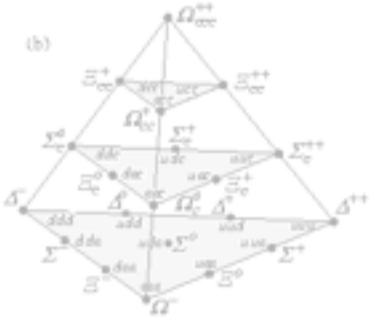
QCD is more than just a perturbative expansion in α_s

The relation between α_s, Feynman diagrams, and the full QCD dynamics is under active investigation. Emergent phenomena:

Jets (the fractal of perturbative QCD) ⟷ amplitude structures in quantum field theory ⟷ factorisation & unitarity. Precision jet (structure) studies.



Strings (strong gluon fields) ⟷ quantum-classical correspondence. String physics. String breaks. Dynamics of hadronization phase transition.



Hadrons ⟷ Spectroscopy (incl excited and exotic states), lattice QCD, (rare) decays, mixing, light nuclei. Hadron beams → multiparton interactions, diffraction, …

> There are more things in heaven and earth, Horatio, than are dreamt of in your philosophy
>
> W. Shakespeare, Hamlet.

LHC RUN 2: STARTS NOW !!!

ALMOST TWICE THE ENERGY (13 TeV compared with 8 TeV) AND MORE INTENSE BEAMS

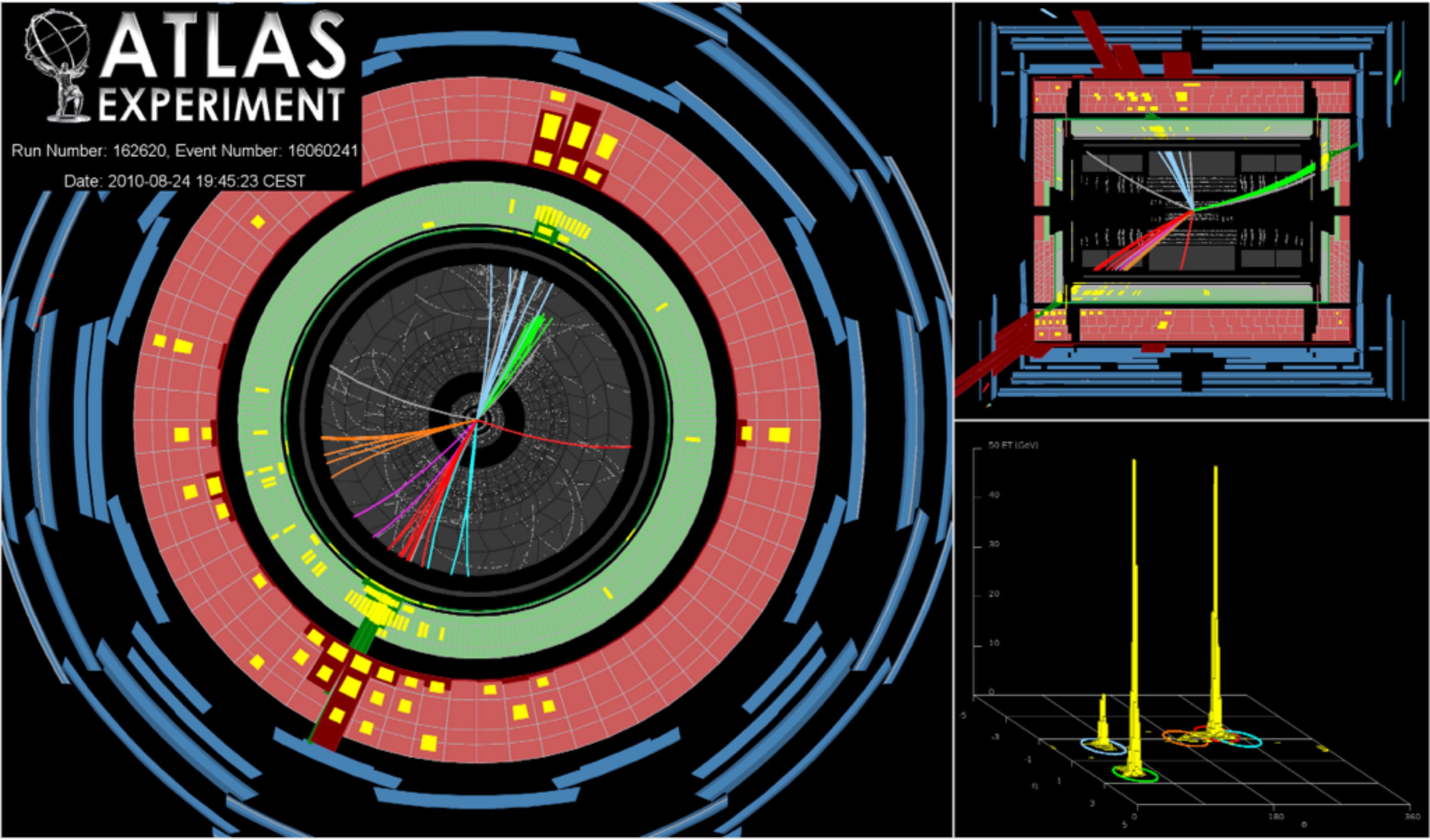
$$\mathcal{L} = \bar{\psi}_q^i (i\gamma^\mu)(D_\mu)_{ij} \psi_q^j - m_q \bar{\psi}_q^i \psi_{qi} - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}$$

$$+ \ldots \ldots \ldots ?$$

LHC Run 1: still no explicit "new physics"

→ we're still looking for *deviations* from SM

Accurate modeling of QCD improve searches & precision



- 1st jet: p_T = 520 GeV, η = -1.4, ϕ = -2.0
- 2nd jet: p_T = 460 GeV, η = 2.2, ϕ = 1.0
- 3rd jet: p_T = 130 GeV, η = -0.3, ϕ = 1.2
- 4th jet: p_T = 50 GeV, η = -1.0, ϕ = -2.9

QCD in the Ultraviolet

The "running" of α_s:

$$Q^2 \frac{\partial \alpha_s}{\partial Q^2} = -\alpha_s^2 (b_0 + b_1 \alpha_s + b_2 \alpha_s^2 + \ldots) \; ,$$

$$b_0 = \frac{11 C_A - 2 n_f}{12\pi}$$

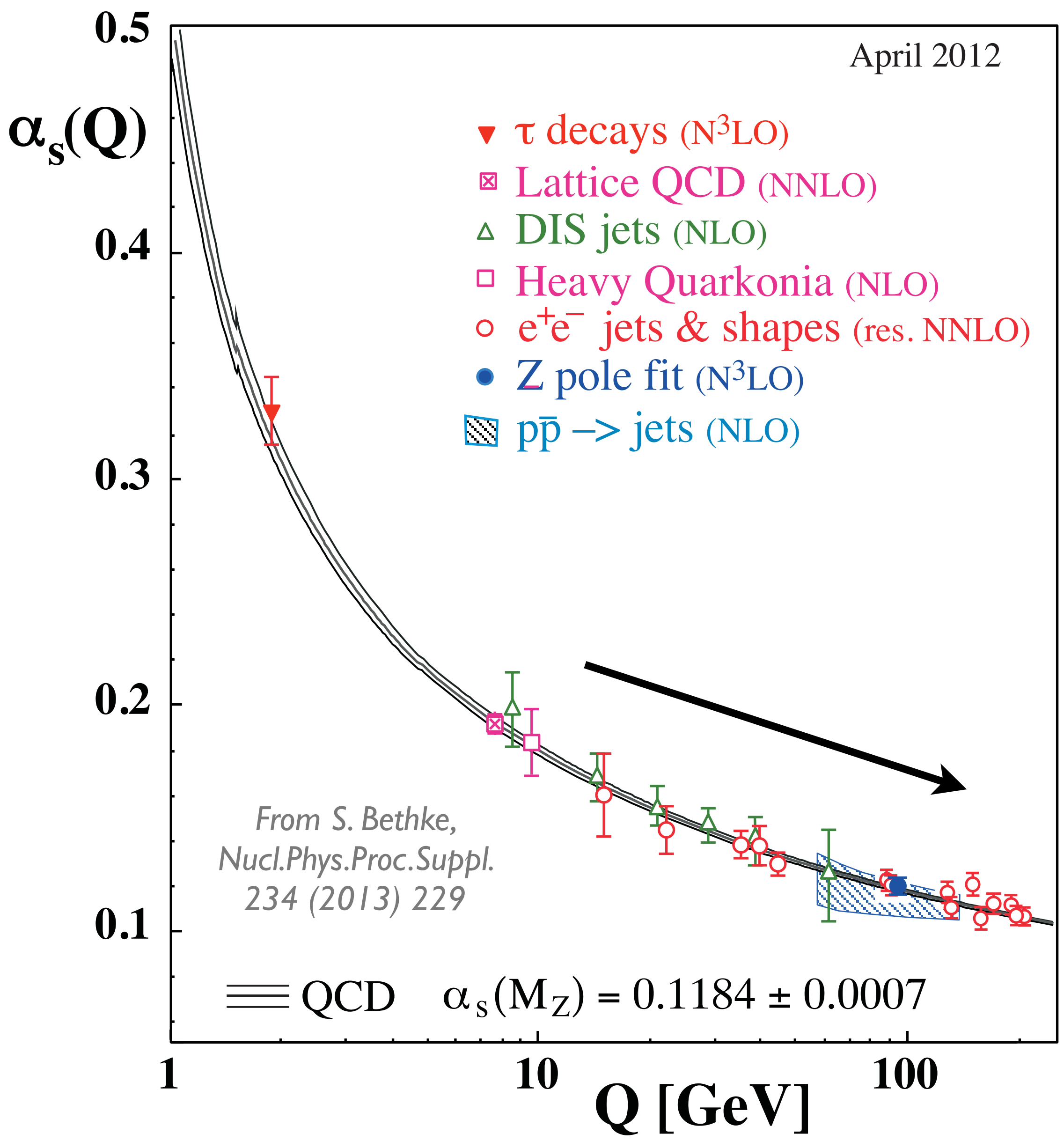
C_A=3 for SU(3)

$$b_1 = \frac{17 C_A^2 - 5 C_A n_f - 3 C_F n_f}{24\pi^2} = \frac{153 - 19 n_f}{24\pi^2}$$

C

$$b_2 = \frac{2857 - 5033 n_f + 325 n_f^2}{128\pi^3}$$

$$b_3 = \text{known}$$



Full symbols are results based on N3LO QCD, open circles are based on NNLO, open triangles and squares on NLO QCD. The cross-filled square is based on lattice QCD.

At high scales Q >> 1 GeV

Coupling $\alpha_s(Q) \ll 1$

Perturbation theory in α_s should be **reliable**: LO, NLO, NNLO, …

E.g., in event shown on previous slide:

- 1st jet: p_T = 520 GeV
- 2nd jet: p_T = 460 GeV
- 3rd jet: p_T = 130 GeV
- 4th jet: p_T = 50 GeV

The Infrared Strikes Back

Naively, QCD radiation suppressed by $\alpha_s \approx 0.1$

Truncate at fixed order = LO, NLO, …

E.g., $\sigma(X+\text{jet})/\sigma(X) \propto \alpha_s$

Example: Pair production of SUSY particles at LHC$_{14}$, with $M_{SUSY} \approx 600$ GeV

LHC - sps1a - m~600 GeV

Plehn, Rainwater, PS PLB645(2007)217

FIXED ORDER pQCD	$\sigma_{\rm tot}$[pb]	$\tilde{g}\tilde{g}$	$\tilde{u}_L\tilde{g}$	$\tilde{u}_L\tilde{u}_L^*$	$\tilde{u}_L\tilde{u}_L$	TT
$p_{T,j} >$ 100 GeV	σ_{0j}	4.83	5.65	0.286	0.502	1.30
inclusive **X + 1 “jet”** →	σ_{1j}	2.89	2.74	0.136	0.145	0.73
inclusive **X + 2 “jets”** →	σ_{2j}	1.09	0.85	0.049	0.039	0.26
$p_{T,j} >$ 50 GeV	σ_{0j}	4.83	5.65	0.286	0.502	1.30
	σ_{1j}	5.90	5.37	0.283	0.285	1.50
	σ_{2j}	4.17	3.18	0.179	0.117	1.21

(Computed with SUSY-MadGraph)

σ for X + jets much larger than naive estimate

$\sigma_{50} \sim \sigma_{\rm tot}$ tells us that there will “always” be a ~ 50-GeV jet “inside” a 600-GeV process

All the scales are high, Q >> 1 GeV, so perturbation theory **should** be OK …

Conformal QCD

The Lagrangian of QCD is **scale invariant**

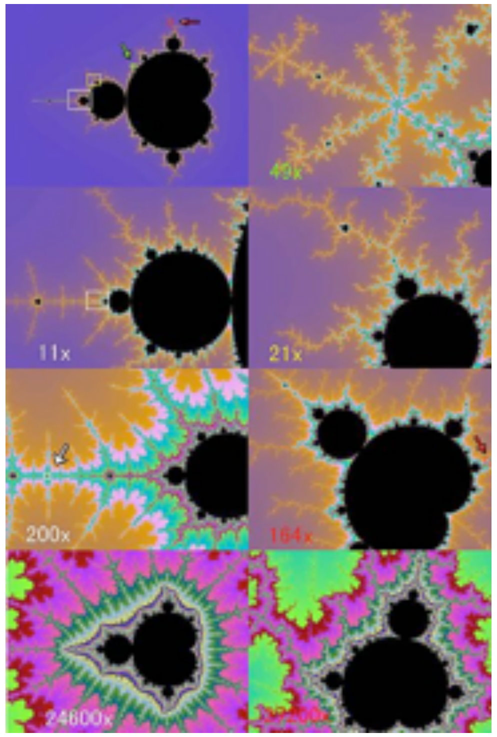
(neglecting small quark masses)

Characteristic of point-like constituents

To first approximation, observables depend only on dimensionless quantities, like **angles** and energy **ratios**

James Bjørken
"Lightcone Scaling"
aka Bjørken Scaling;
Conformal invariance

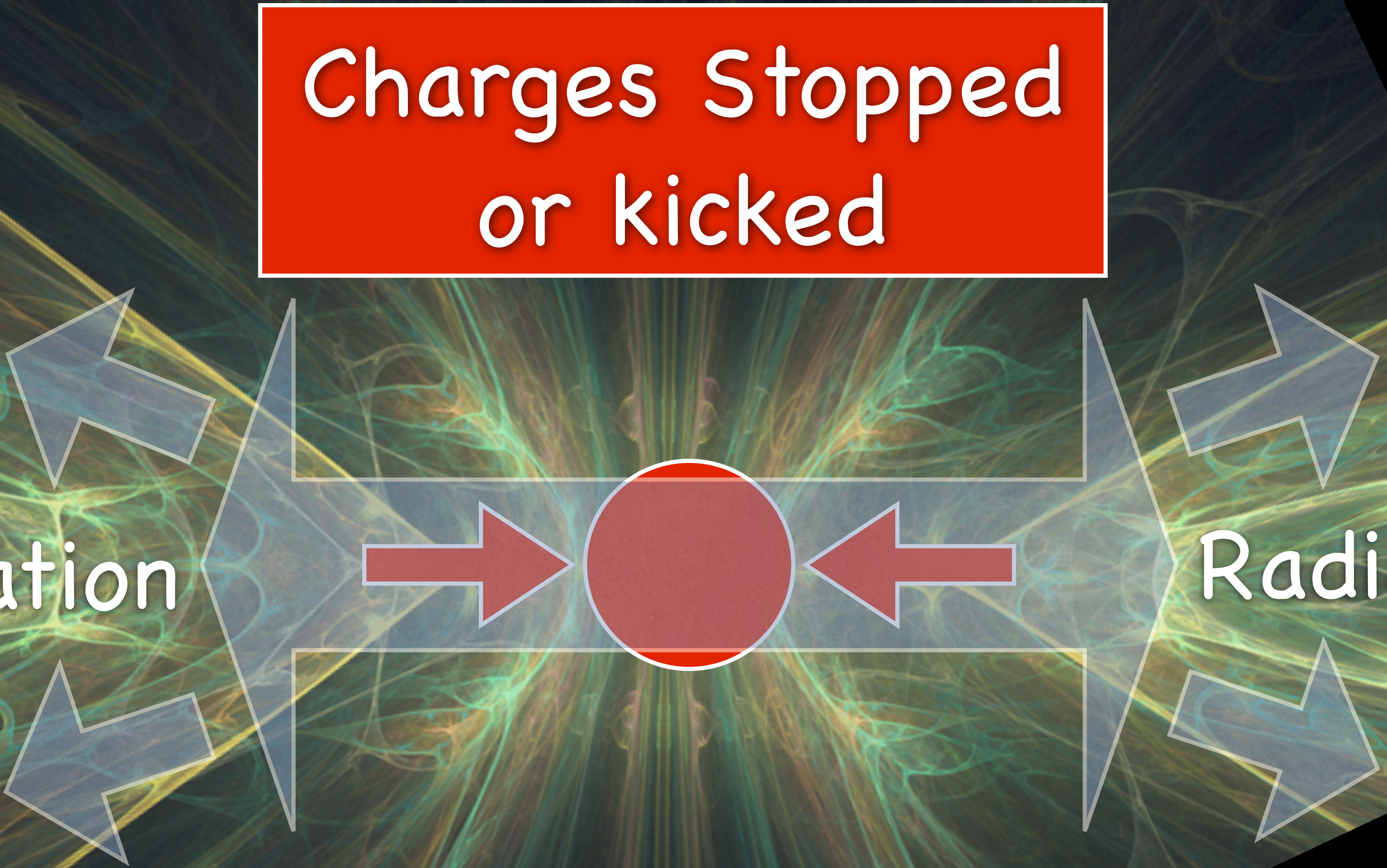
Also means that when we look closer at these constituents, they must generate ever self-similar patterns = fractals



Note: scaling **violation** *is* induced in full QCD, but only by renormalization: $g_s^2 = 4\pi\alpha_s(\mu)$

(some) Physics

cf. equivalent-photon
approximation
Weiszäcker, Williams
~ 1934


Charges Stopped or kicked

Radiation

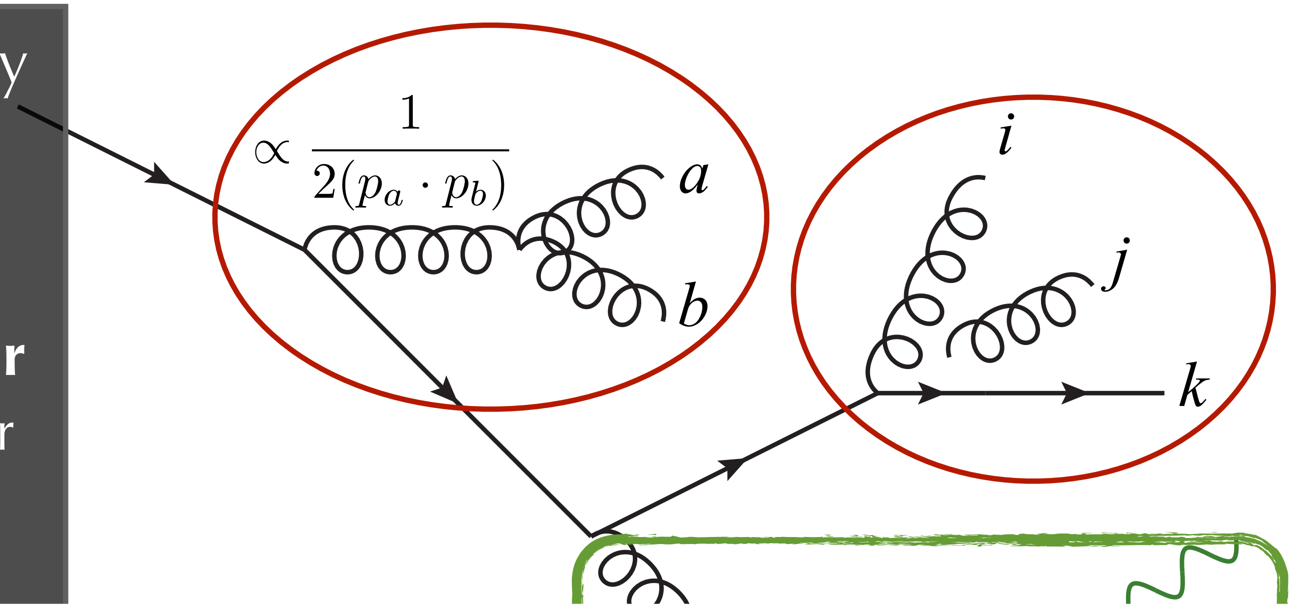
Radiation

a.k.a.
Bremsstrahlung
Synchrotron Radiation

The harder they stop, the harder the fluctations that continue to become radiation

Jets ≈ Fractals

- **Most bremsstrahlung** is driven by divergent propagators → simple structure
- **Amplitudes factorize in singular limits** (→ universal "conformal" or "fractal" structure)



Partons ab → "collinear":

P(z) = DGLAP splitting kernels, with z = energy fraction = $E_a/(E_a+E_b)$

$$|\mathcal{M}_{F+1}(\ldots,a,b,\ldots)|^2 \overset{a||b}{\rightarrow} g_s^2\mathcal{C}\,\frac{P(z)}{2(p_a\cdot p_b)}|\mathcal{M}_F(\ldots,a+b,\ldots)|^2$$

Coherence → Parton j really emitted by (i,k) "colour antenna" (in leading colour approximation)

Gluon j → "soft":

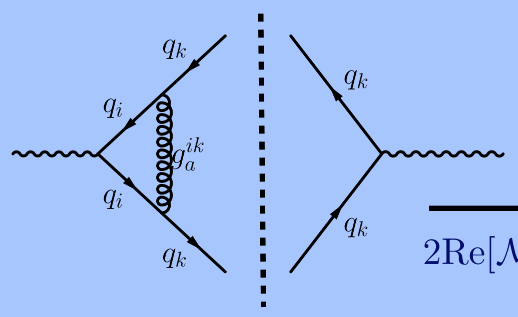
$$|\mathcal{M}_{F+1}(\ldots,i,j,k\ldots)|^2 \overset{j_g\to 0}{\rightarrow} g_s^2\mathcal{C}\,\frac{(p_i\cdot p_k)}{(p_i\cdot p_j)(p_j\cdot p_k)}|\mathcal{M}_F(\ldots,i,k,\ldots)|^2$$

+ scaling violation: $g_s^2 \rightarrow 4\pi\alpha_s(Q^2)$

Can apply this many times → nested factorizations
Jets-within-jets-within-jets …

From Legs to Loops

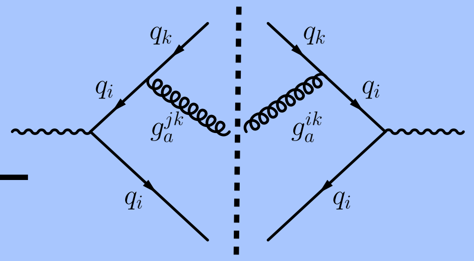
Unitarity: sum(probability) = 1



Kinoshita-Lee-Nauenberg:

(sum over degenerate quantum states = finite: infinities must cancel!)

$$\xrightarrow[2\mathrm{Re}[\mathcal{M}^{(1)}\mathcal{M}^{(0)*}]]{\qquad} \mathrm{Loop} \;=\; -\int \mathrm{Tree} \;+\; F \;\xleftarrow[\left|\mathcal{M}^{(0)}_{+1}\right|^2]{\qquad}$$



Neglect non-singular piece, F → "Leading-Logarithmic" (LL) Approximation

→ **Can also include loops-within-loops-within-loops …**

→ Bootstrap for approximate All-Orders Quantum Corrections!

Parton Showers: reformulation of pQCD corrections as gain-loss diff eq.

Iterative (Markov-Chain) evolution algorithm, based on universality and unitarity

With evolution kernel $\sim \frac{|\mathcal{M}_{n+1}|^2}{|\mathcal{M}_n|^2}$ (or soft/collinear approx thereof)

Generate explicit fractal structure across all scales (via Monte Carlo Simulation)

Evolve in some measure of *resolution* ~ hardness, virtuality, 1/time … ~ fractal scale

+ account for scaling violation via quark masses and $g_s^2 \to 4\pi\alpha_s(Q^2)$

Our Research

Parton Showers are based on 1→2 splittings

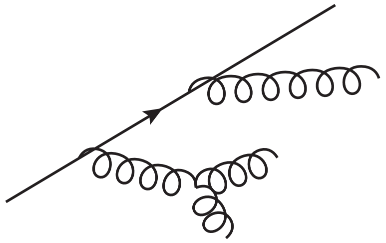
I.e., each **parton** undergoes a sequence of splittings

Multi-parton coherence effects can be included via “angular ordering”

Or via “dipole radiation functions”

(~ partitions dipole radiation pattern into 2 monopole terms)

Recoil effects needed to impose (E,p) conservation (“local” or “global”)



E.g., **PYTHIA** (also HERWIG, SHERPA)

At Monash, we develop an **Antenna Shower**, in which splittings are fundamentally 2→3

Each colour **dipole/antenna** undergoes a sequence of splittings

+ Intrinsically includes dipole coherence (leading N_C)

+ Lorentz invariance and explicit local (E,p) conservation

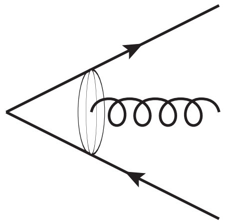
+ The non-perturbative limit of a colour dipole is a string piece

Roots in Lund ~ mid-80ies: Gustafson & Petterson, Nucl.Phys. B306 (1988) 746

What’s new in our approach?

Higher-order perturbative effects can be introduced via calculable corrections in an elegant and very efficient way

+ Writing a genuine antenna shower also for the initial state evolution

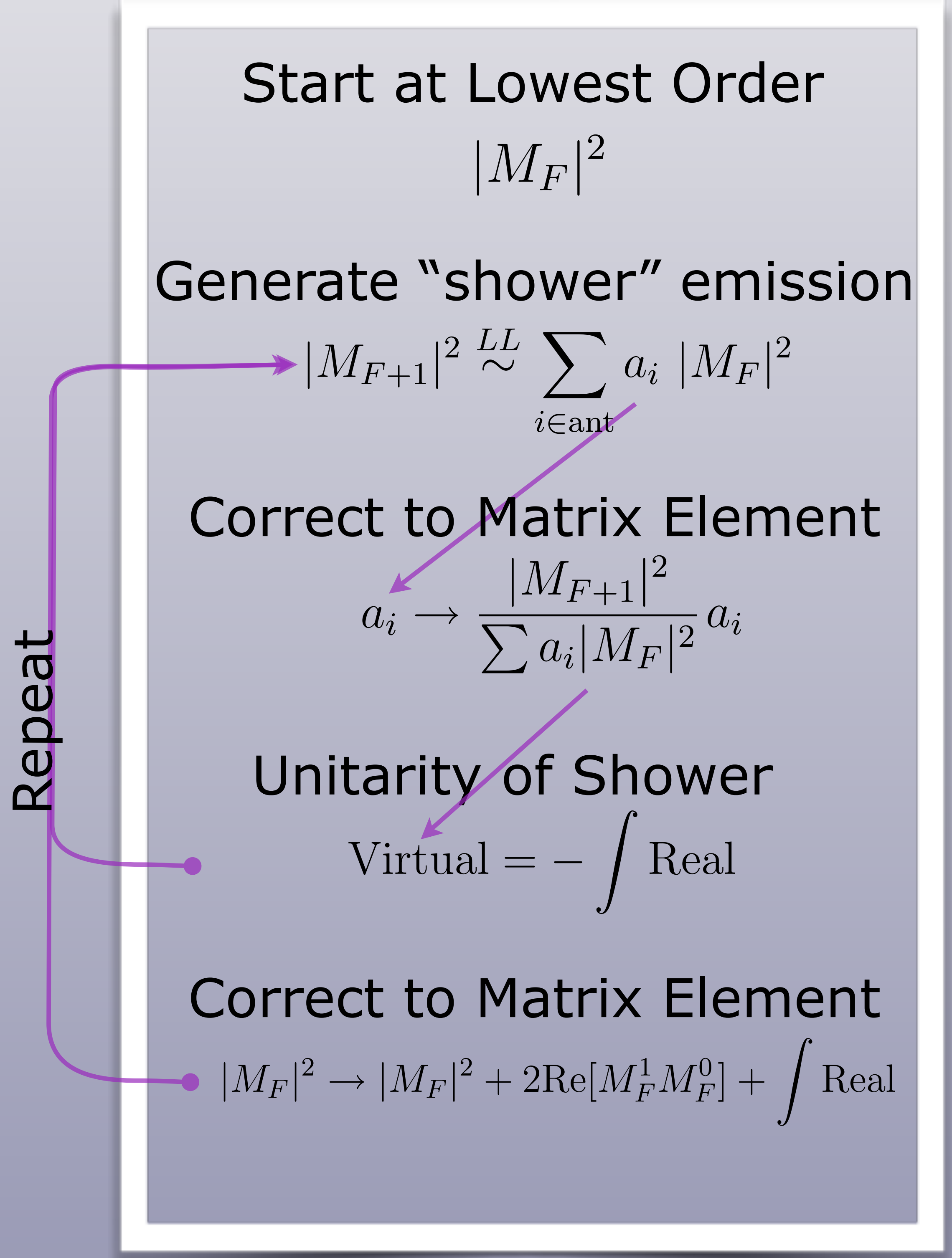


E.g., **VINCIA**
(also ARIADNE)

Cf a lattice and its dual lattice
Can either perceive of lattice **sites**
or lattice **links**. Equivalent (dual) representations.

VINCIA: Markovian pQCD*

*)pQCD : perturbative QCD



Start at Lowest Order

$$|M_F|^2$$

Generate "shower" emission

$$|M_{F+1}|^2 \overset{LL}{\sim} \sum_{i\in\text{ant}} a_i \; |M_F|^2$$

Correct to Matrix Element

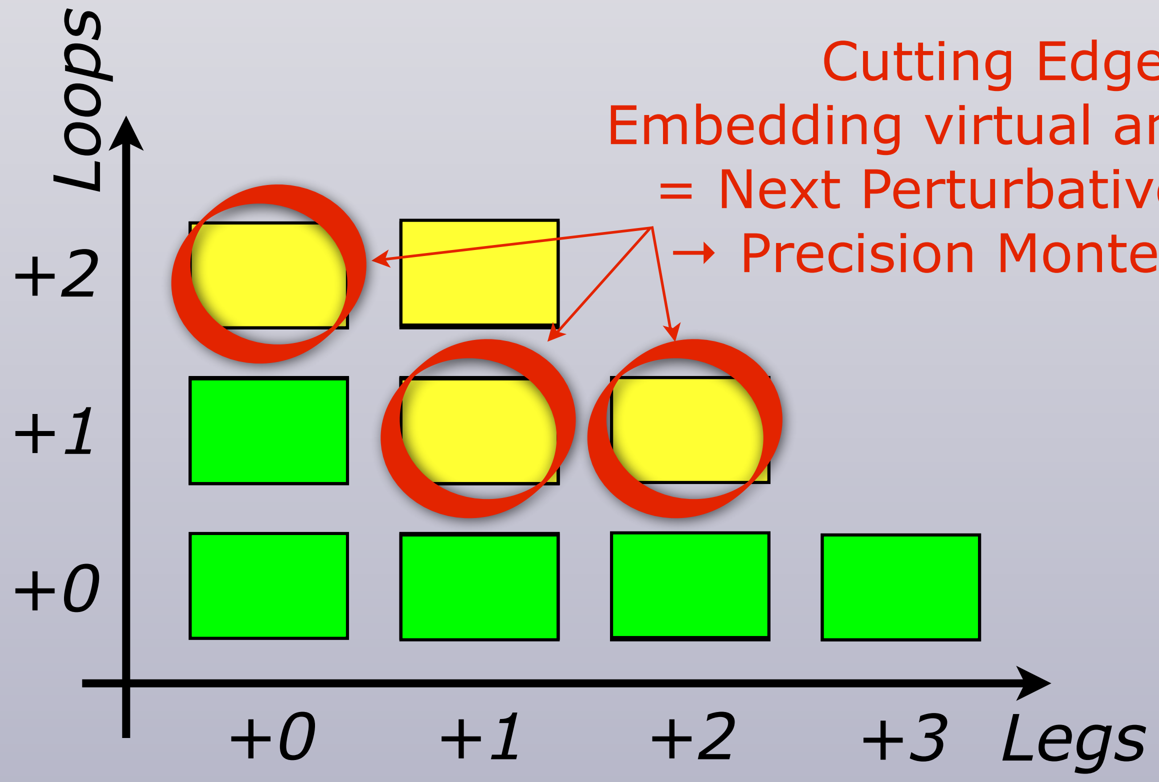
$$a_i \to \frac{|M_{F+1}|^2}{\sum a_i |M_F|^2} a_i$$

Unitarity of Shower

$$\text{Virtual} = -\int \text{Real}$$

Correct to Matrix Element

$$|M_F|^2 \to |M_F|^2 + 2\text{Re}[M_F^1 M_F^0] + \int \text{Real}$$



Cutting Edge:
Embedding virtual amplitudes
= Next Perturbative Order
→ Precision Monte Carlos

"Higher-Order Corrections To Timelike Jets"
GeeKS: Giele, Kosower, Skands, PRD 84 (2011) 054003

Quo Vadis?

All sights are on **Run 2 of the LHC**

- Next order of precision for jet rates and structure
  - Aid precision measurements and enhance discovery reach
  - Vast multi-jet phase spaces to explore with LHC
- \+ higher calculational efficiencies : SPEED
  - (has become a major issue for highly complicated final states)
  - Test runs in e^+e^- show factors 10^2 - 10^3 increases over conventional schemes
- \+ systematic and automated theory uncertainties
  - Part of being precise is knowing **how** precise. Our job to give an answer.

Understanding the fractal

- Unitarity and the structure of perturbative QCD
- Beyond the Leading-Logarithmic approximation?
- Beyond the Leading-Colour approximation?
- The Structure of the proton (parton distributions)

&

Get this research going in Australia

+ Applications

Example: The Top Quark

Heaviest known elementary particle:
m_t ~ 187 *u* (~m_{Au})

Lifetime: 10^{-24} s

Complicated decay chains:

$$t \to bW^+ \qquad \bar{t} \to \bar{b}W^-$$

$$W \to \{q\bar{q}', \ell\nu\}$$

quarks → jets
b-quarks → b-jets

$$m_t^2 \approx (p_b + p_{W^+})^2$$

$$\approx (p_{b-\mathrm{jet}} + p_{q-\mathrm{jet}} + p_{\bar{q}-\mathrm{jet}})^2$$

Accurate jet energy calibrations → m_t

Analogously for any process / measurement involving coloured partons

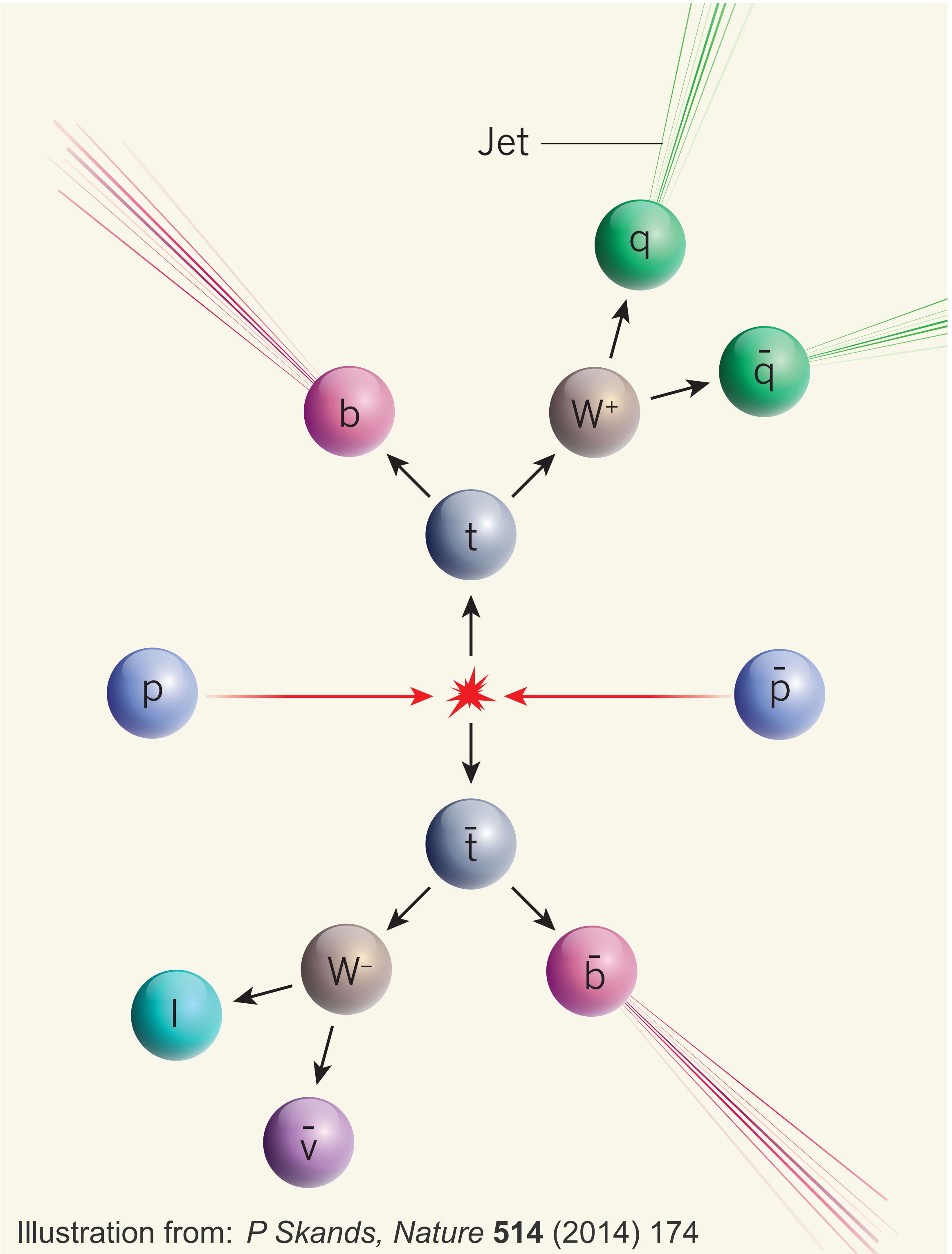


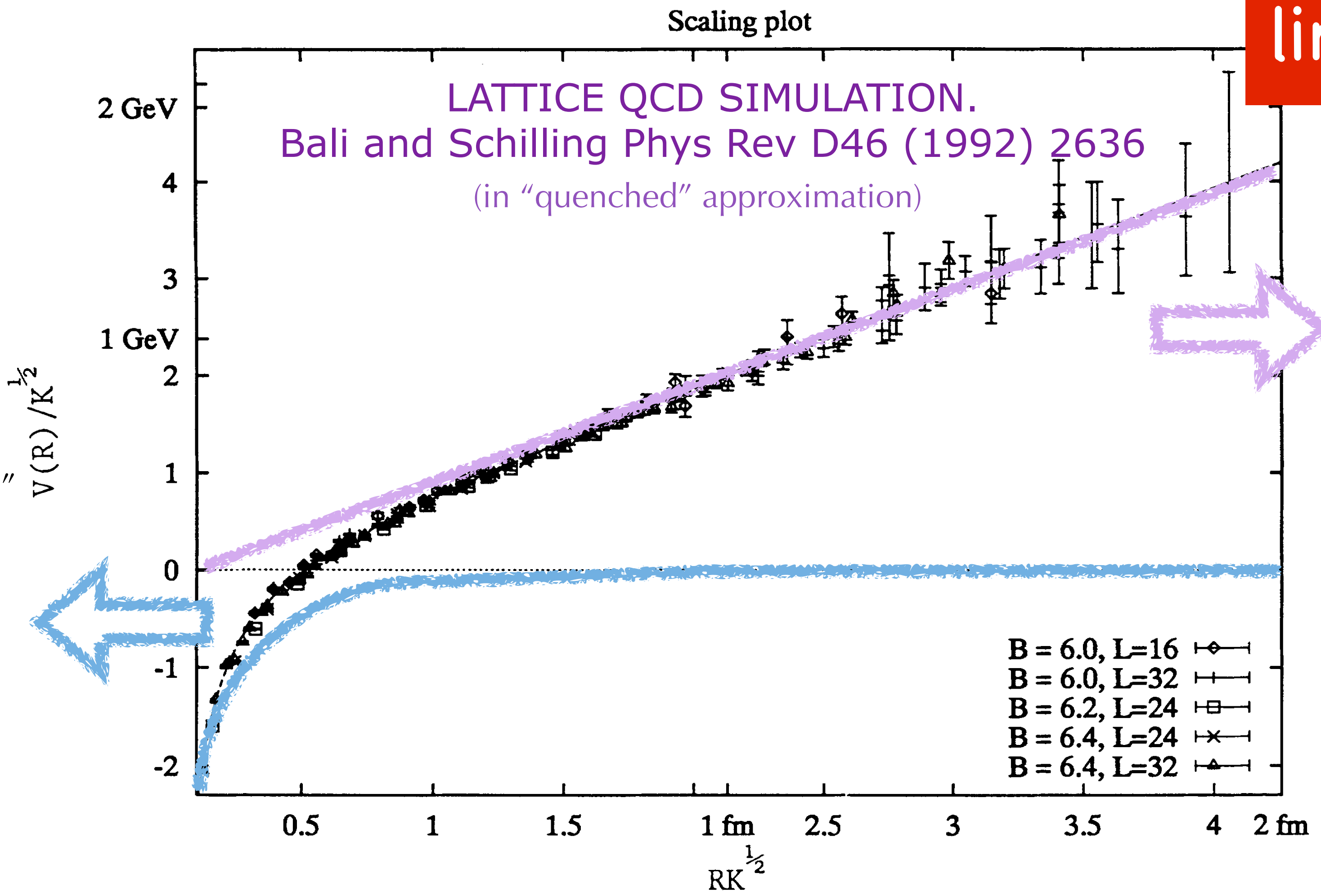
Illustration from: *P Skands, Nature* **514** (2014) 174

Long Wavelengths > 10^{-15} m

Quark-Antiquark Potential

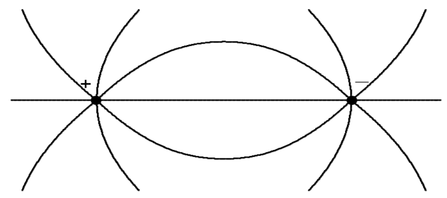
As function of separation distance

What physical system has a linear potential?



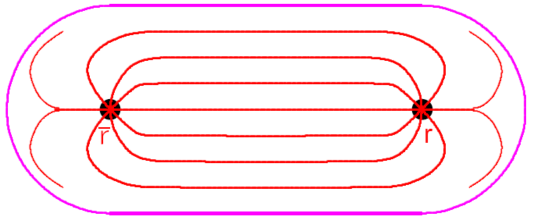
LATTICE QCD SIMULATION.
Bali and Schilling Phys Rev D46 (1992) 2636
(in “quenched” approximation)

Short Distances ~ “Coulomb”



“Free” Partons

Long Distances ~ Linear Potential



“Confined” Partons
(a.k.a. Hadrons)

$$F(r) \approx \text{const} = \kappa \approx 1\ \text{GeV/fm} \iff V(r) \approx \kappa r$$

~ Force required to lift a 16-ton truck

String Breaks

In QCD, strings can (and do) break!

(In superconductors, would require magnetic monopoles)

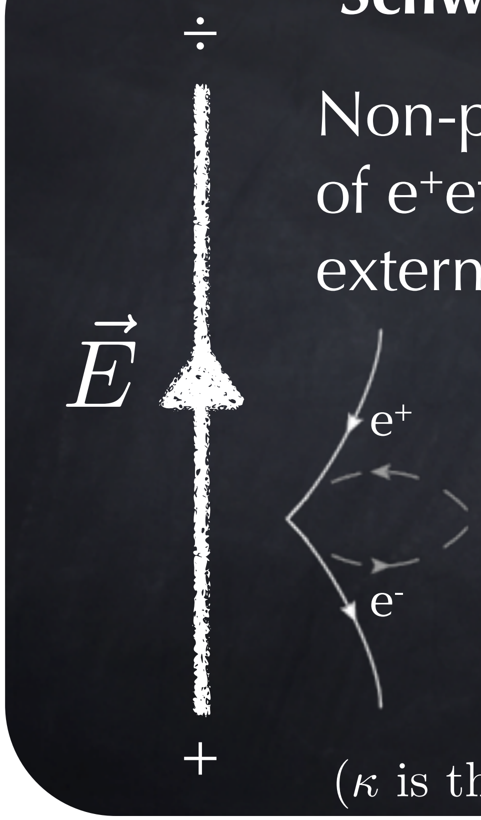
In QCD, the roles of electric and magnetic are reversed

Quarks (and antiquarks) are "chromoelectric monopoles"

There are at least two possible analogies ~ tunneling:

CANONICAL

Schwinger Effect



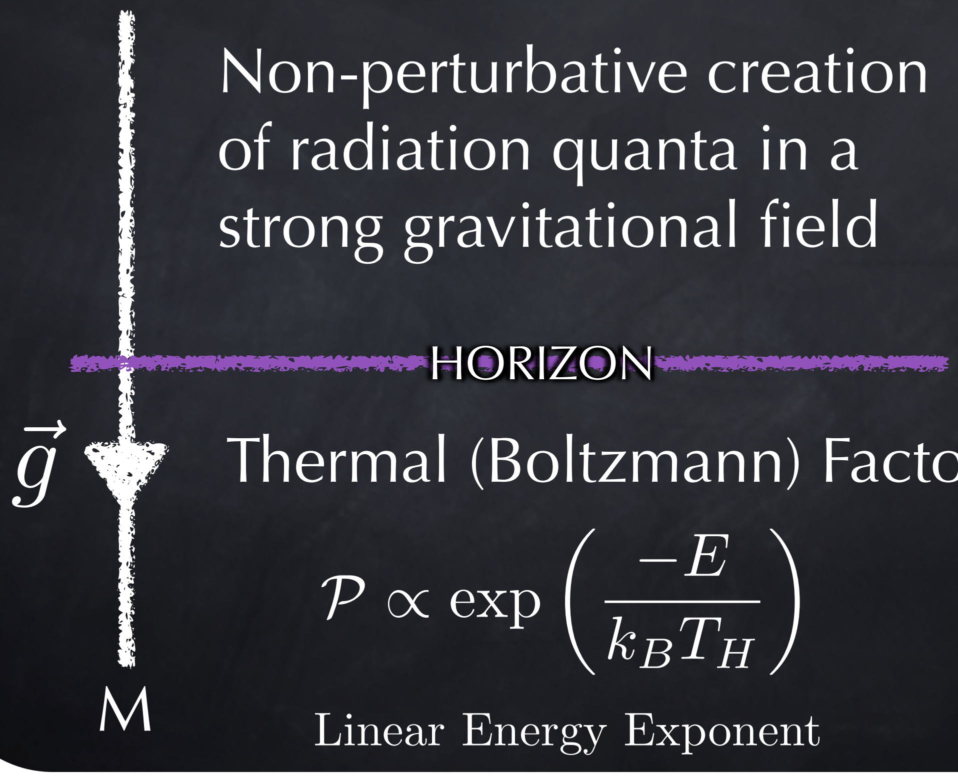
Non-perturbative creation of e^+e^- pairs in a strong external Electric field

Probability from Tunneling Factor

$$\mathcal{P} \propto \exp\left(\frac{-m^2 - p_\perp^2}{\kappa/\pi}\right)$$

(κ is the string tension equivalent)

Hawking Radiation



Non-perturbative creation of radiation quanta in a strong gravitational field

Thermal (Boltzmann) Factor

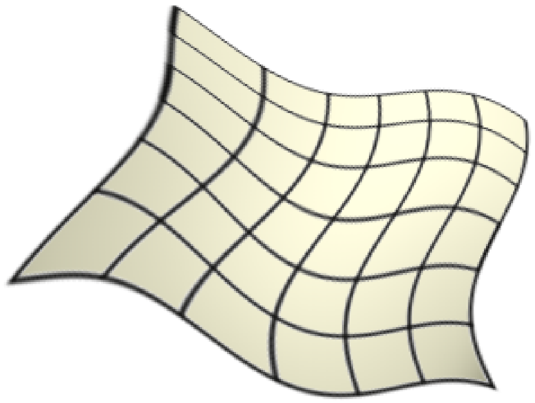
$$\mathcal{P} \propto \exp\left(\frac{-E}{k_B T_H}\right)$$

Linear Energy Exponent

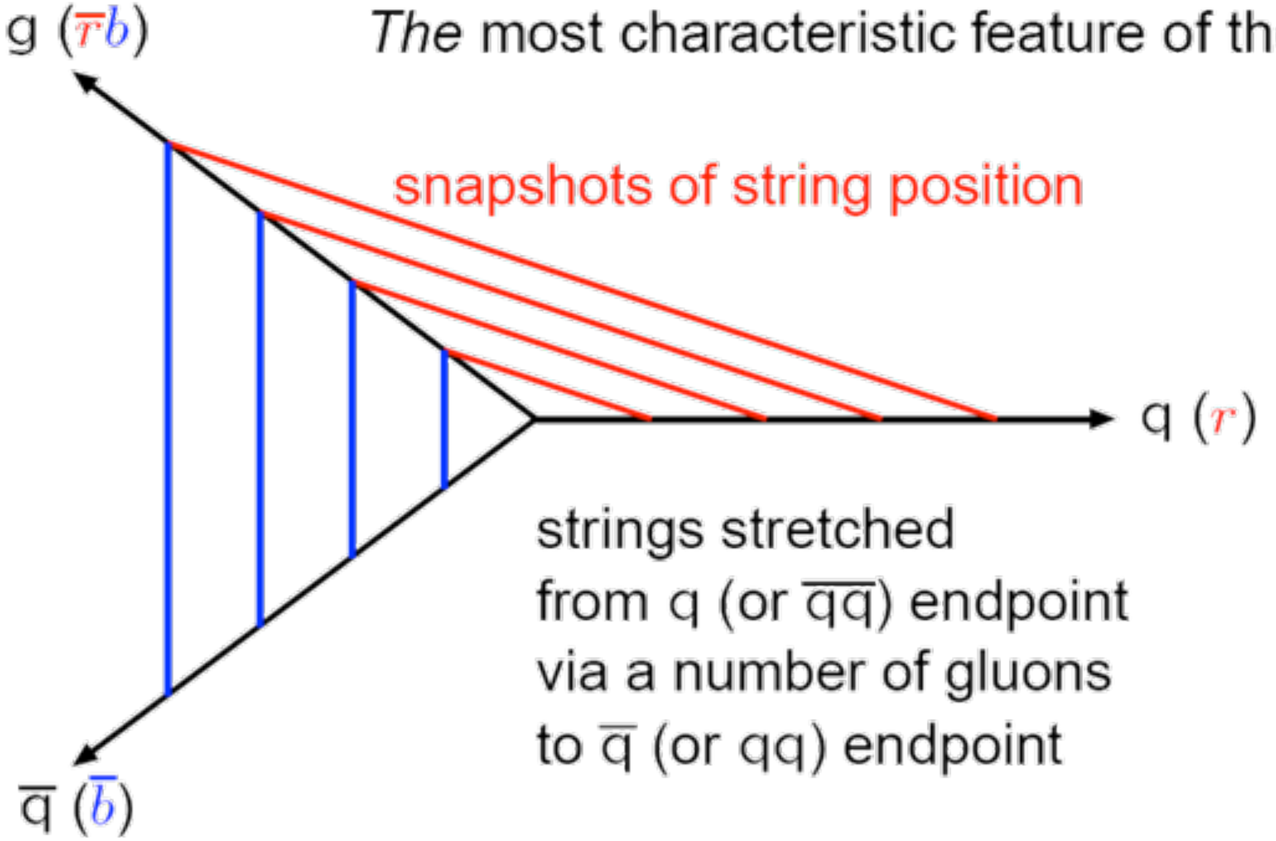
ALTERNATIVE?

The "Lund" String

- **Quarks** → String Endpoints
- **Gluons** → Transverse Excitations (kinks)



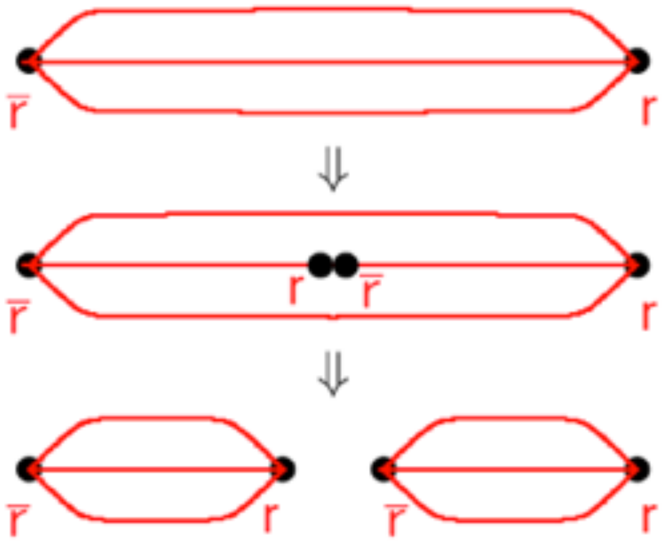
The most characteristic feature of the Lund model



strings stretched
from q (or $\overline{qq}$) endpoint
via a number of gluons
to $\bar{q}$ (or qq) endpoint

Gluon = kink on string, carrying energy and momentum

String Breaks
by Tunneling (Schwinger Type)

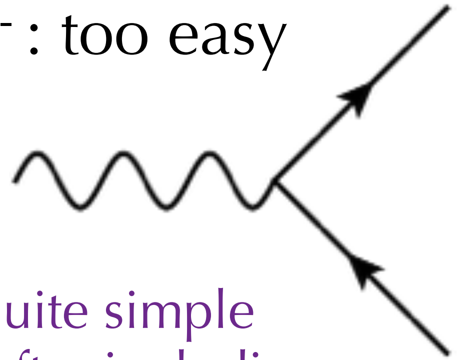


- Probability of string break constant per unit area → **AREA LAW**
- Breakup vertices causally disconnected → order is irrelevant → iterative algorithm

Colour Confusion

Between which partons do confining potentials arise?

e^+e^- : too easy



(still quite simple even after including bremsstrahlung etc.)

At e^+e^- colliders (eg LEP) - We generally find quite good agreement between **measured** particle spectra and **models** based on parton/antenna showers + strings

(with a couple of interesting exceptions, not covered here)

"Leading Colour" dipole decomposition works well

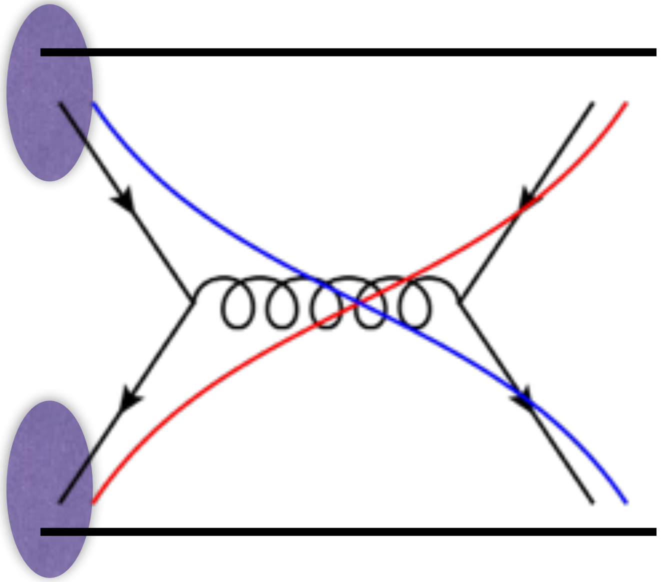
→ re-use same models as input for LHC (universality) ?

Proton-Proton (LHC)

More colour kicked around (& also colour in initial state)

Include "Beam Remnants"

Still might look relatively simple, to begin with



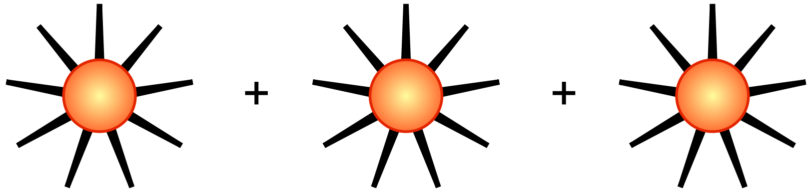
But no law against *several* parton-parton interactions

In fact, can easily be shown to happen frequently
Included in all (modern) Monte Carlo models
But how to make sense of the colour structure?

Collective Effects?

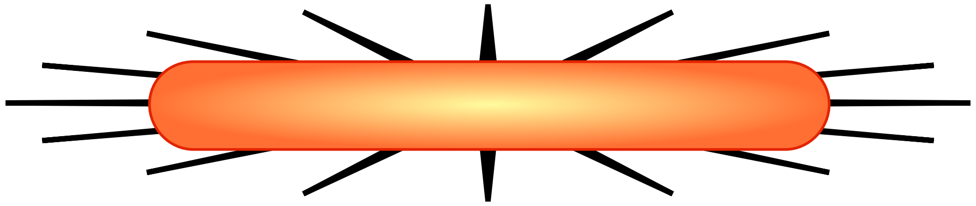
A rough indicator of how much colour gets kicked around, should be the number of particles produced

So we study event properties as a function of "N_{ch}" = N_{tracks}



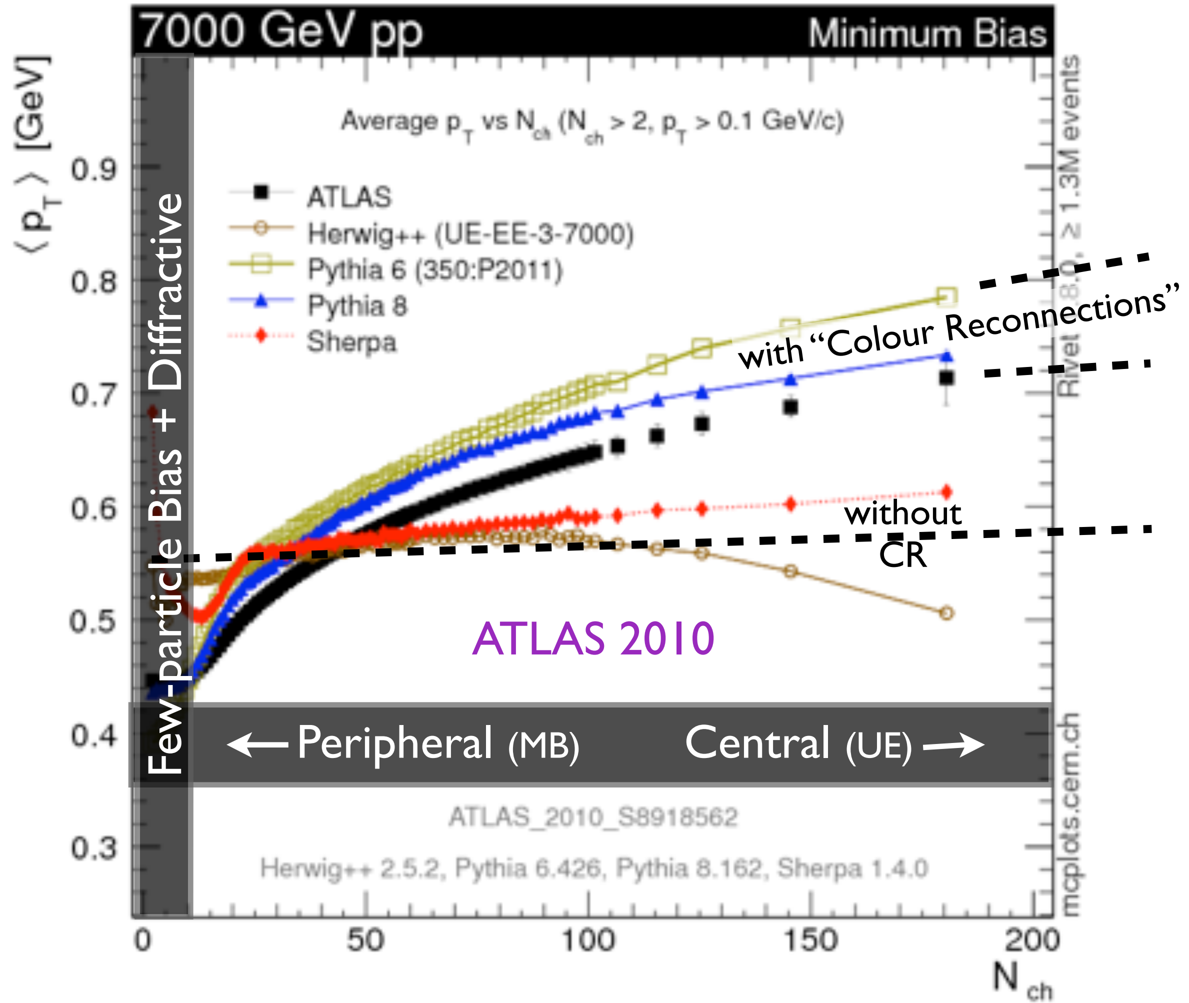
Independent Particle Production:

→ **averages stay the same**



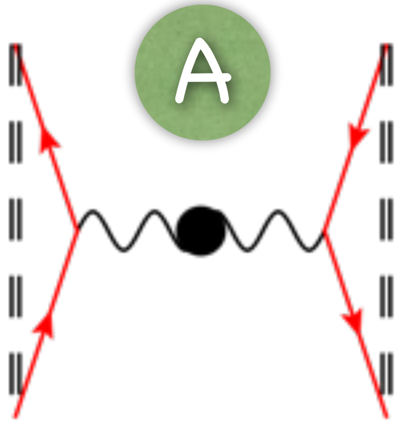
Correlations / Collective effects:

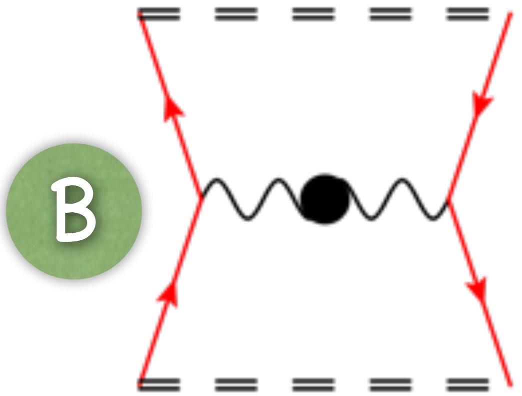
→ **averages depend on N_{ch}**



Plot shows the average transverse momentum versus N_{ch}

What are “Colour Reconnections”?





Simple example: $e^+e^- \to W^+W^- \to q_1\bar{q}_2q_3\bar{q}_4$

- Intensely studied at LEP2.
  - CR implied a non-perturbative uncertainty on the W mass measurement, ΔMW ~ 40 MeV
- CR constrained to ~ 10% ~ 1/NC2
- Simple two-string system. What about pp?

Several modelling attempts

- Based on minimising the string action
  - String interactions (Khoze, Sjostrand)
  - Generalized Area Law (Rathsman et al.)
  - Colour Annealing (Skands, Wicke)
  - Gluon Move Model (Sjostrand et al.)
- Based on SU(3)$_C$ group multiplet weights
  - Dipole Swing (Lonnblad et al.) $3 \otimes \bar{3} = 8 \oplus 1$
  - Generalized colour coherence (Christensen, Skands)

$$3 \otimes 3 = 6 \oplus \bar{3}\,.$$

$$8 \otimes 8 = 27 \oplus 10 \oplus \overline{10} \oplus 8 \oplus 8 \oplus 1$$

$$3 \otimes 8 = 15 \oplus 6 \oplus 3\,,$$

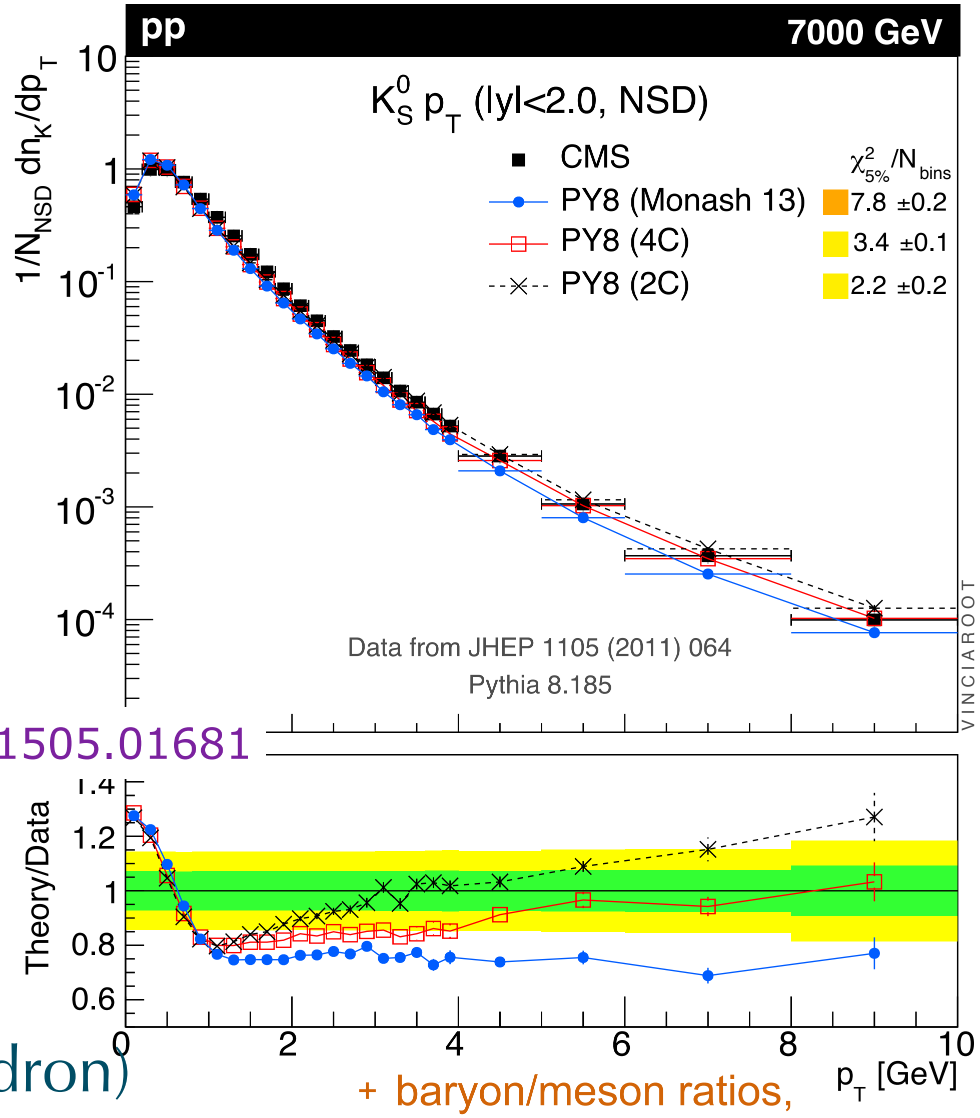
Collective Effects?

There is now quite a lot of confusion in the field

- Old-fashioned string models are having trouble at LHC
  - Eg need "CR" and don't reproduce low-pT identified-particle spectra
- Quark-gluon plasma inspired models?
  - Using hydrodynamics (eg EPOS)
  - Statistical (Thermal) Distributions
  - Good fits … even for ee … but … thermal???
    - **And how to reconcile with string picture?**
- Colour-(re)connection / String Effects?
  - Subleading colour effects?
    - **Multi-parton coherence? Colour accidents?**

Christensen, Skands: *String Formation Beyond Leading Colour*, arXiv:1505.01681

    - **Soft-gluon exchanges?**
  - String-string interaction effects?
  - More colour charge: strings with higher tension?
  - Rescattering Effects (parton-parton or hadron-hadron)



+ baryon/meson ratios, correlations, strangeness, …

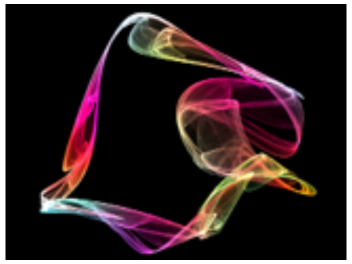
Summary

Jets

- Discovered at SPEAR (SLAC '72) and DORIS (DESY '73): E_{CM} ~ 5 GeV
  - Collimated sprays of nuclear matter (hadrons).
- Quasi-fractal structure of jets-within-jets & loops-within-loops
  - Simulated by parton-, dipole-, or **antenna** showers
  - Complementary to usual perturbative (LO, NLO, …) matrix elements
    - **Showers are most precise for relatively soft/collinear radiation**
    - **Fixed-order calculations are most precise for relatively "hard" radiation**
    - **Much focus on how to *combine* the two consistently and efficiently: "matching"**
  - Unitarity is a key aspect of both approaches; sums & detailed balance.

Strings enforce confinement

- ~ well understood in "dilute" environments ~ vacuum
- Many indications that confinement is more complicated in pp
  - LHC Run 1 provided a treasure trove of data.
  - We are learning which questions to ask; what to measure in **Run 2** !



New research at Monash

Precision LHC **phenomenology**

PYTHIA & **VINCIA**

NLO Event Generators

QCD strings, hadronisation

Outreach and Citizen Science

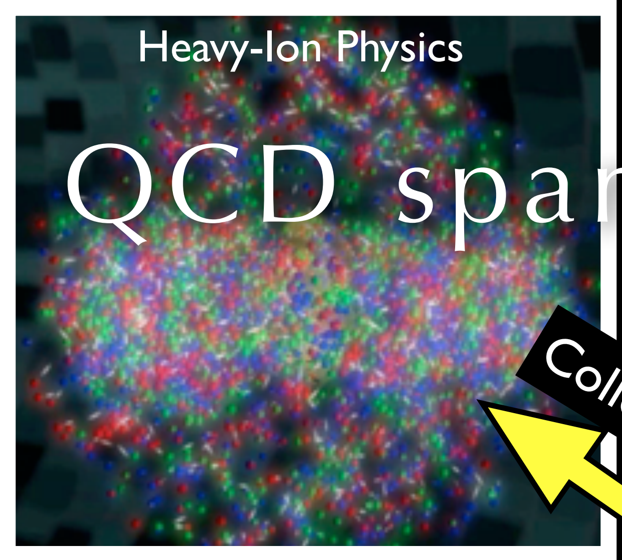
Support LHC **experiments,**
astro-particle community,
and **Future** accelerators

+ Warwick Alliance

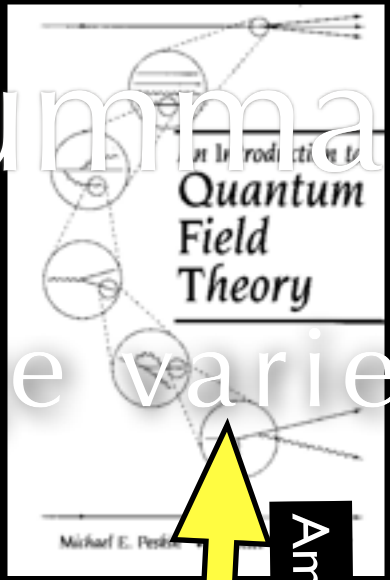
New joint research program with Warwick ATLAS, on developing and testing advanced colllider-QCD models. **PHD studentship open now**: based at Monash + 1 year in the UK/CERN.

Summary

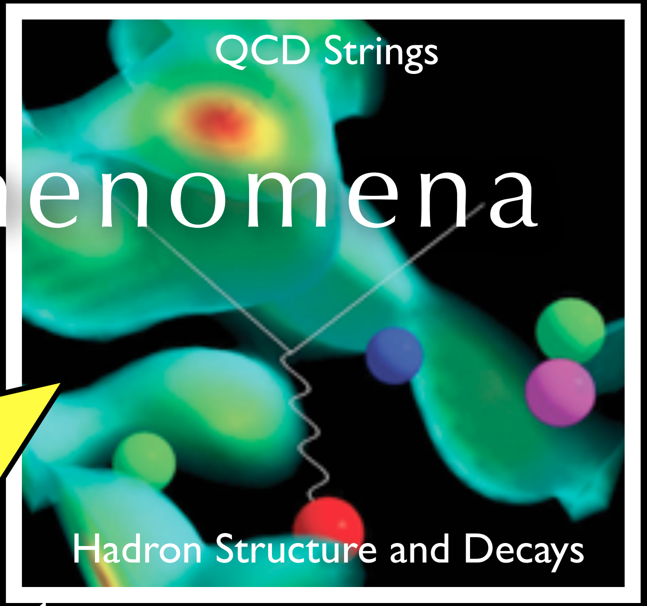
QCD spans a huge variety of phenomena


Heavy-Ion Physics

Collective Effects



Amplitudes

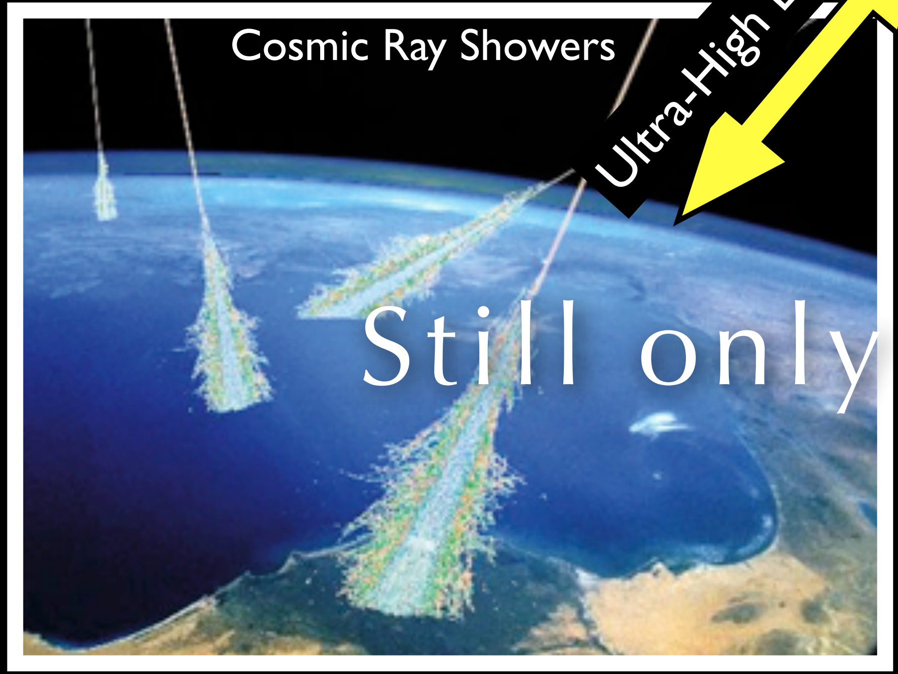

QCD Strings

Hadron Structure and Decays

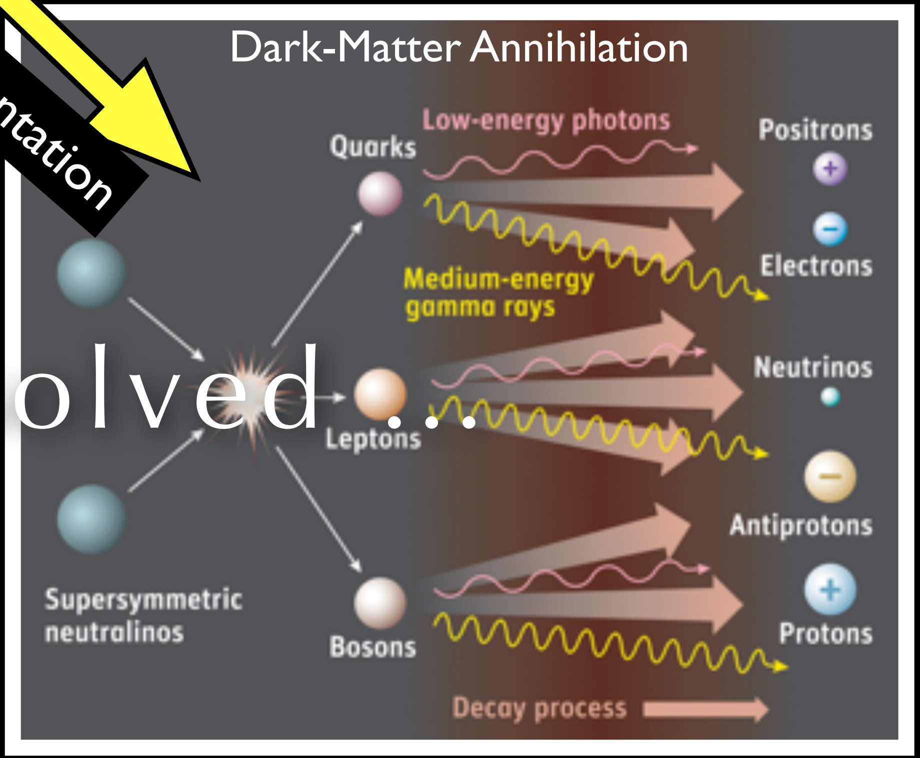
Confinement

$$\mathcal{L} = \bar{\psi}_q^i (i\gamma^\mu)(D_\mu)_{ij}\psi_q^j - m_q \bar{\psi}_q^i \psi_{qi} - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}$$

Ultra-High Energies


Cosmic Ray Showers

Fragmentation


Dark-Matter Annihilation

Still only partially solved…

Asymptotic Freedom

"What this year's Laureates discovered was something that, at first sight, seemed completely contradictory. The interpretation of their mathematical result was that the closer the quarks are to each other, the *weaker* is the 'colour charge'. When the quarks are really close to [*1] each other, the ~~force~~ charge is so weak that they behave almost as free particles. This phenomenon is called 'asymptotic freedom'. The converse is true when the quarks move apart: [*2] the ~~force~~ potential becomes stronger when the distance increases."

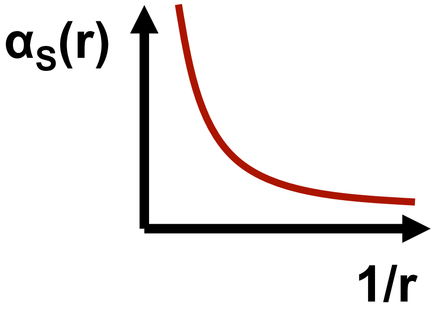
Nobelprize.org
The Official Web Site of the Nobel Prize

The Nobel Prize in Physics 2004
David J. Gross, H. David Politzer, Frank Wilczek

David J. Gross H. David Politzer Frank Wilczek

The Nobel Prize in Physics 2004 was awarded jointly to David J. Gross, H. David Politzer and Frank Wilczek *"for the discovery of asymptotic freedom in the theory of the strong interaction"*.

Photos: Copyright © The Nobel Foundation



[*1] The force still goes to ∞ as $r \to 0$ (Coulomb potential), just less slowly

[*2] The potential grows linearly as $r \to \infty$, so the force actually becomes constant (even this is only true in "quenched" QCD. In real QCD, the force eventually vanishes for r>>1fm)

Evolution Equations

What we need is a differential equation

Boundary condition: a few partons defined at a high scale (Q_F)

Then evolves (or "runs") that parton system down to a low scale (the hadronization cutoff ~ 1 GeV) → It's an evolution equation in Q_F

Close analogue: nuclear decay

Evolve an unstable nucleus. Check if it decays + follow chains of decays.

Decay constant

$$\frac{\mathrm{d}P(t)}{\mathrm{d}t} = c_N$$

Probability to remain undecayed in the time interval $[t_1,t_2]$

$$\Delta(t_1,t_2) = \exp\left(-\int_{t_1}^{t_2} c_N\,\mathrm{d}t\right) = \exp\left(-c_N\,\Delta t\right)$$

$$= 1 - c_N\Delta t + \mathcal{O}(c_N^2)$$

Decay probability per unit time

$$\frac{\mathrm{d}P_{\mathrm{res}}(t)}{\mathrm{d}t} = \frac{-\mathrm{d}\Delta}{\mathrm{d}t} = c_N\,\Delta(t_1,t)$$

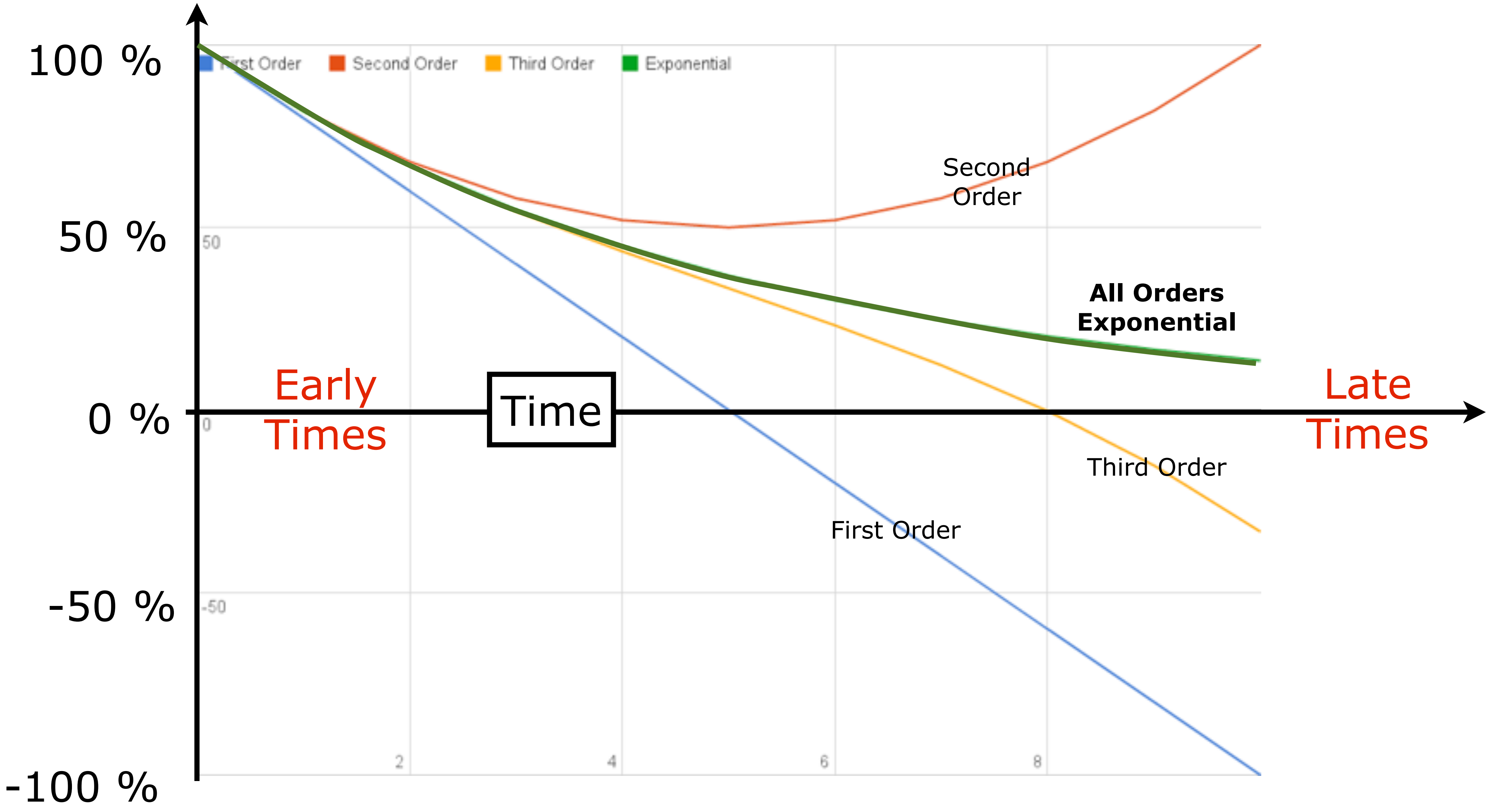
(requires that the nucleus did not already decay)

$\Delta(t_1,t_2)$: "Sudakov Factor"

Nuclear Decay

Nuclei remaining undecayed after time t

$$= \Delta(t_1, t_2) = \exp\left(-\int_{t_1}^{t_2} \mathrm{d}t\, \frac{\mathrm{d}\mathcal{P}}{\mathrm{d}t}\right)$$



The Sudakov Factor

In nuclear decay, the Sudakov factor counts:

How many nuclei remain undecayed after a time t

Probability to remain undecayed in the time interval [t_1,t_2]

$$\Delta(t_1,t_2) = \exp\left(-\int_{t_1}^{t_2} c_N \,\mathrm{d}t\right) \;=\; \exp\left(-c_N\,\Delta t\right)$$

The Sudakov factor for a parton system counts:

The probability that the parton system doesn't evolve (branch) when we run the factorization scale (~1/time) from a high to a low scale

Evolution probability per unit "time"

$$\frac{\mathrm{d}P_{\mathrm{res}}(t)}{\mathrm{d}t} = \frac{-\mathrm{d}\Delta}{\mathrm{d}t} = c_N\,\Delta(t_1,t)$$

(replace *t* by shower evolution scale)

(replace c_N by proper shower evolution kernels)

What's the evolution kernel?

cf. conformal (fractal) QCD, Lecture 1
(and PDF evolution, Lecture 2)

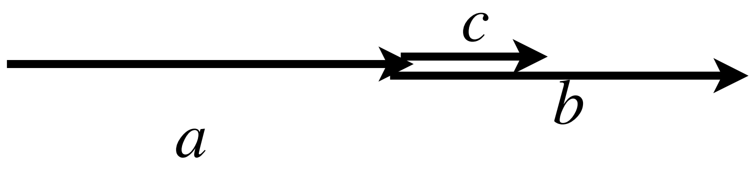
DGLAP splitting functions

Can be derived from *collinear limit* of MEs $(p_b+p_c)^2 \rightarrow 0$

+ evolution equation from invariance with respect to Q_F → RGE

DGLAP
(E.g., PYTHIA)

$$\mathrm{d}\mathcal{P}_a = \sum_{b,c} \frac{\alpha_{abc}}{2\pi} P_{a\to bc}(z)\,\mathrm{d}t\,\mathrm{d}z \ .$$



$p_b = z\, p_a$

$p_c = (1\text{-}z)\, p_a$

$$\begin{aligned}
P_{\mathrm{q}\to\mathrm{qg}}(z) &= C_F \frac{1+z^2}{1-z} \ , \\
P_{\mathrm{g}\to\mathrm{gg}}(z) &= N_C \frac{(1-z(1-z))^2}{z(1-z)} \ , \\
P_{\mathrm{g}\to\mathrm{q\bar{q}}}(z) &= T_R\,(z^2+(1-z)^2) \ , \\
P_{\mathrm{q}\to\mathrm{q}\gamma}(z) &= e_{\mathrm{q}}^2 \frac{1+z^2}{1-z} \ , \\
P_{\ell\to\ell\gamma}(z) &= e_\ell^2 \frac{1+z^2}{1-z} \ ,
\end{aligned}$$

$$\mathrm{d}t = \frac{\mathrm{d}Q^2}{Q^2} = \mathrm{d}\ln Q^2$$

… with Q^2 some measure of "hardness"
= event/jet resolution
measuring parton virtualities / formation time / …

Note: there exist now also alternatives to AP kernels (with same collinear limits!): dipoles, antennae, …