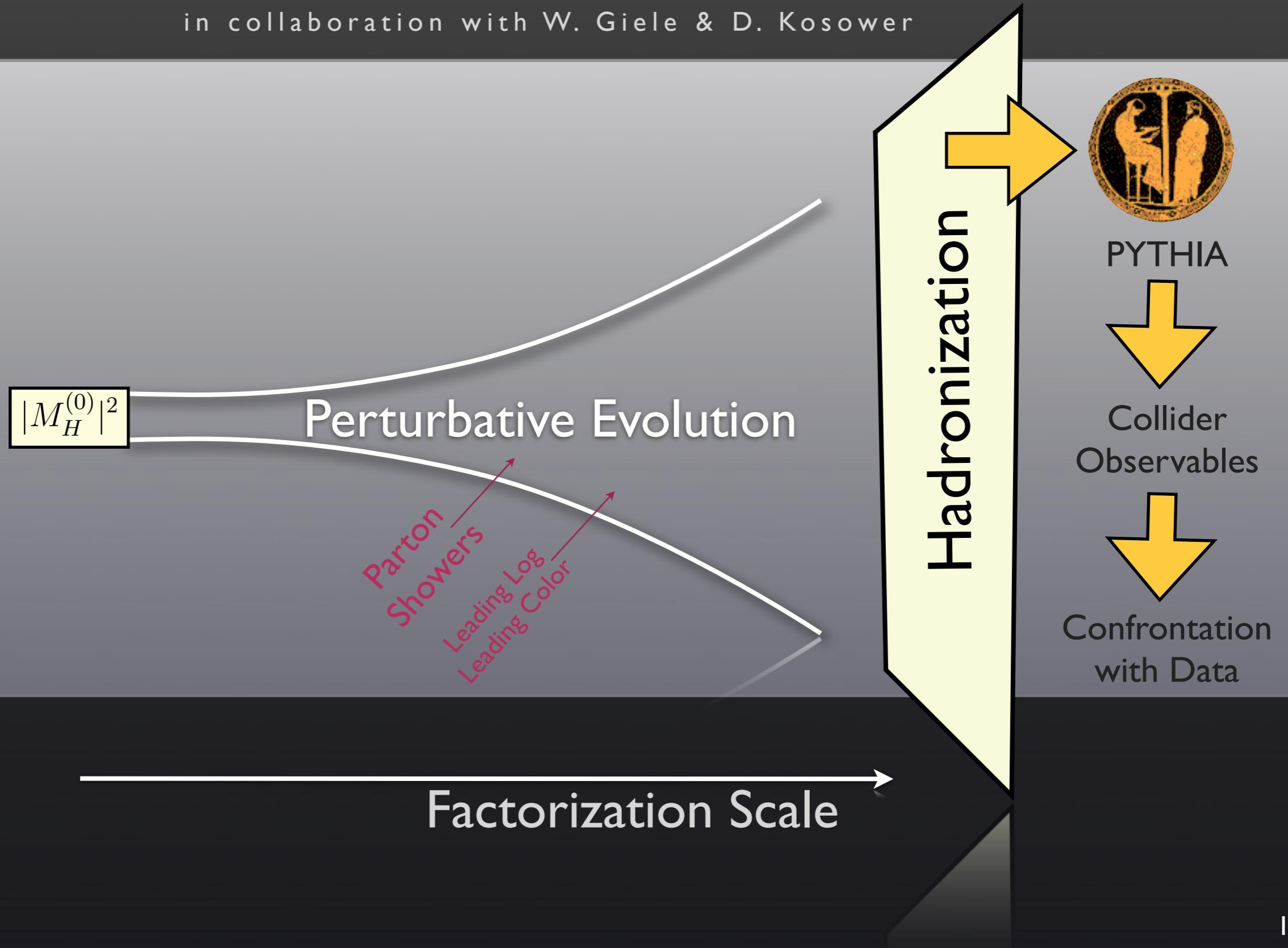


# A New Formalism for LO Matching

P. Skands & J. Lopez-Villarejo (CERN)

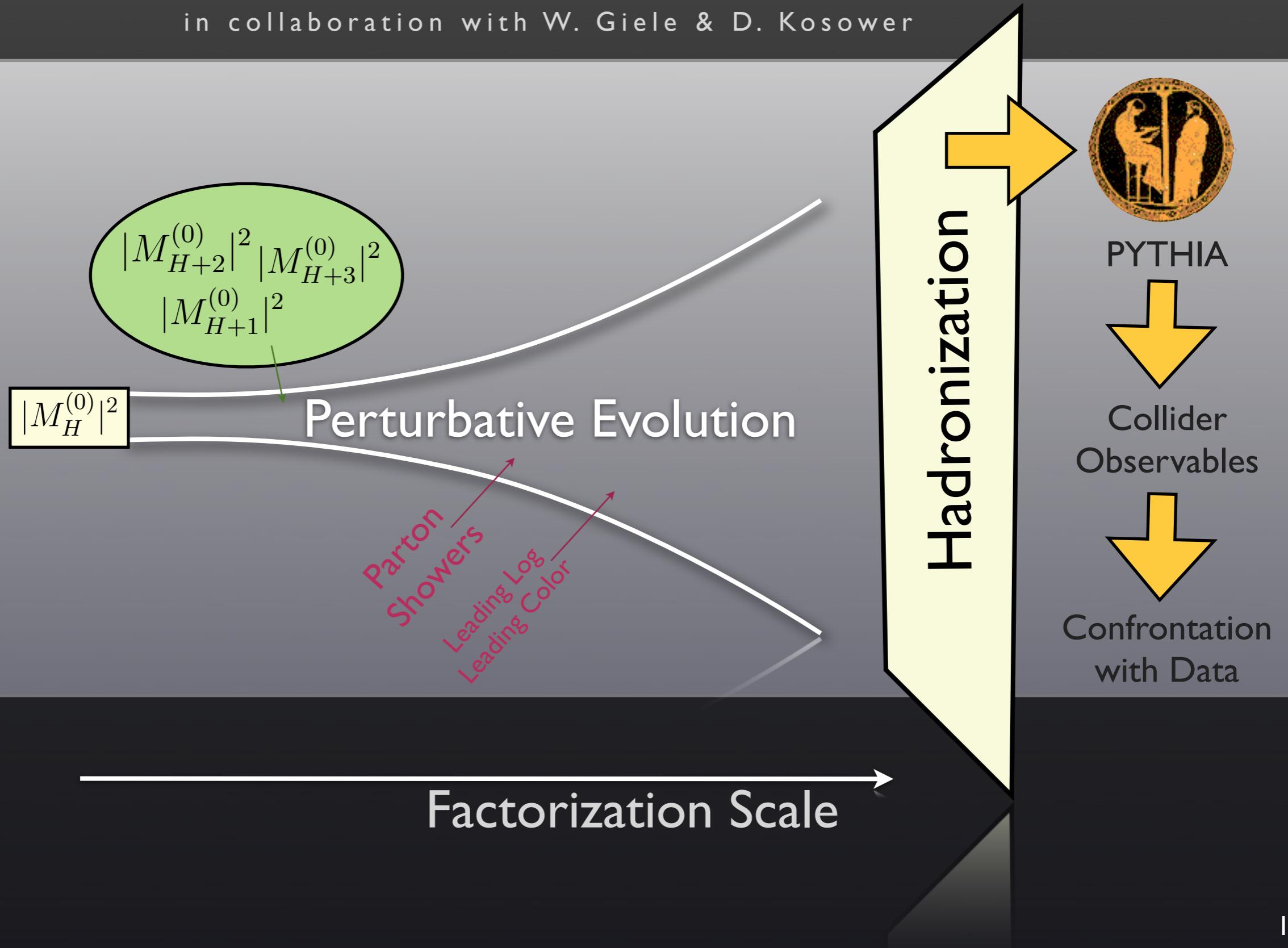
in collaboration with W. Giele & D. Kosower



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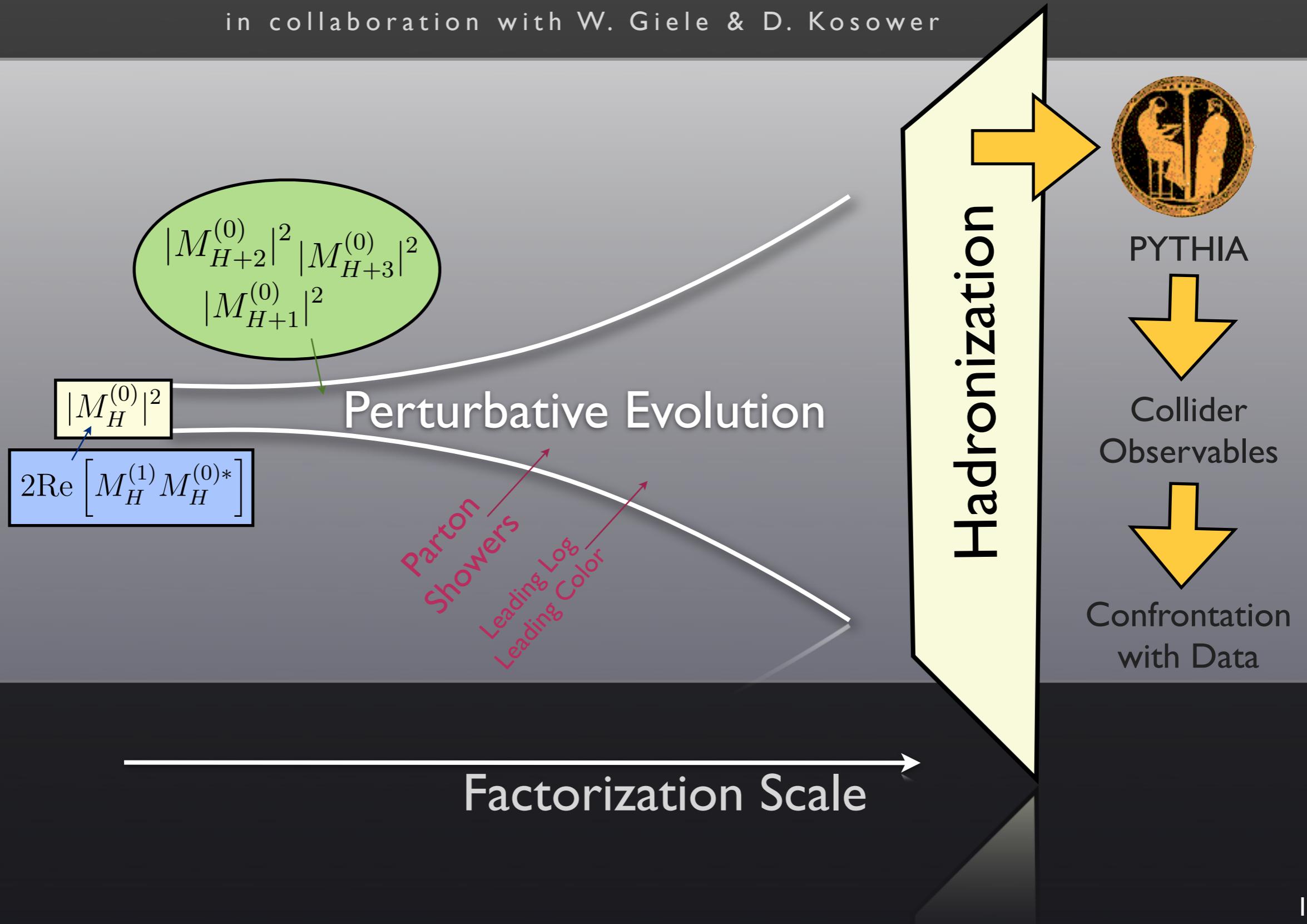
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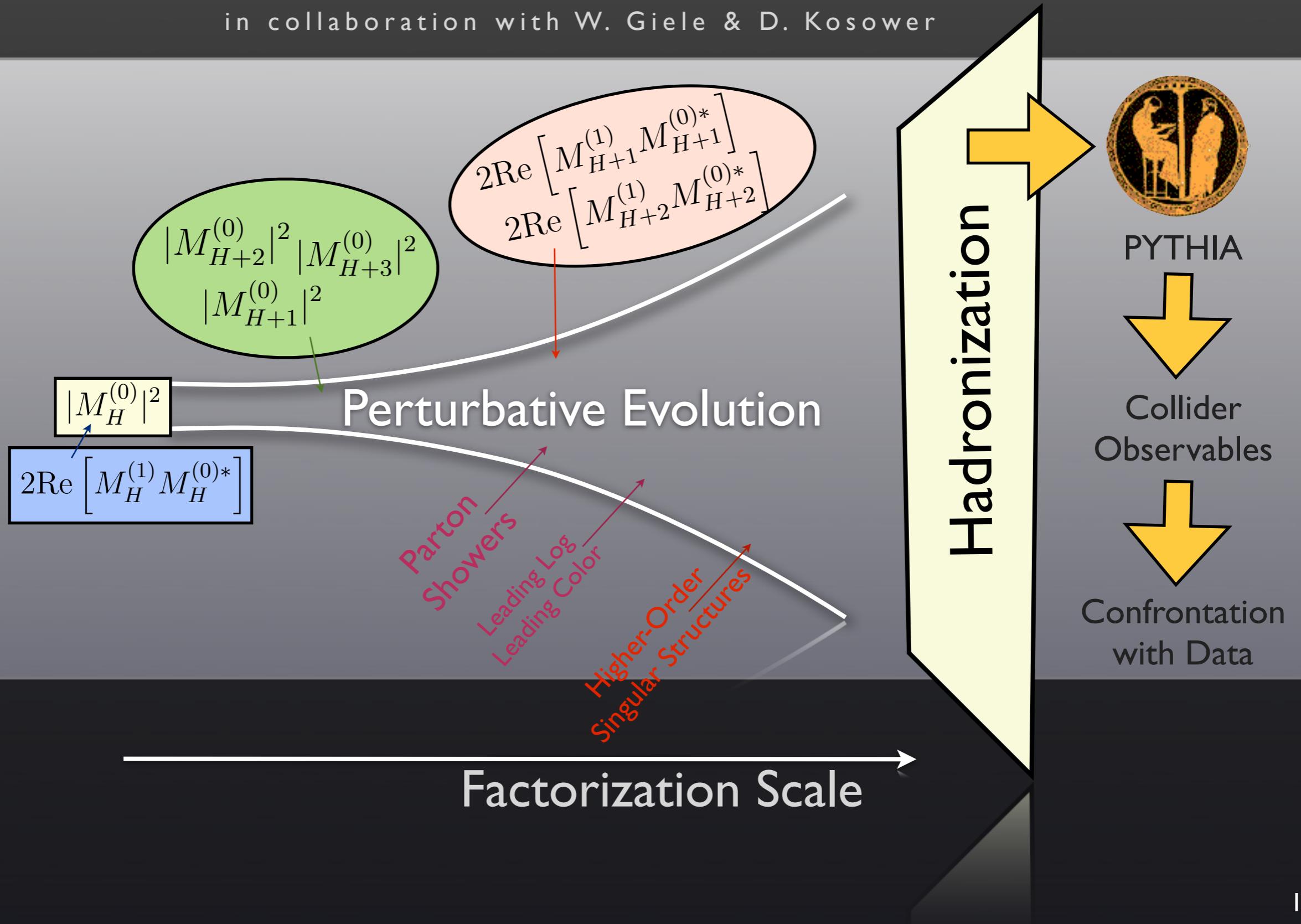
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# A New Formalism for LO Matching

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# “New” ?

## For matching to the first emission:

= PYTHIA scheme

(reformulated for antennae)

Sjöstrand & Bengtsson, Phys.Lett. B185 (1987) 435, Nucl.Phys. B289 (1987) 810

## For matching to the first loop:

= POWHEG scheme

(real-emission part same as PYTHIA, hence compatible)

Nason, JHEP 0411 (2004) 040; Nason, Ridolfi, JHEP 0608 (2006) 077; ...

## What is new (apart from antennae):

Giele, Kosower, Skands, arXiv:1102.2126 (accepted, PRD)

Repeating this for the next emission, and the next, ...

*GKS ~ multileg scheme (unitary) that reduces to PYTHIA/POWHEG at 1<sup>st</sup> order*

*Unitarity → No “matching scale” needed*

Substantially faster than MLM, CKKW (no initialization, no separate n-parton phase-spaces)

*The calculation also yields ~10 automatic uncertainty estimates at a moderate speed penalty (less than running the program twice)*

# VINCIA

## What is it?

Plug-in to PYTHIA 8 <http://projects.hepforge.org/vincia>

## What does it do?

“Matched Markov antenna showers”

*Improved parton showers*

- + Re-interprets tree-level matrix elements as  $2 \rightarrow n$  antenna functions
- + Extends matching to soft region (no “matching scale”)

Extensive (and automated) uncertainty estimates

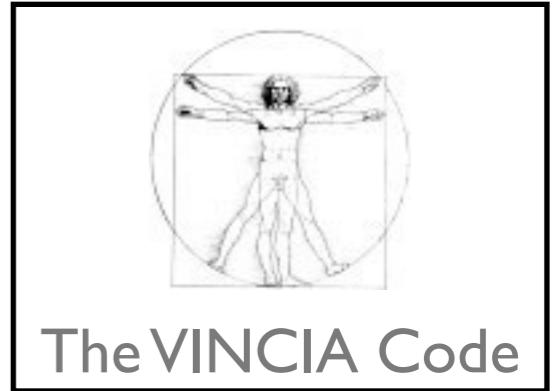
*Systematic variations of shower functions, evolution variables,  $\mu_R$ , etc.*

→ A vector of output weights for each event (central value = unity = unweighted)

## Who is doing it?

GEEKS: Giele, Kosower, Skands

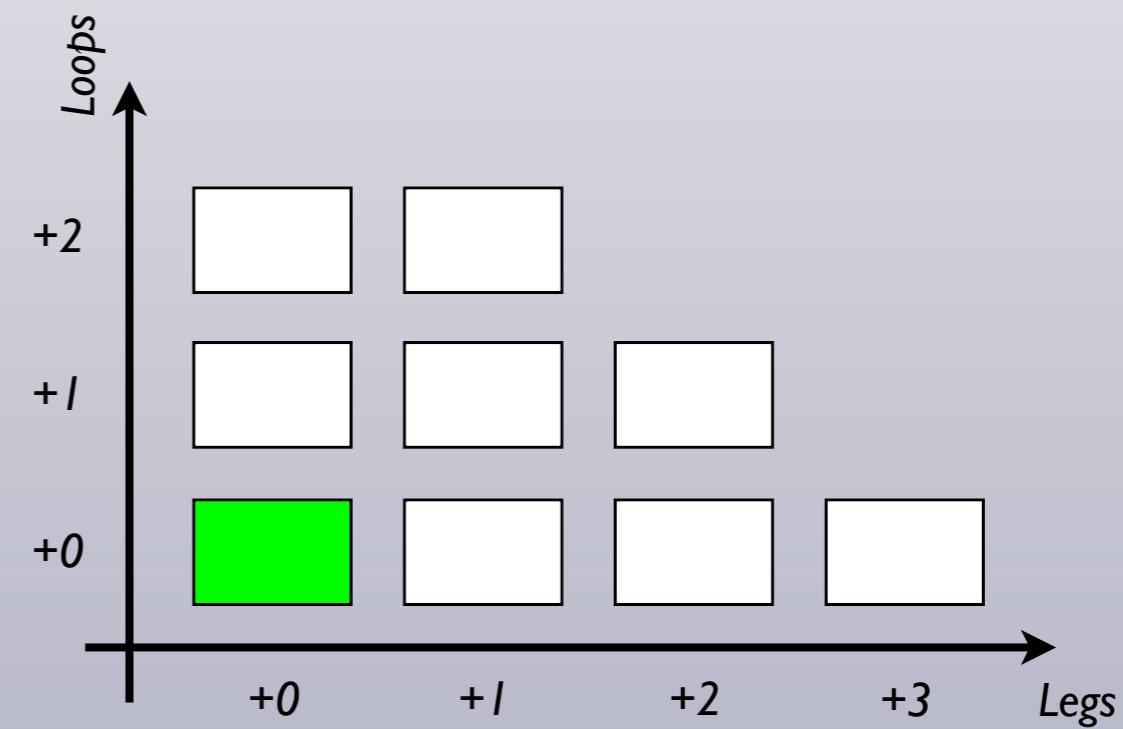
- + Collaborations with Gehrmann-de-Ridder & Ritzmann (mass effects), Lopez-Villarejo (“sector showers”), Hartgring & Laenen (NLO multileg), Diana (ISR), Larkoski (Polarization), Bravi & Volunteers (Tuning)



# Markov pQCD

Start at Born level

$$|M_F|^2$$



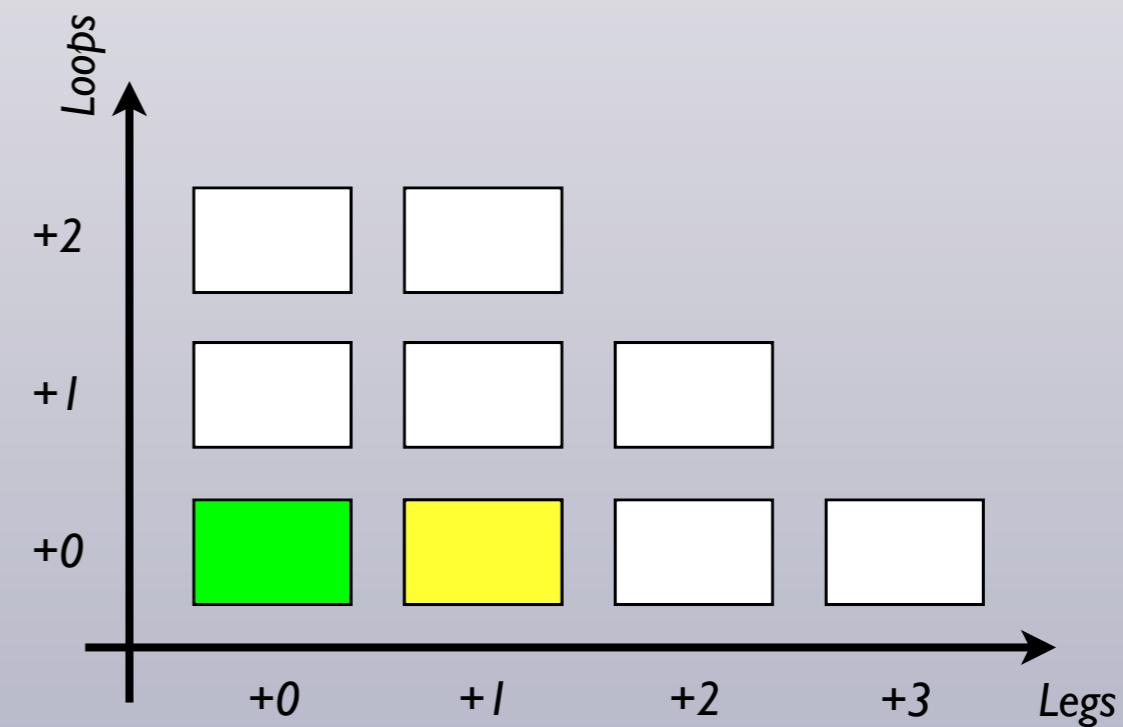
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$$|M_{F+1}|^2 \stackrel{LL}{\sim} \sum_{i \in \text{ant}} a_i |M_F|^2$$



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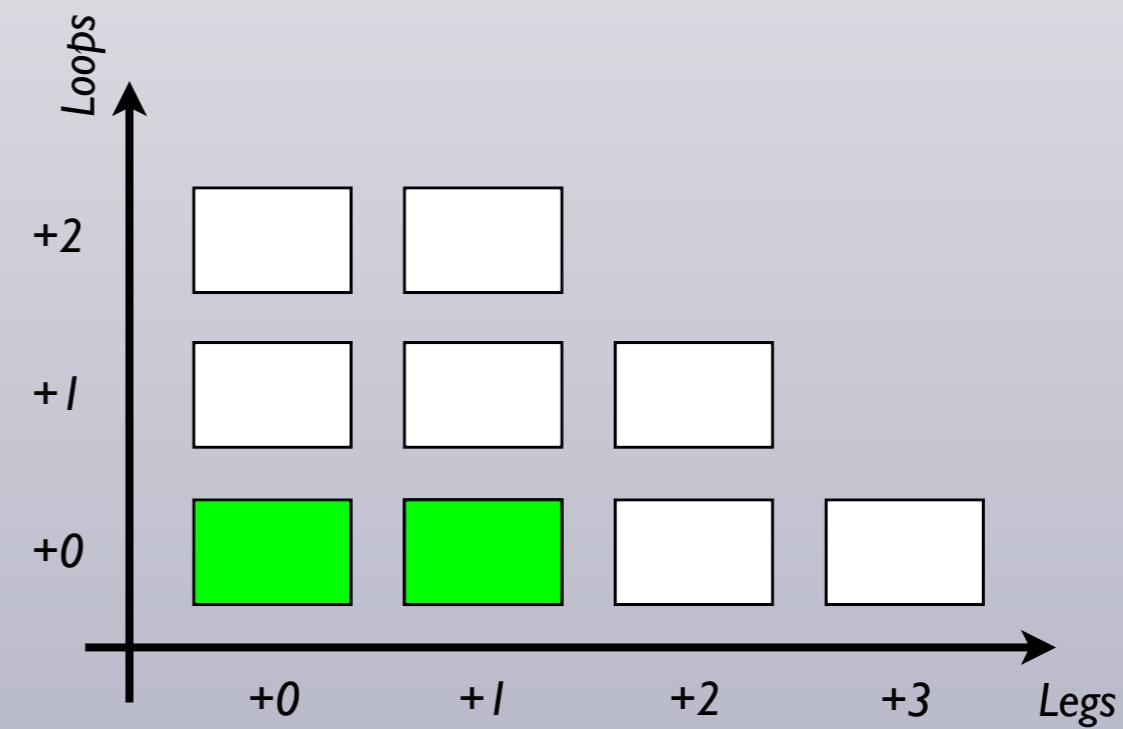
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Correct to Matrix Element

PYTHIA trick

$$a_i \rightarrow \frac{|M_{F+1}|^2}{\sum a_i |M_F|^2}$$



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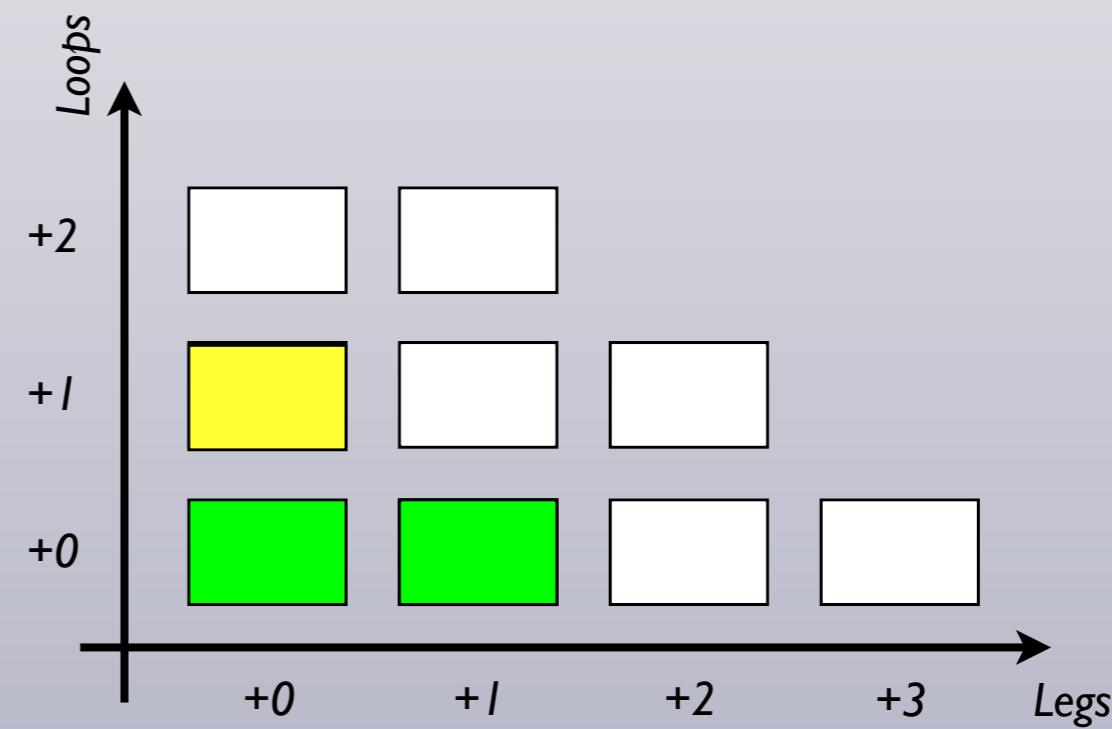
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Unitarity of Shower

$$\text{Virtual} = - \int \text{Real}$$



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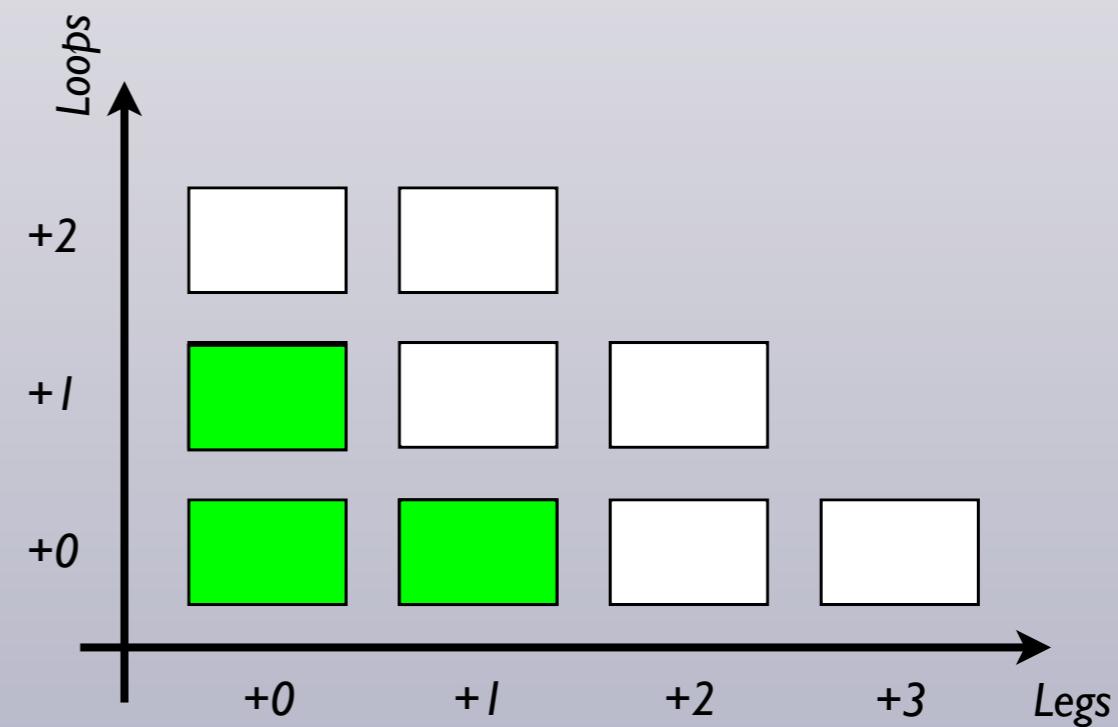
Unitarity of Shower

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$$|M_F|^2 \rightarrow |M_F|^2 + 2\text{Re}[M_F^1 M_F^0] + \int \text{Real}$$

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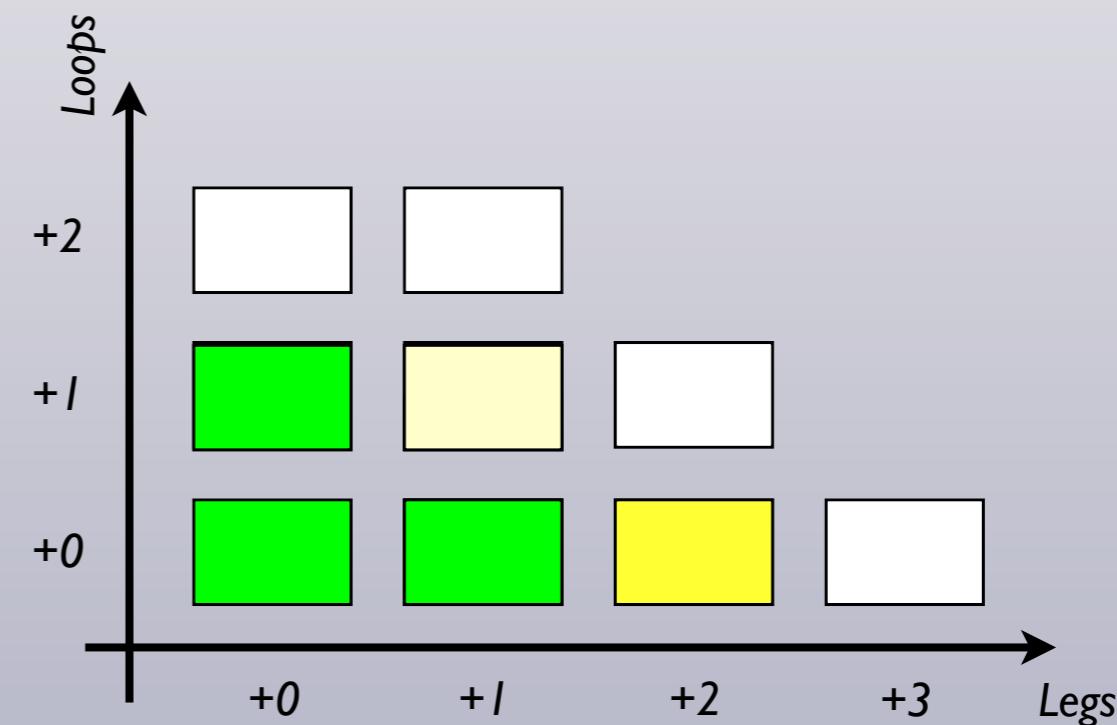
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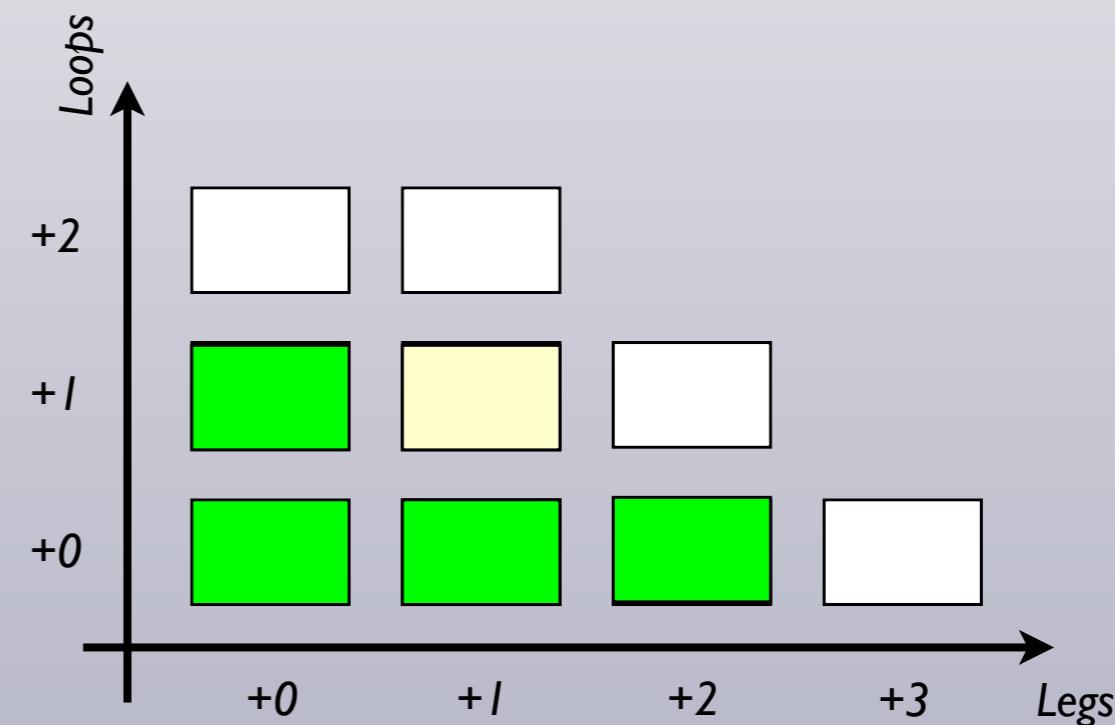
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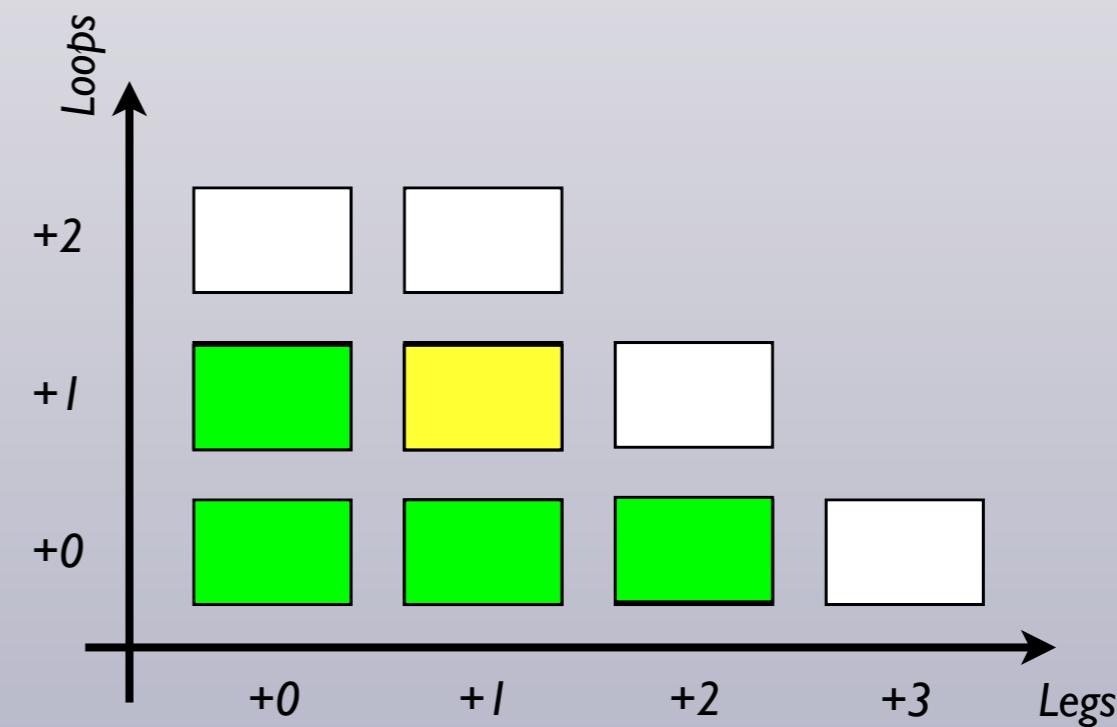
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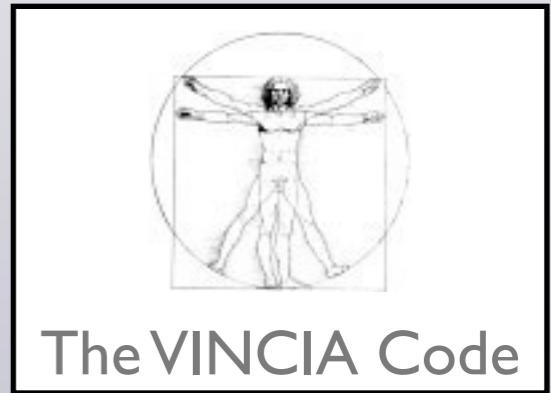
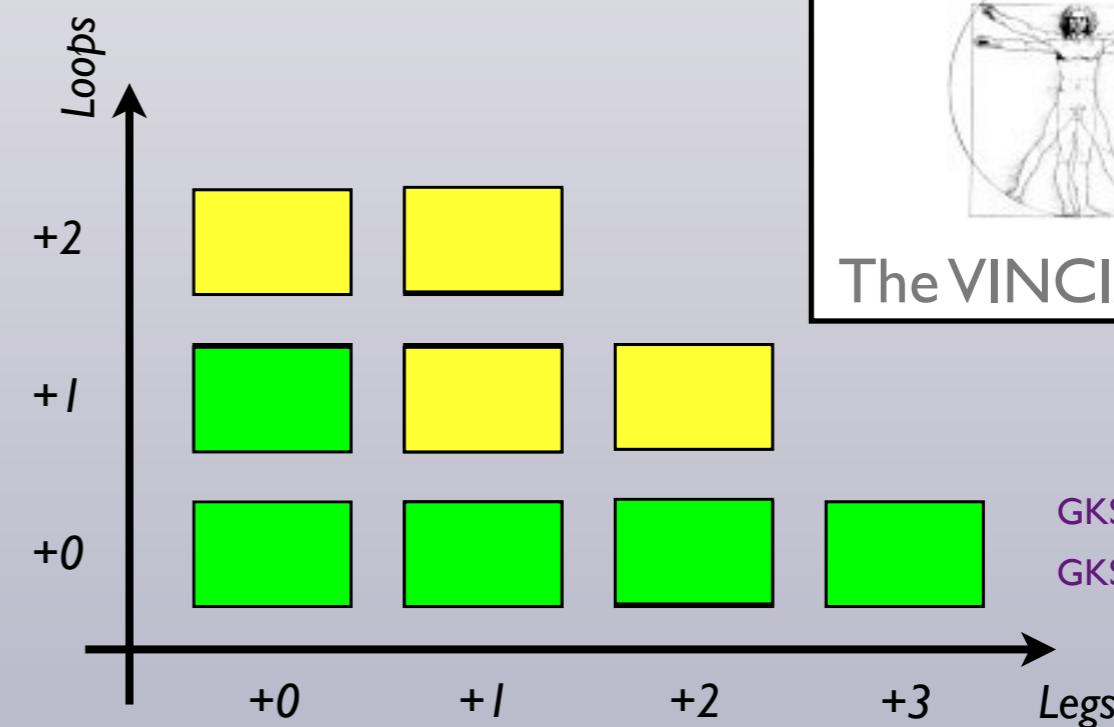
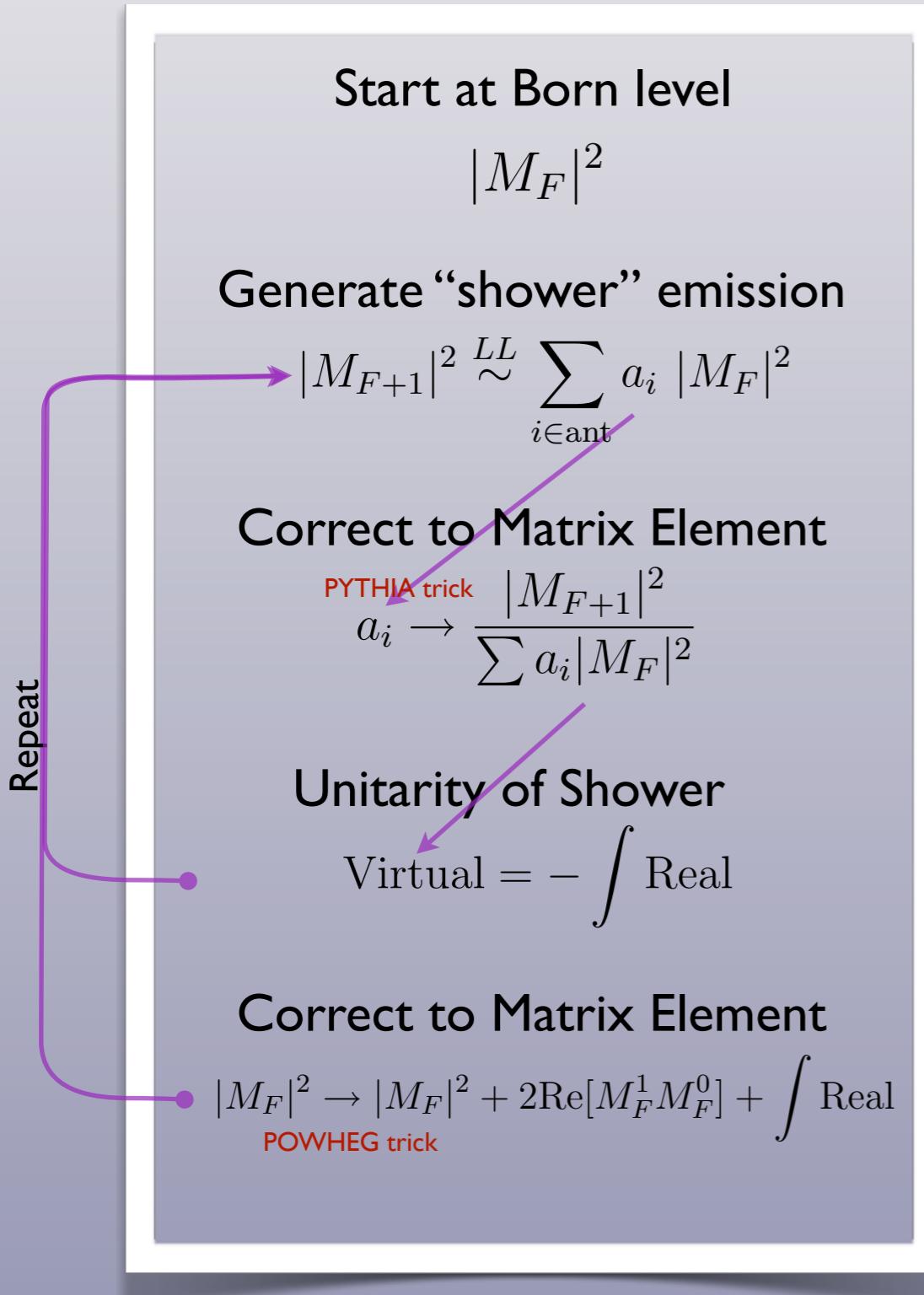
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POWHEG trick

Repeat

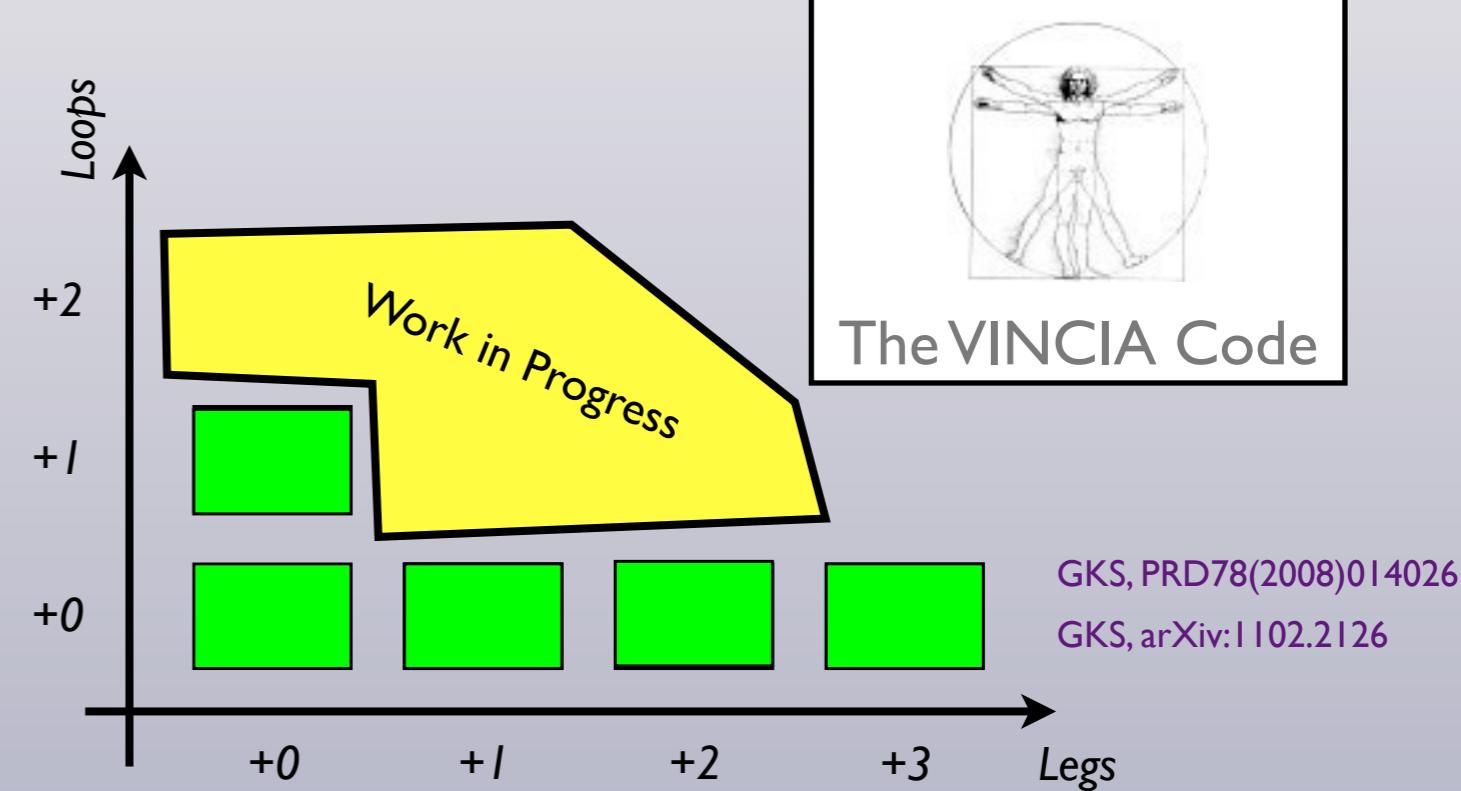
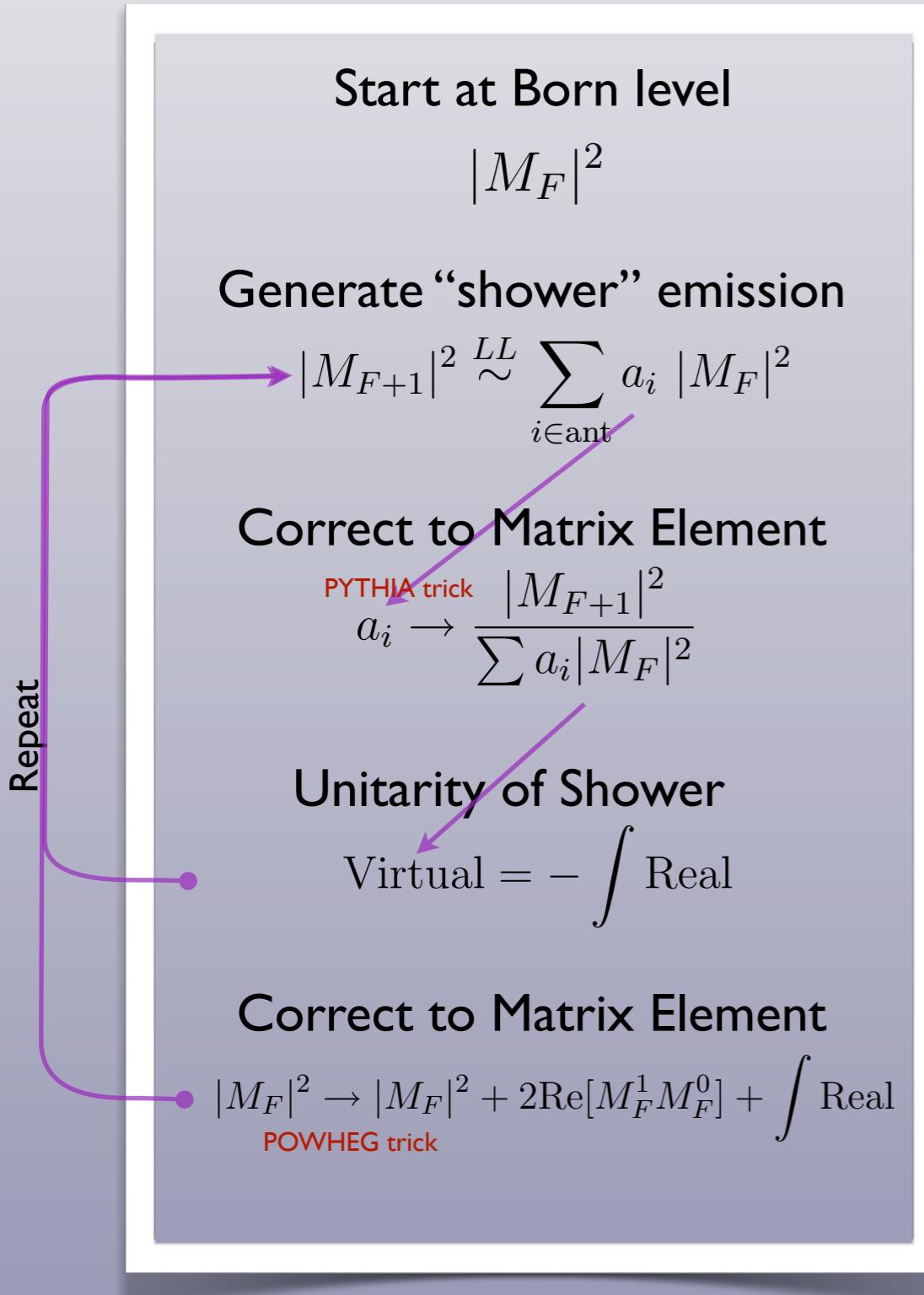


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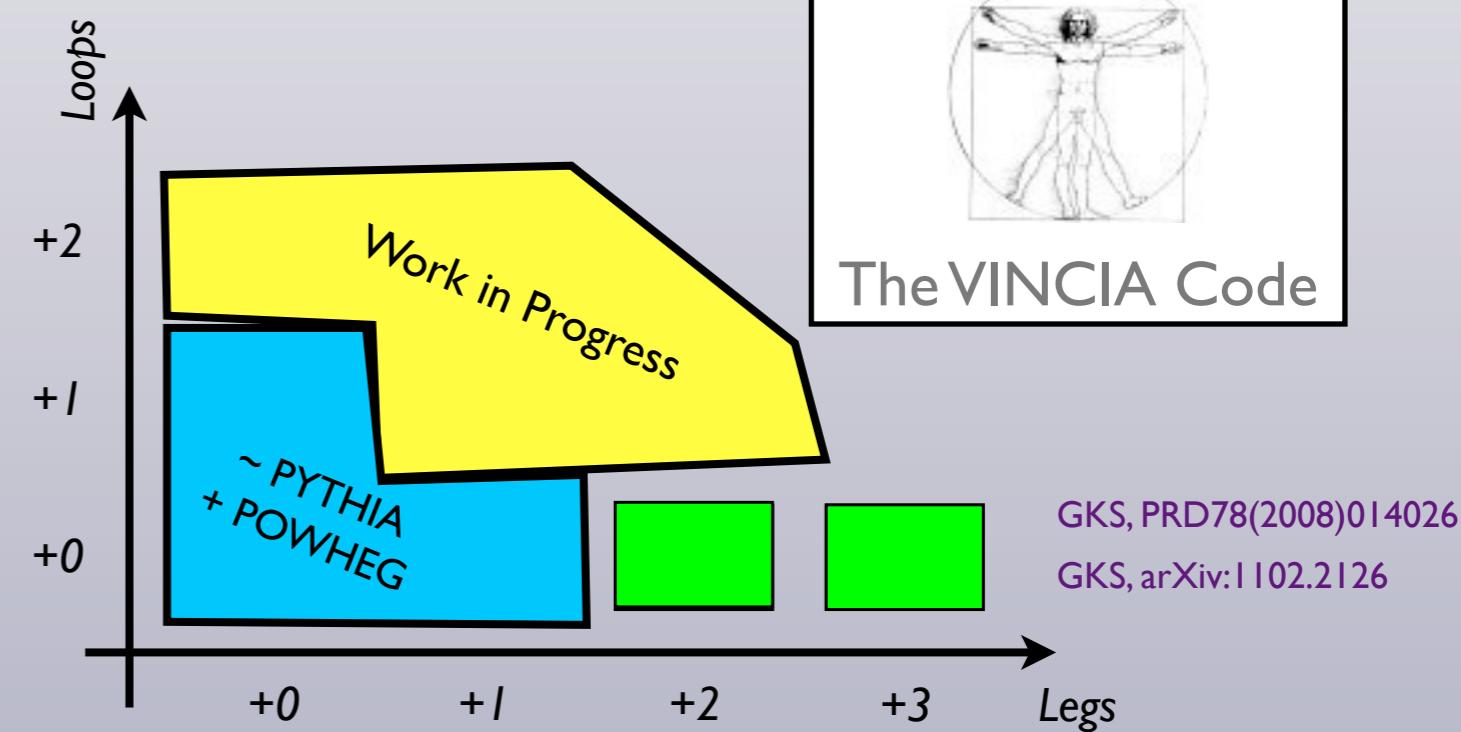
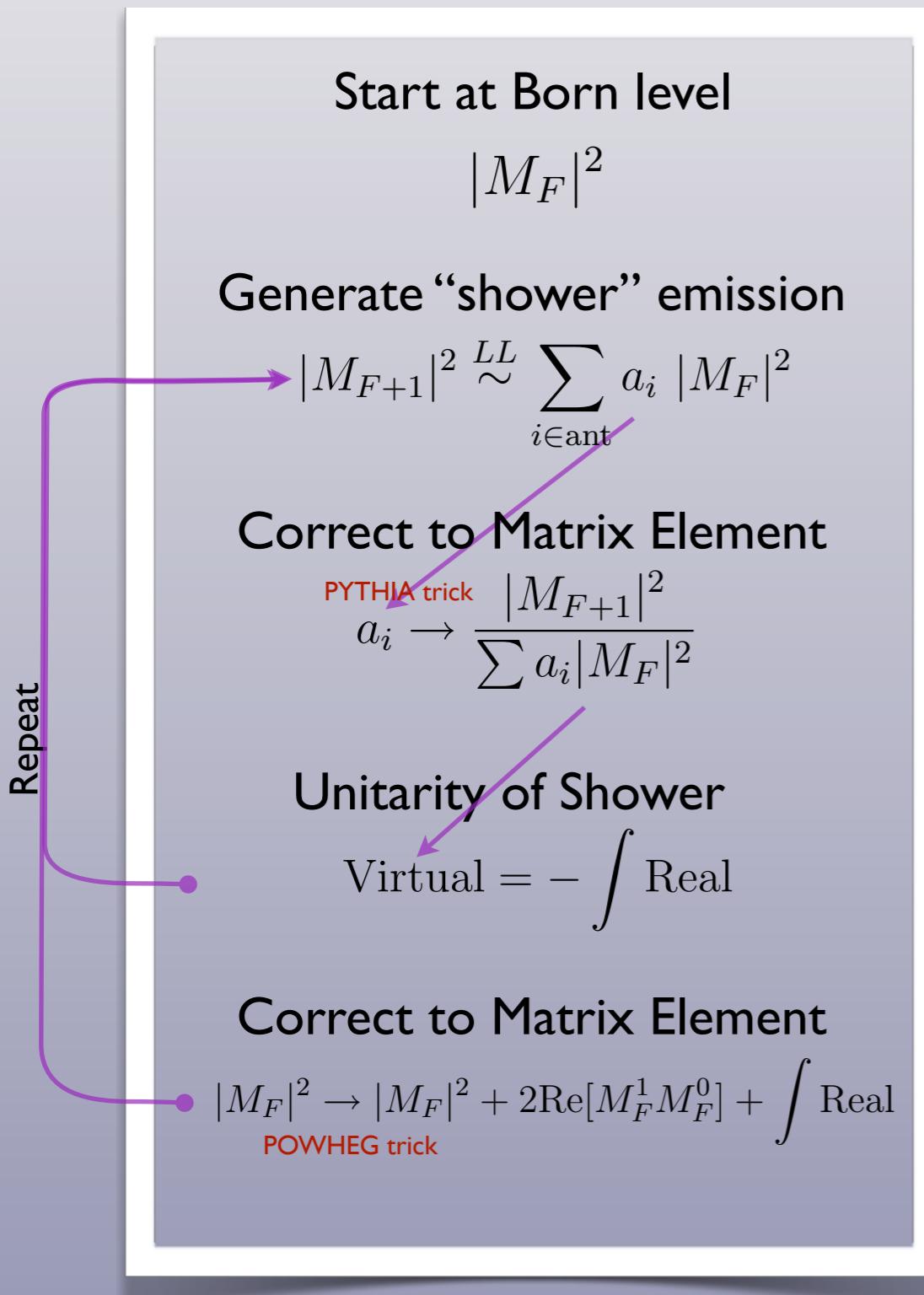


GKS, PRD78(2008)014026  
 GKS, arXiv:1102.2126

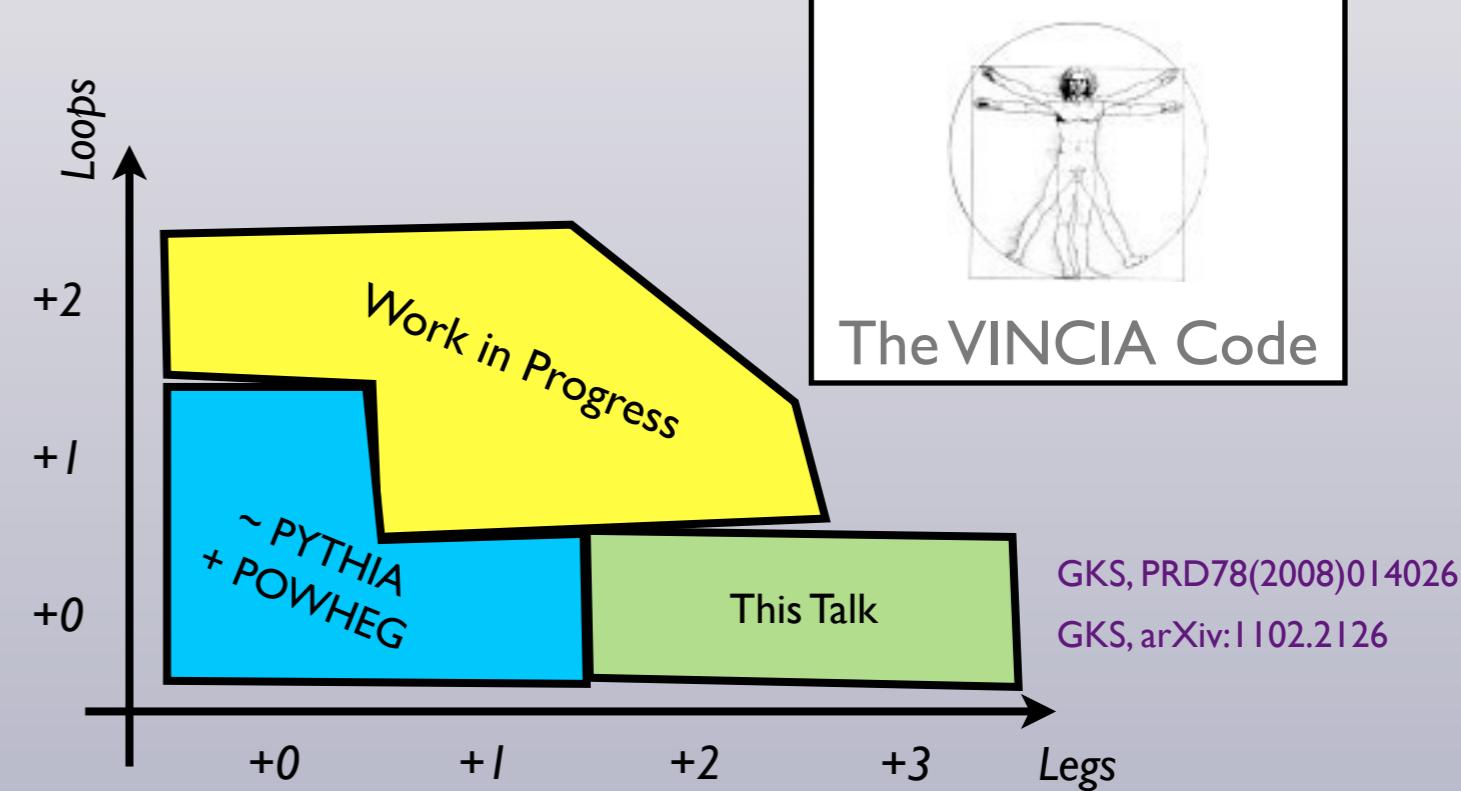
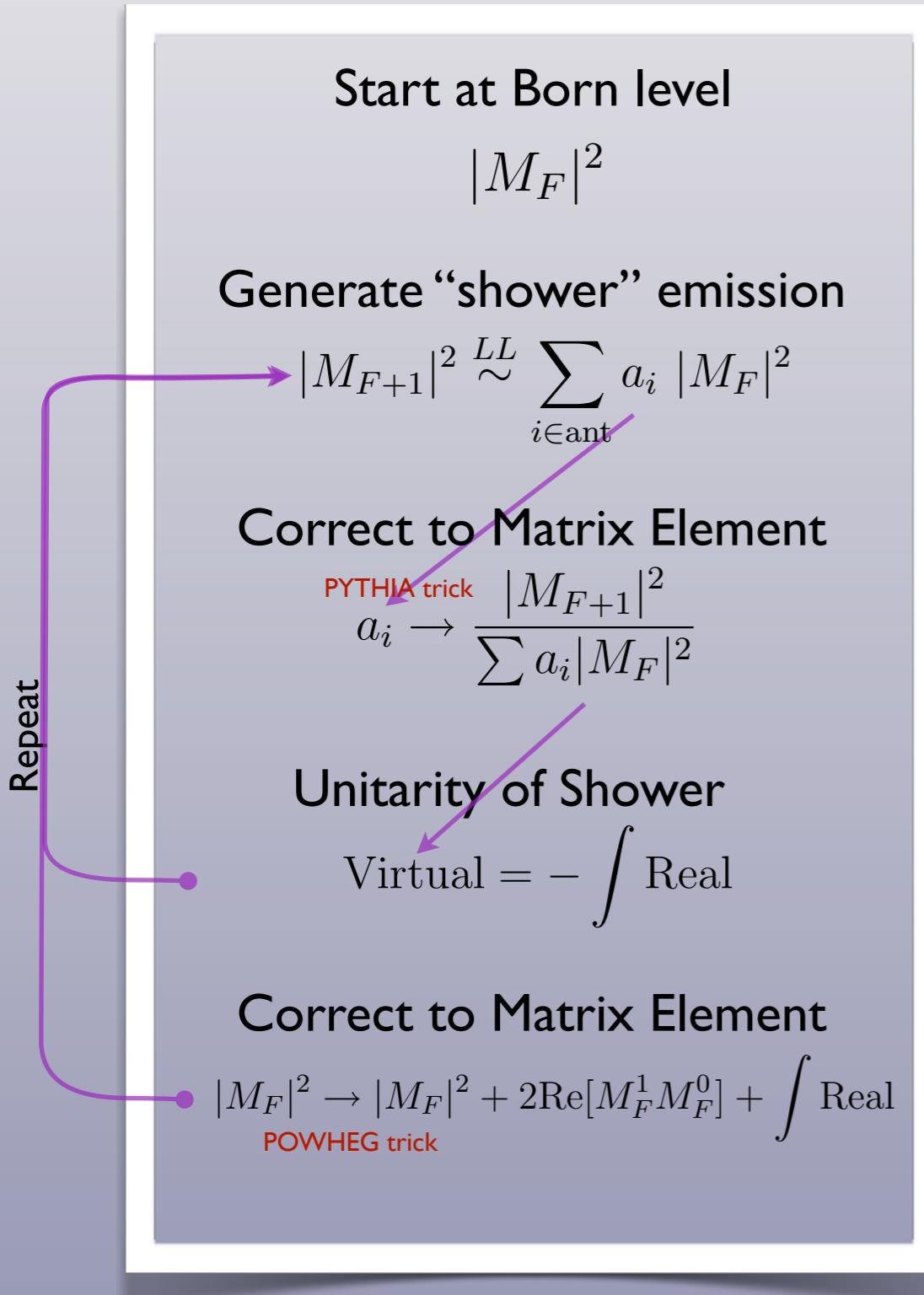
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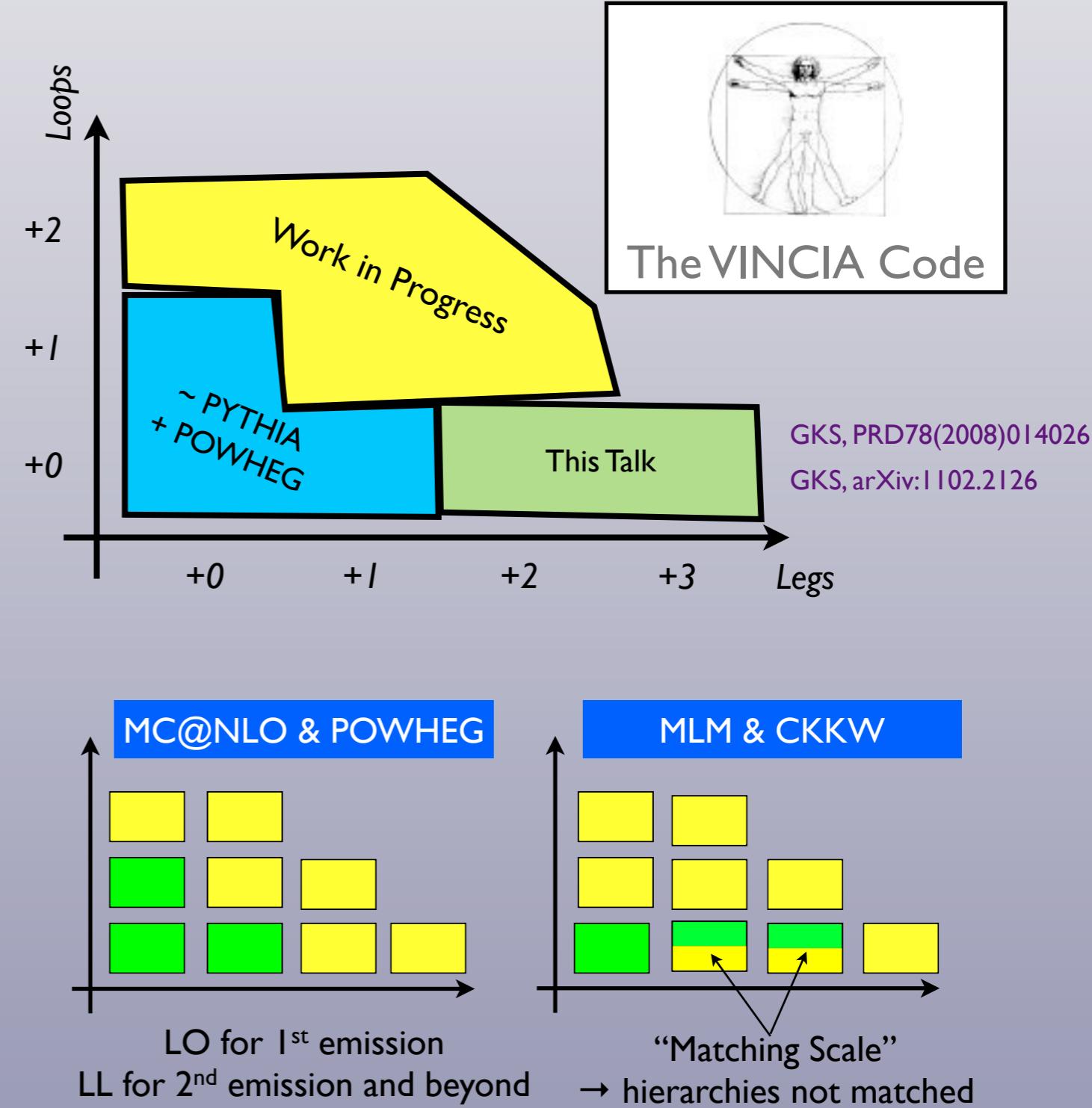
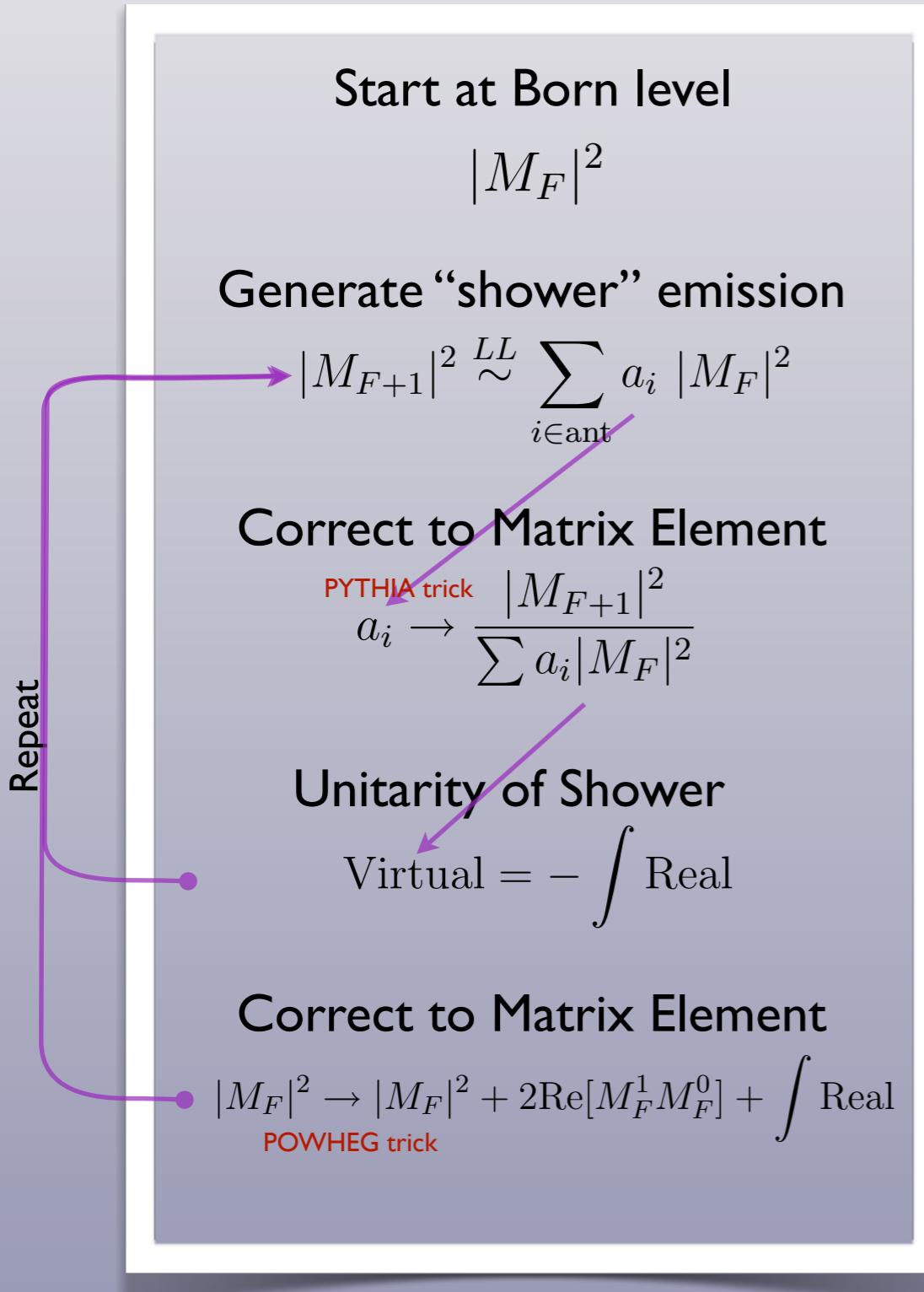
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# The Denominator

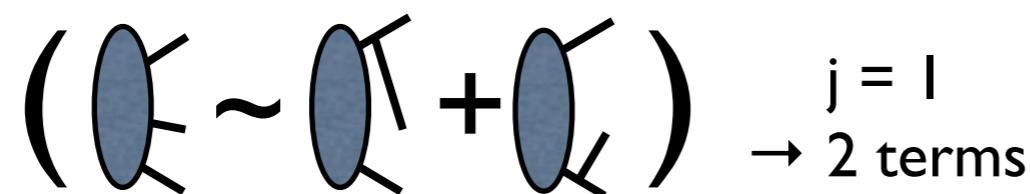
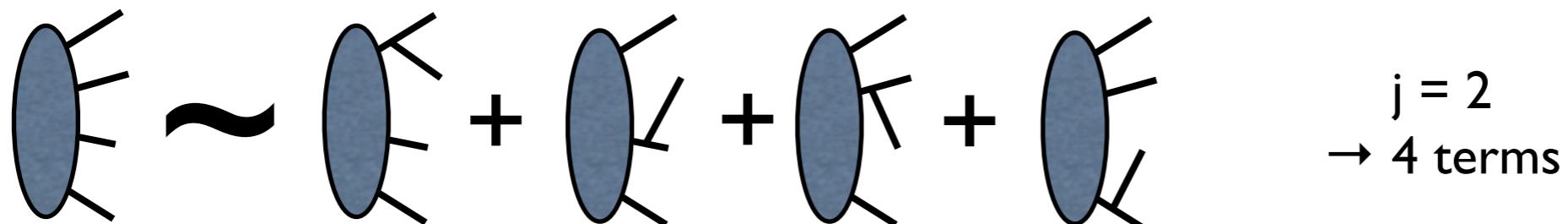
$$a_i \rightarrow \frac{|M_{F+1}|^2}{\sum a_i |M_F|^2}$$

**In a traditional parton shower, you would face the following problem:**

Existing parton showers are *not* really Markov Chains

*Further evolution (restart scale) depends on which branching happened last  
→ proliferation of terms*

Number of histories contributing to  $n^{\text{th}}$  branching  $\propto 2^n n!$



(+ parton showers have complicated and/or frame-dependent phase-space mappings, especially at the multi-parton level)

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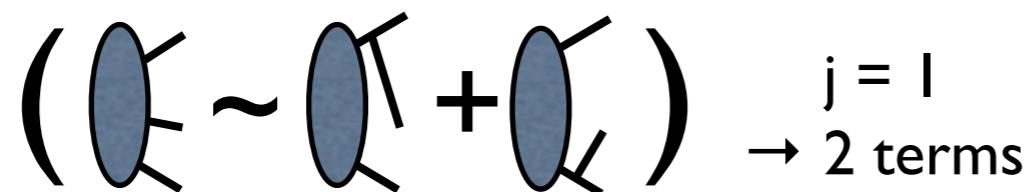
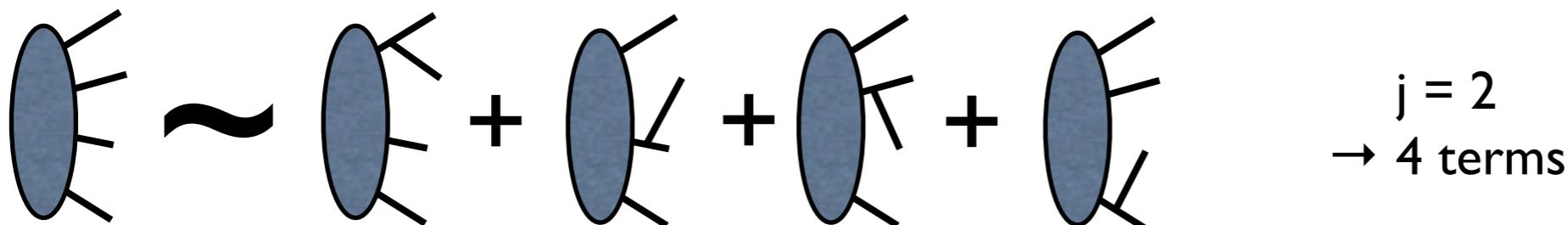
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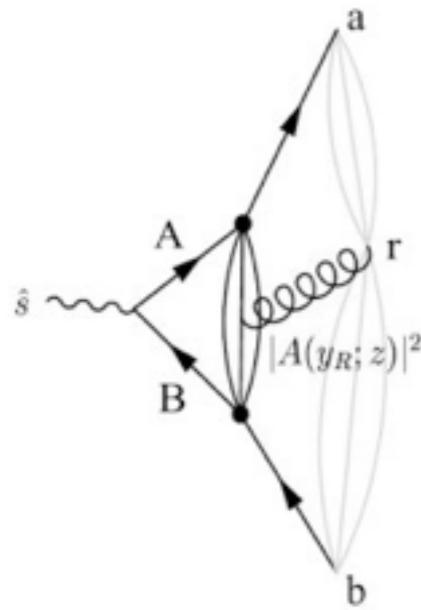
**Parton- (or Catani-Seymour) Shower:**  
After 2 branchings: 8 terms  
After 3 branchings: 48 terms  
After 4 branchings: 384 terms

(+ parton showers have complicated and/or frame-dependent phase-space mappings, especially at the multi-parton level)

# Matched Markovian Antenna Showers

**Antenna showers:** one term per parton pair

$$2^n n! \rightarrow n!$$



(+ generic Lorentz-invariant and on-shell phase-space factorization)

+ Change “shower restart” to Markov criterion:

Given an  $n$ -parton configuration, “ordering” scale is

$$Q_{ord} = \min(Q_{E1}, Q_{E2}, \dots, Q_{En})$$

Unique restart scale, independently of how it was produced

+ Matching:  $n! \rightarrow n$

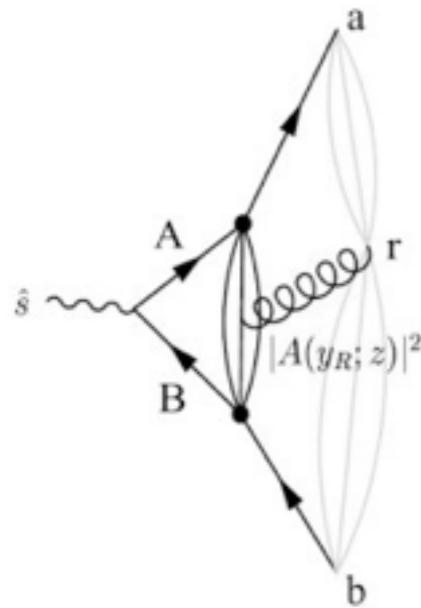
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Matched Markovian Antenna Shower:

After 2 branchings: 2 terms

After 3 branchings: 3 terms

After 4 branchings: 4 terms

+ J. Lopez-Villarejo  $\rightarrow$  1 term at any order

Parton- (or Catani-Seymour) Shower:

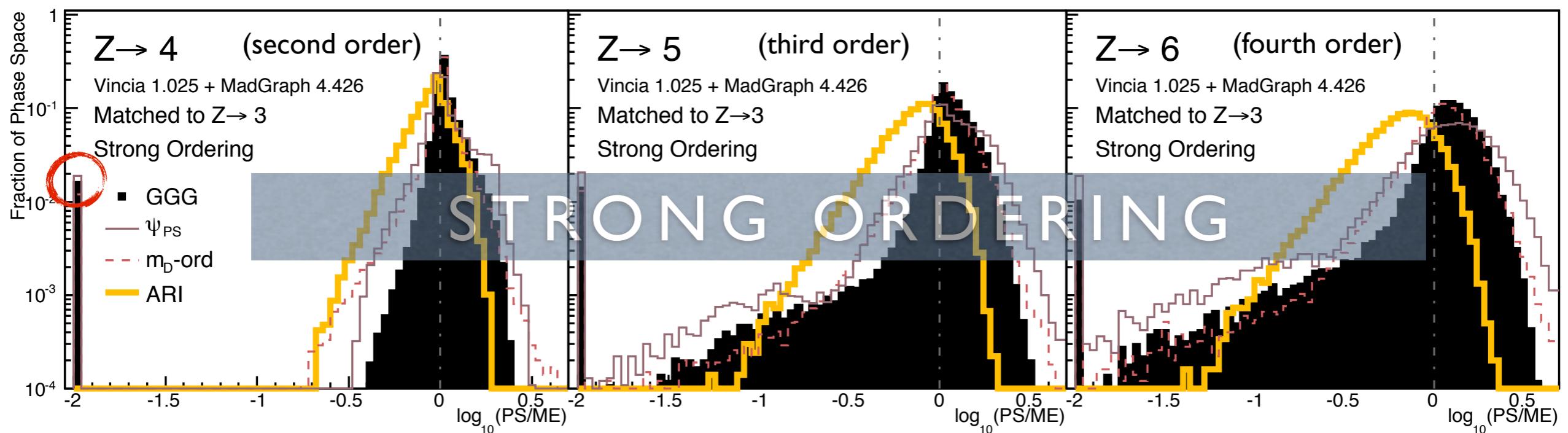
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# Approximations

Distribution of  $\log_{10}(\text{PS}_{\text{LO}}/\text{ME}_{\text{LO}})$  (inverse  $\sim$  matching coefficient)

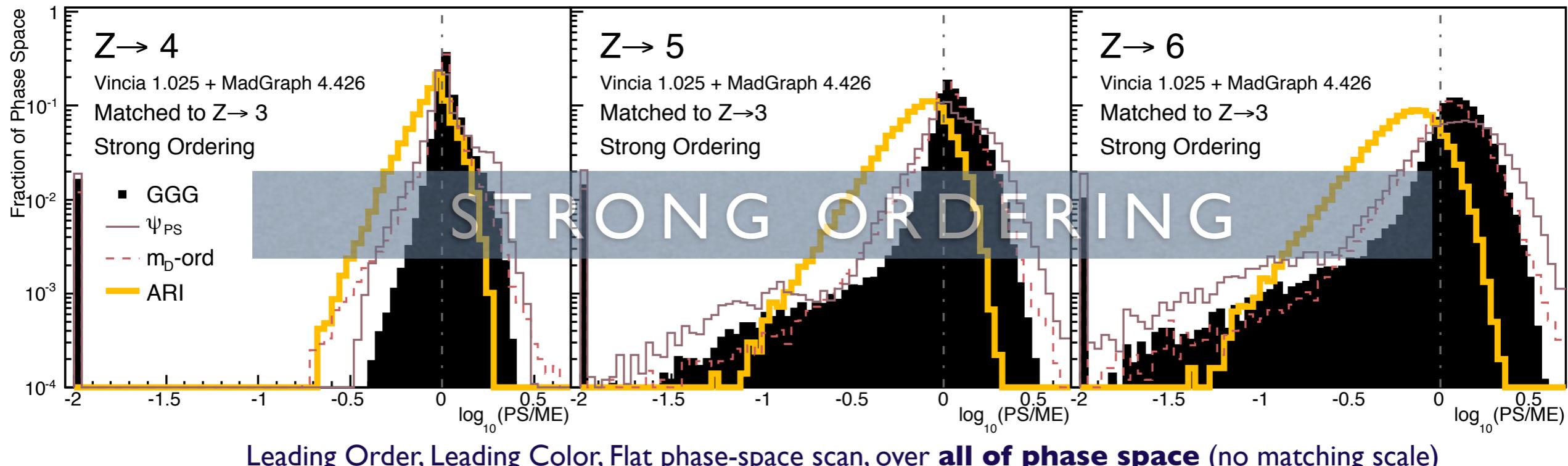


Dead Zone: 1-2% of phase space have no strongly ordered paths leading there\*

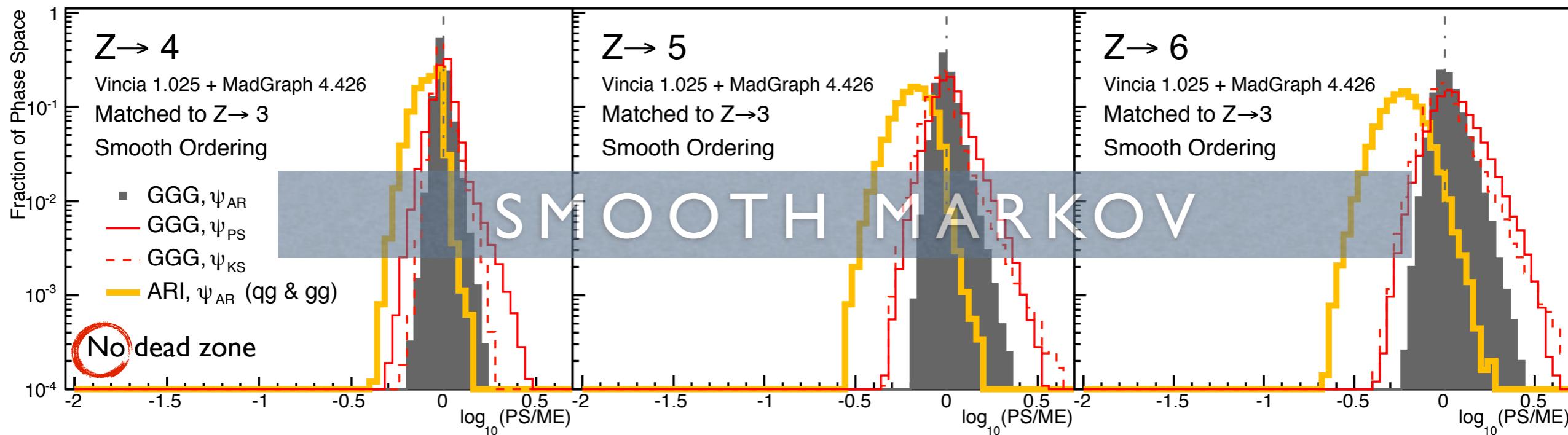
\*fine from strict LL point of view: those points correspond to “unordered” non-log-enhanced configurations

# → Better Approximations

Distribution of  $\text{Log}_{10}(\text{PS}_{\text{LO}}/\text{ME}_{\text{LO}})$  (inverse ~ matching coefficient)



Leading Order, Leading Color, Flat phase-space scan, over **all of phase space** (no matching scale)



GEEKS (Giele, Kosower, Skands): arXiv:1102.2126

## Generate Trials **without** imposing strong ordering

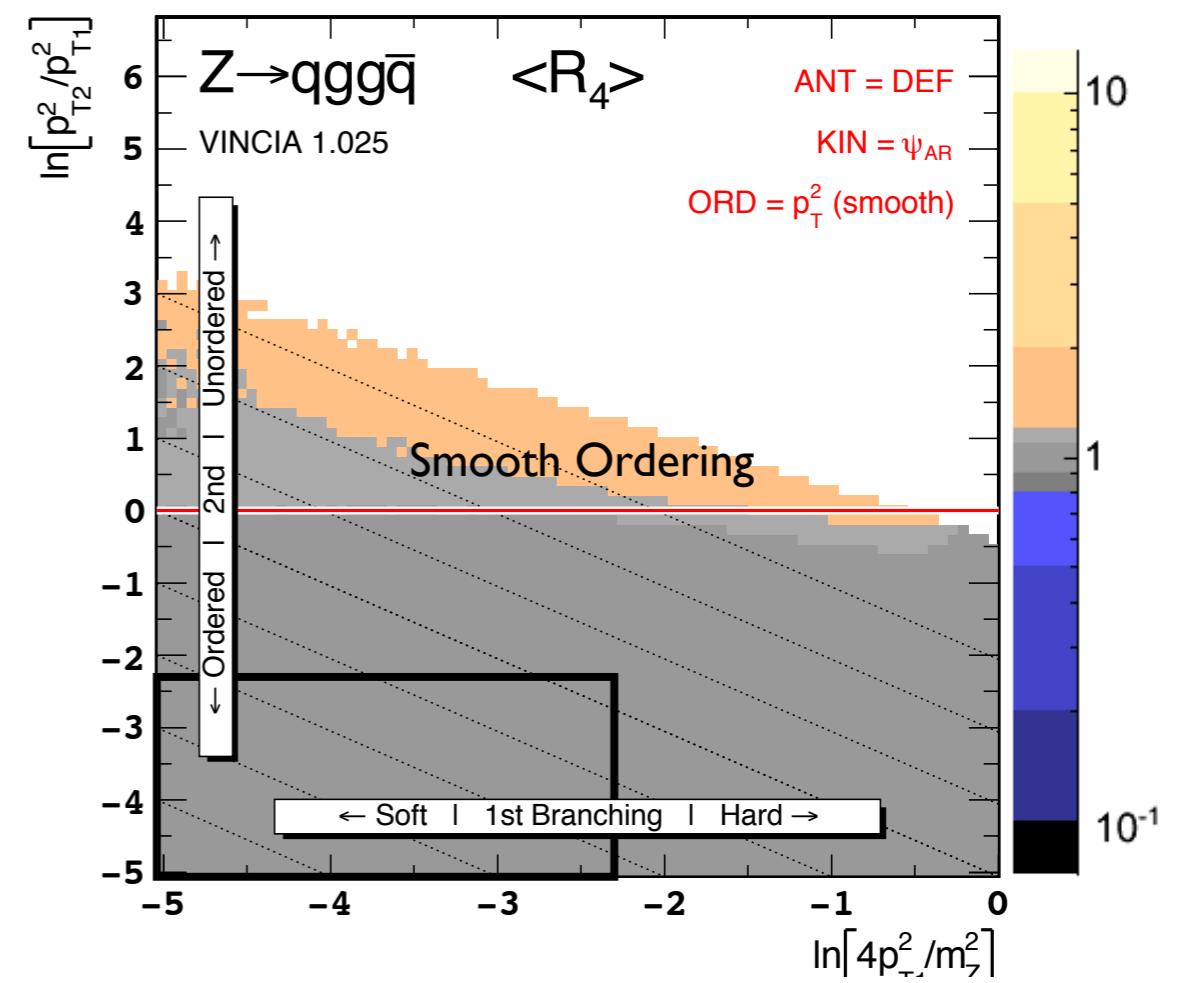
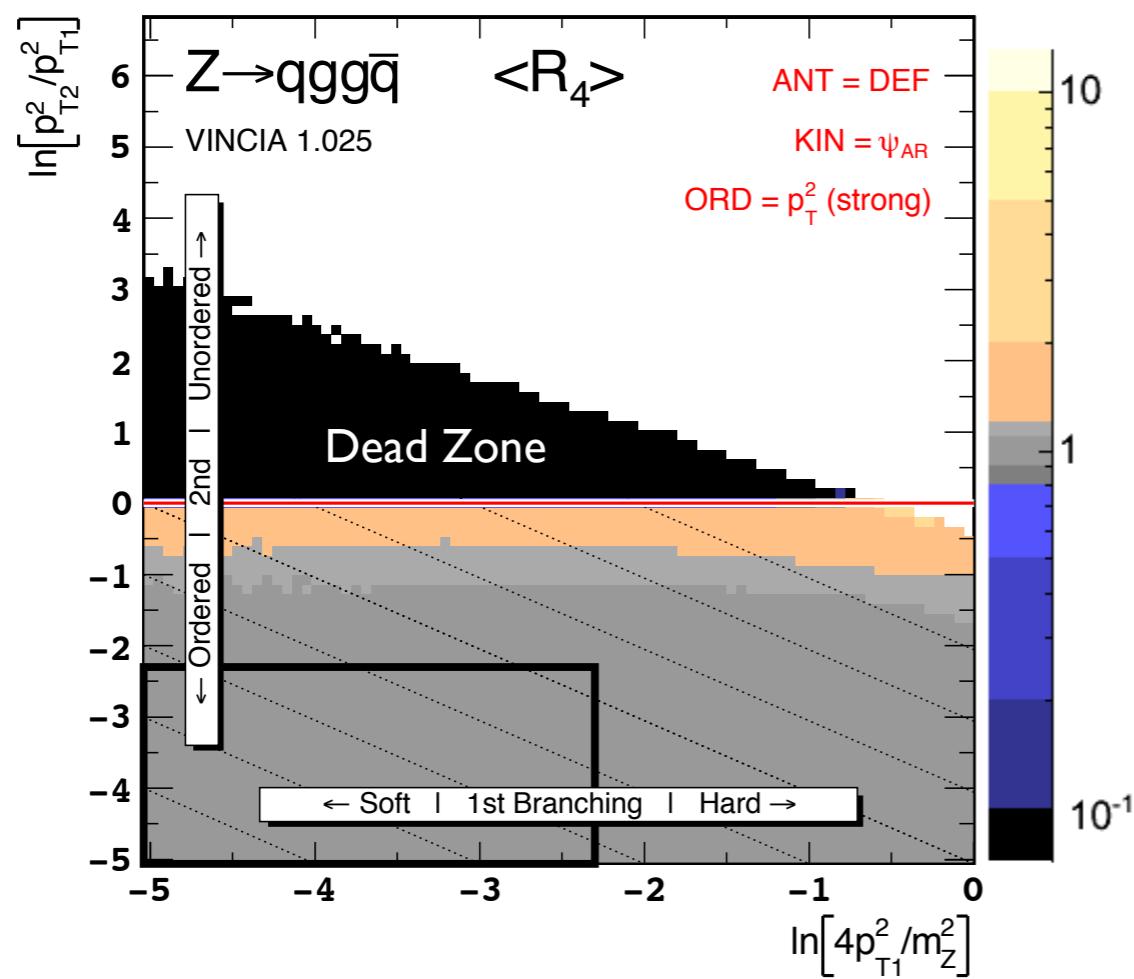
At each step, each dipole allowed to fill its entire phase space

*Overcounting removed by matching*

+ smooth ordering beyond matched multiplicities

$$\frac{\hat{p}_\perp^2}{\hat{p}_\perp^2 + p_\perp^2} P_{LL} \quad \frac{\hat{p}_\perp^2}{p_\perp^2} \text{ last branching}$$

$$\frac{\hat{p}_\perp^2}{p_\perp^2} \text{ current branching}$$



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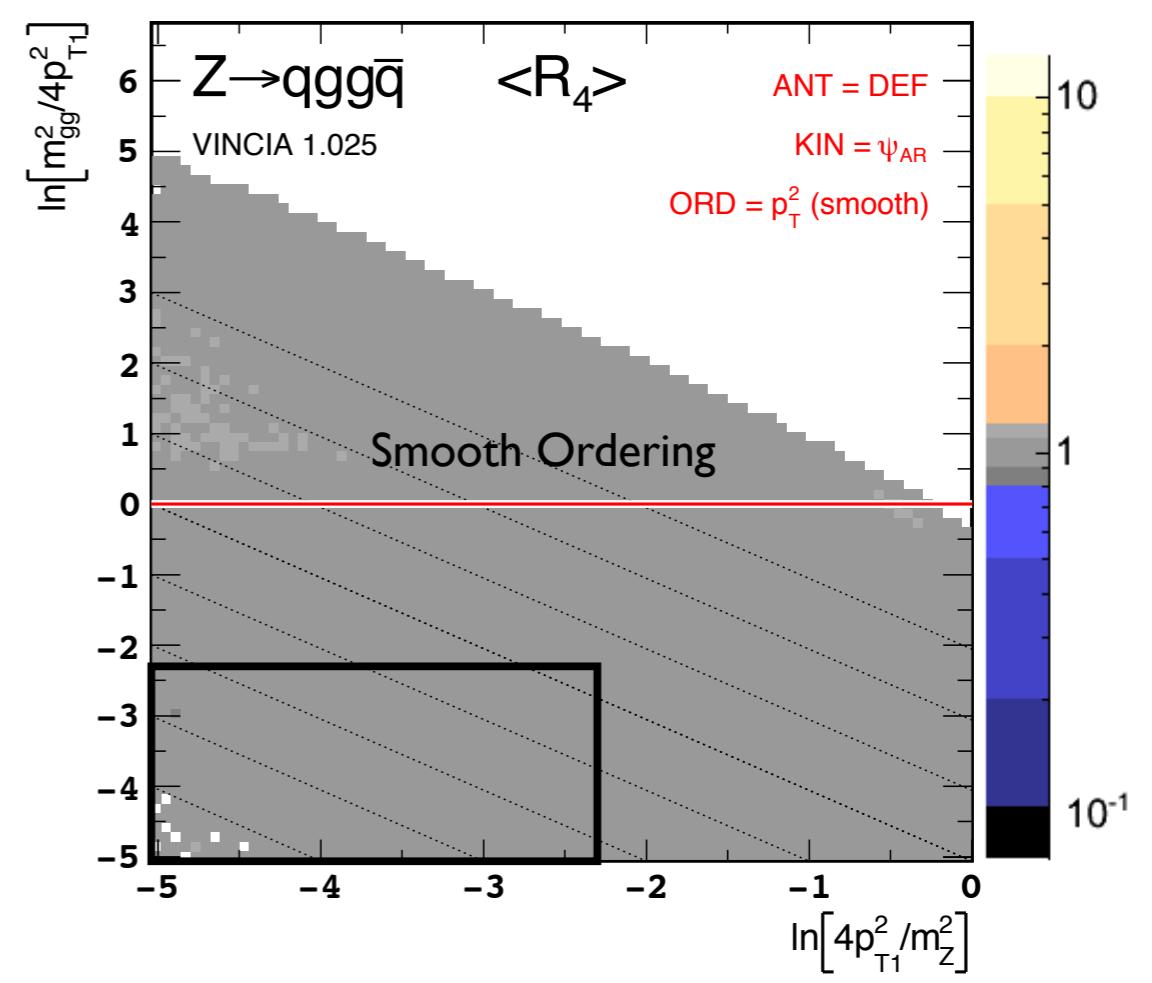
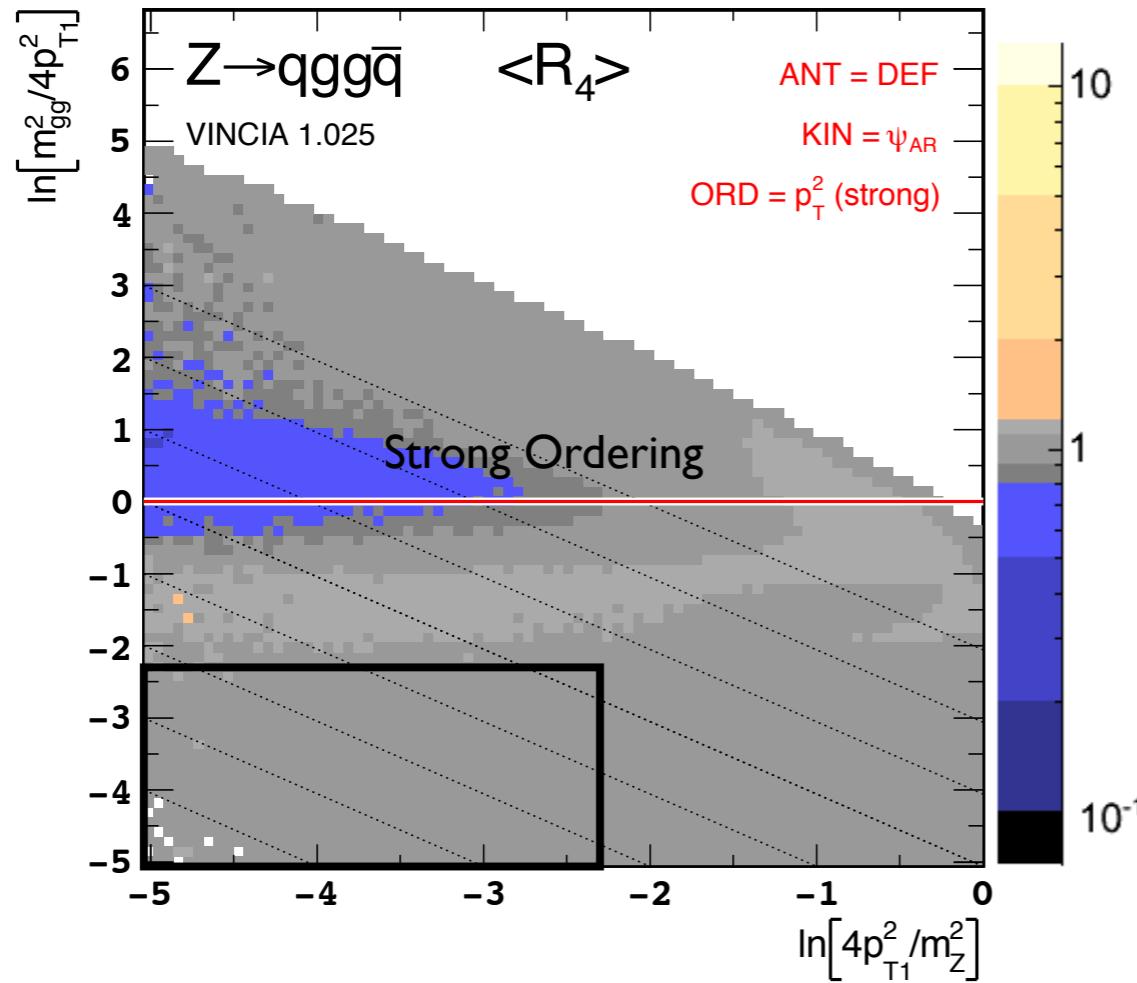
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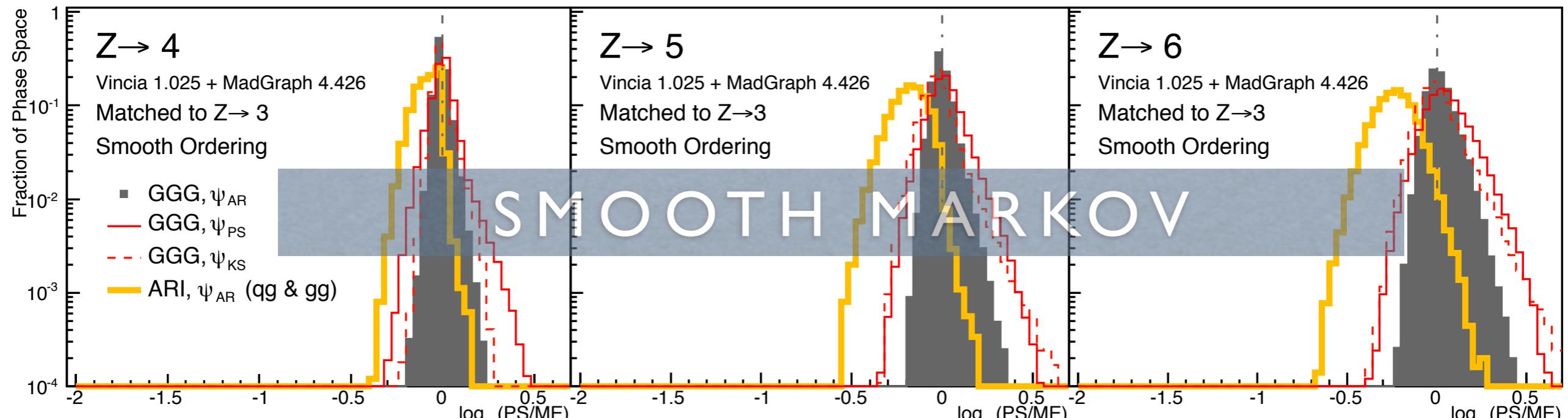
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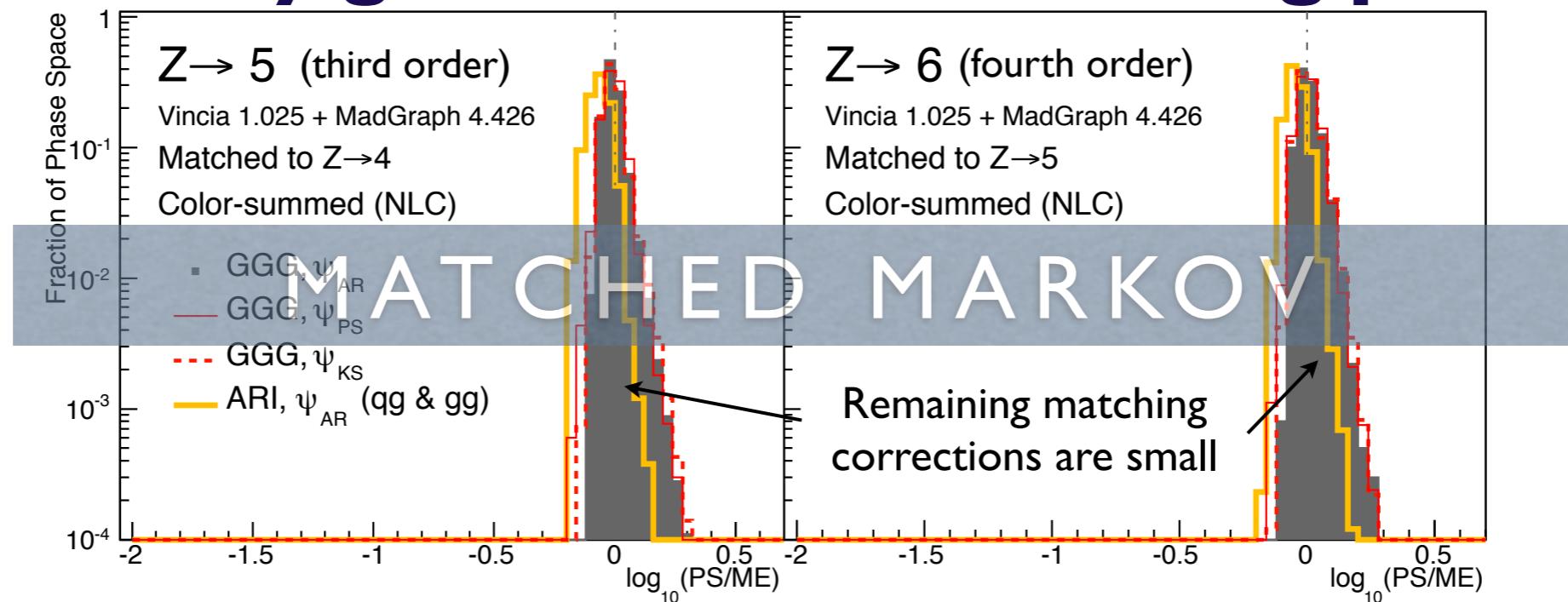
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# + Matching (+ full colour)



→ A very good all-orders starting point



A photograph of a desert landscape at sunset. A two-lane road with yellow center and edge lines curves from the bottom left towards the horizon. The sky is filled with dramatic, dark clouds, with bright orange and yellow sunlight filtering through and reflecting off the horizon. In the foreground, there's dry, light-colored brush and a small, dark tree on a hill to the right.

# Uncertainties

# Uncertainty Variations

**A result is only as good as its uncertainty**

Normal procedure:

*Run MC  $2N+1$  times (for central +  $N$  up/down variations)*

Takes  $2N+1$  times as long

+ uncorrelated statistical fluctuations

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**Automate and do everything in one run**

VINCIA: all events have weight = 1

Compute *unitary* alternative weights on the fly

→ sets of alternative weights representing variations (all with  $\langle w \rangle = 1$ )

Same events, so only have to be hadronized/detector-simulated ONCE!

MC with Automatic Uncertainty Bands

# Uncertainties

**For each branching,  
recompute weight for:**

- Different renormalization scales
- Different antenna functions
- Different ordering criteria
- Different subleading-color treatments

	Weight
Nominal	1
Variation	$P_2 = \frac{\alpha_s 2 a_2}{\alpha_s 1 a_1} P_1$

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**+ Unitarity**

For each *failed* branching:

$$P_{2;\text{no}} = 1 - P_2 = 1 - \frac{\alpha_s 2 a_2}{\alpha_s 1 a_1} P_1$$

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**+ Matching**

Differences explicitly matched out

(Up to matched orders)

(Can in principle also include variations of matching scheme...)

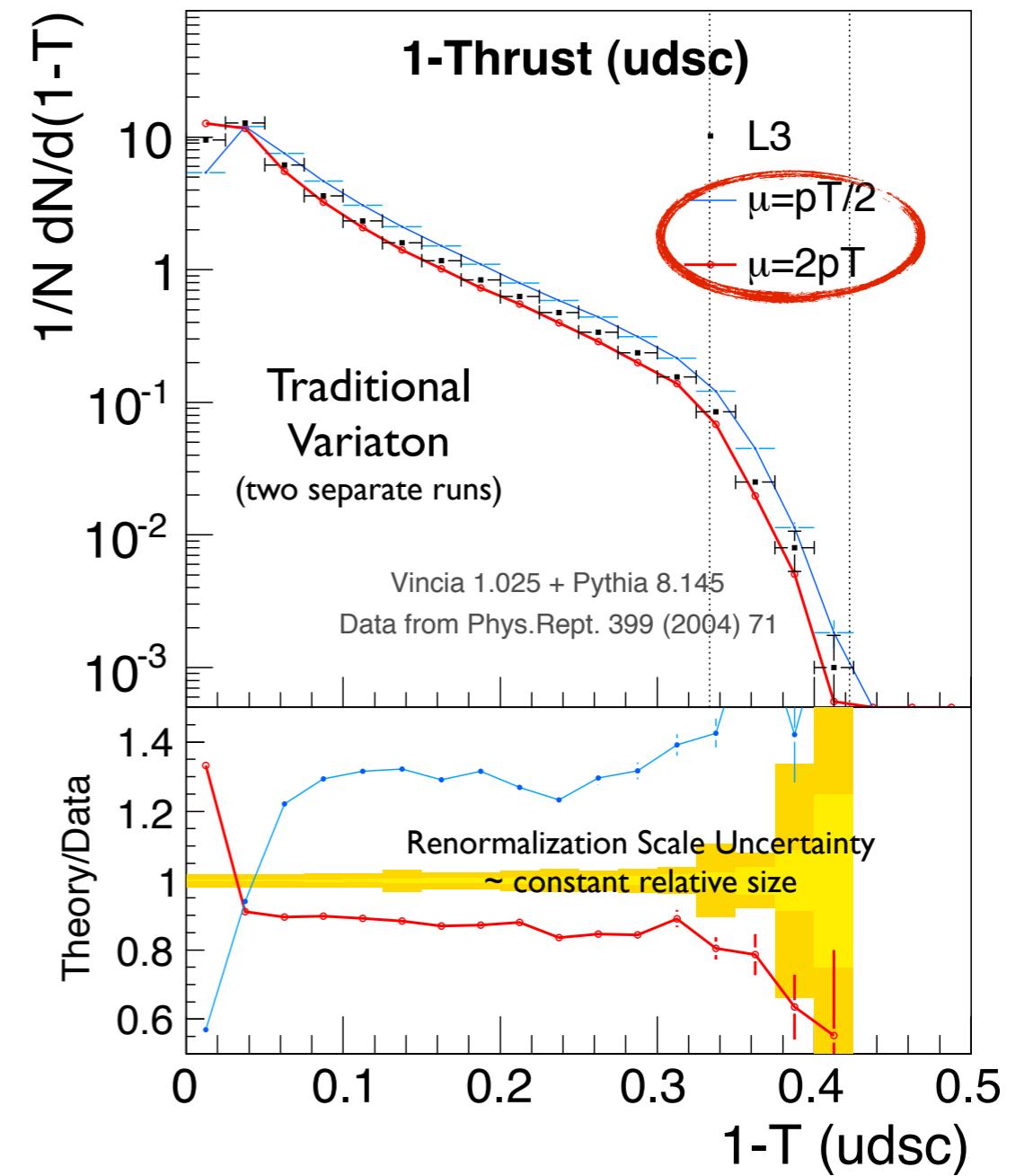
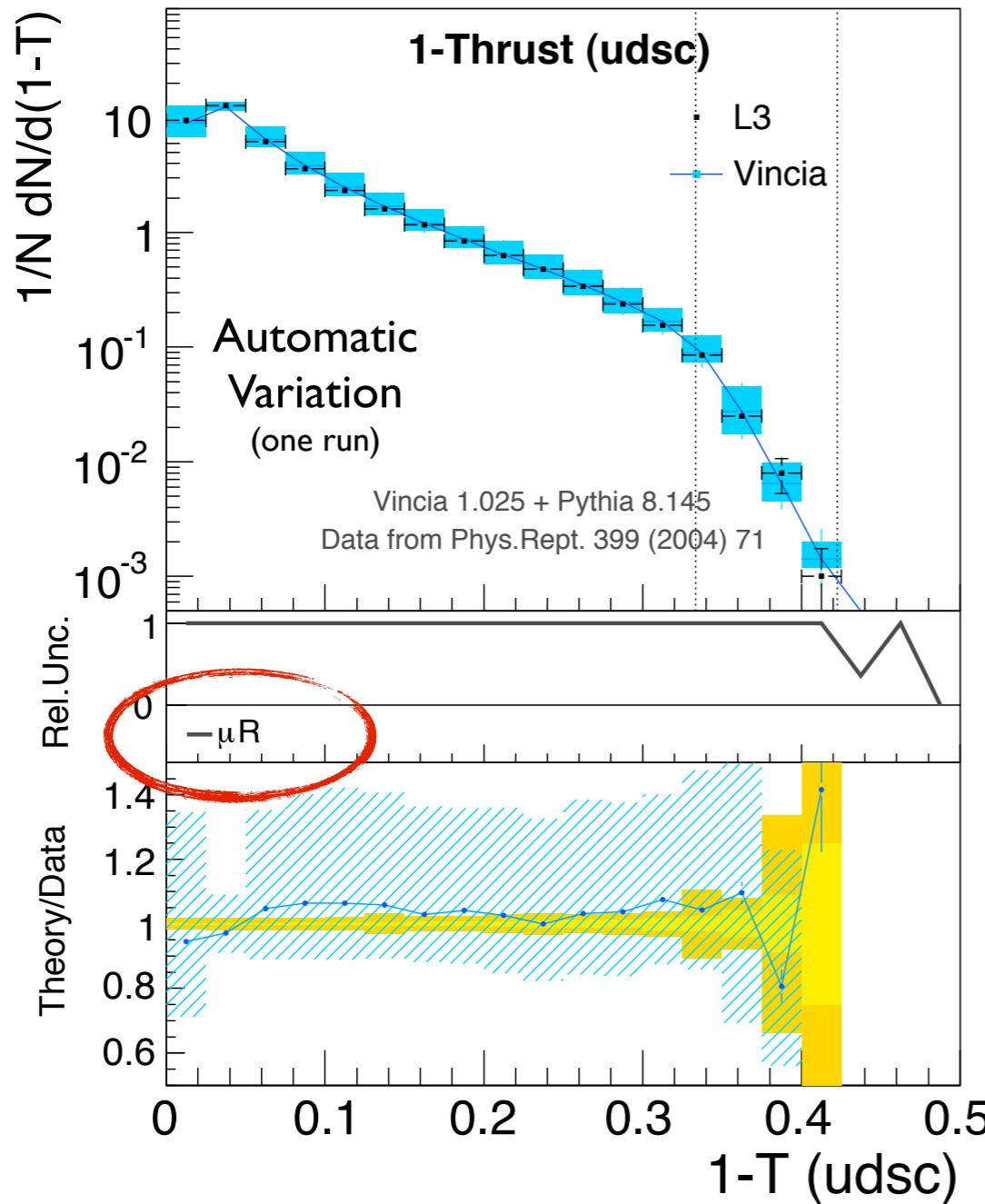
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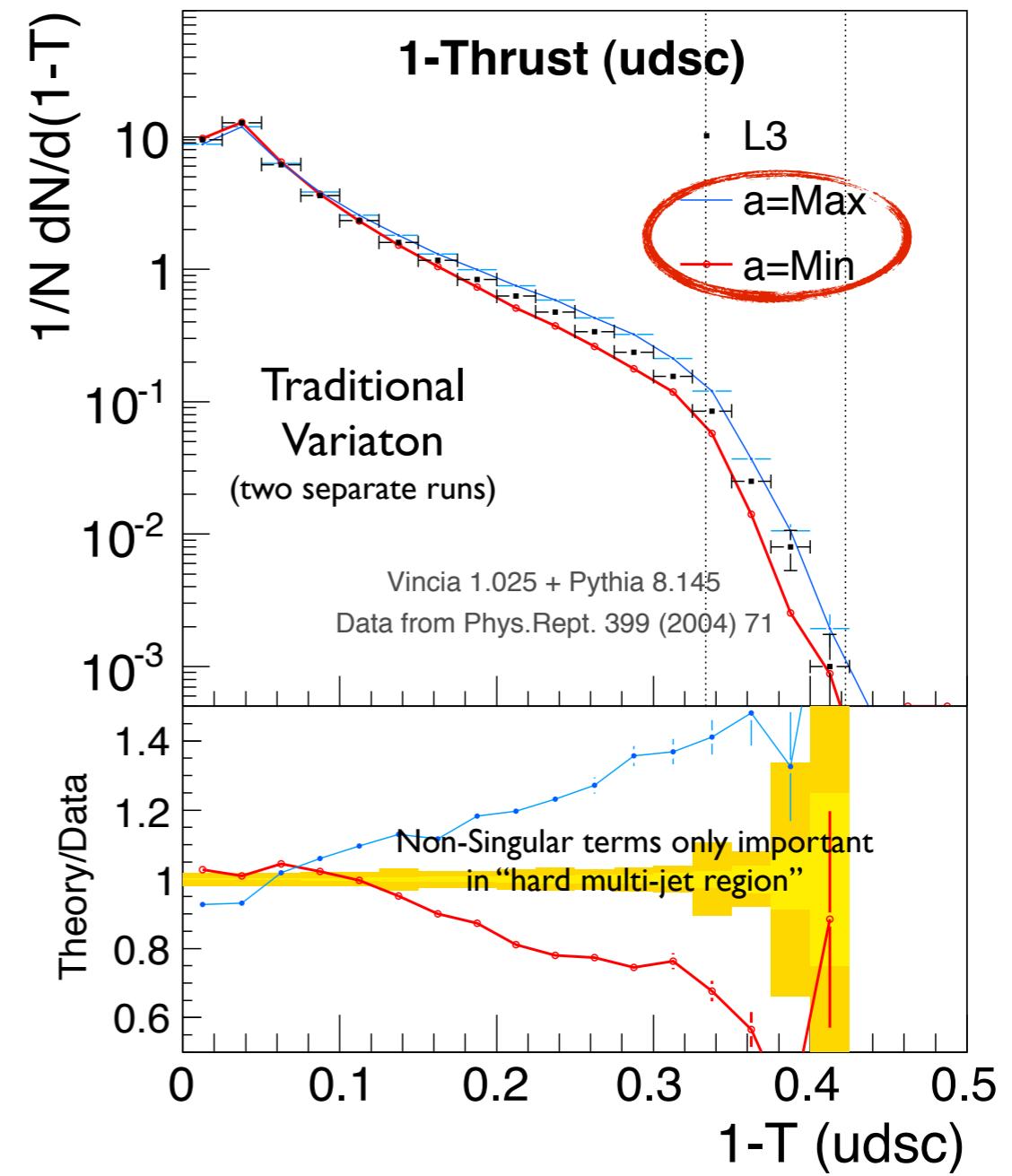
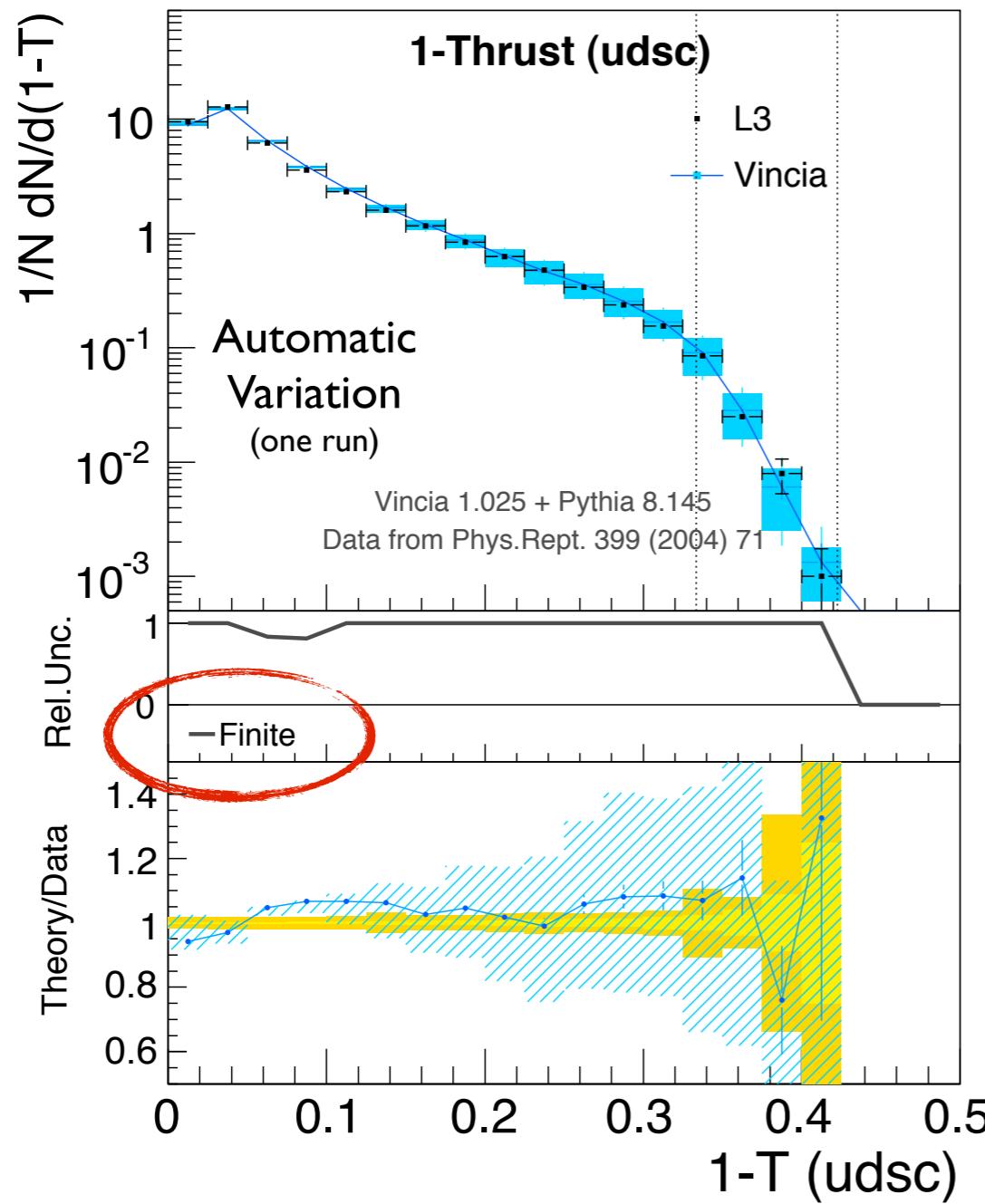
Vincia:uncertaintyBands = on



Variation of renormalization scale (no matching)

# Automatic Uncertainties

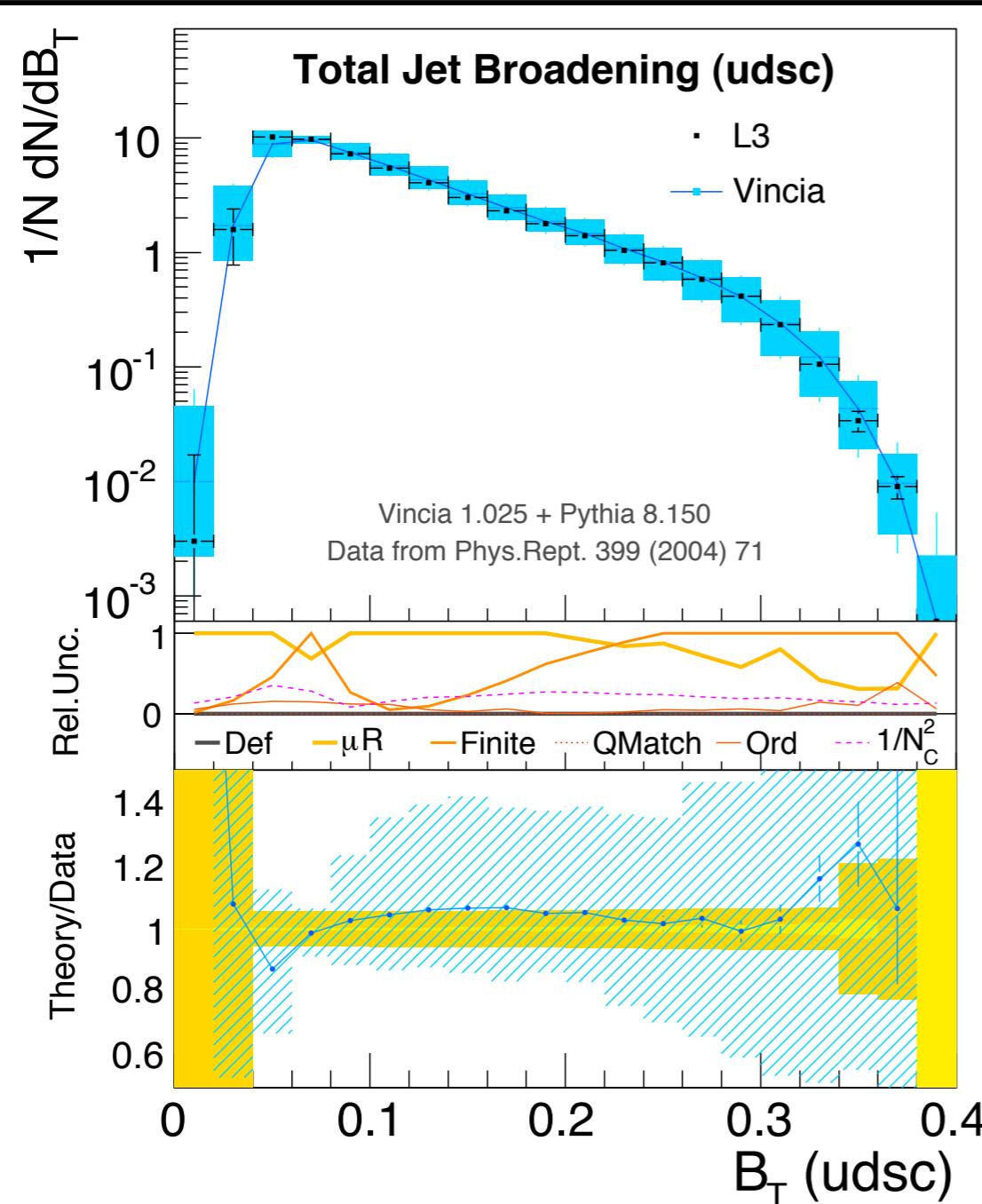
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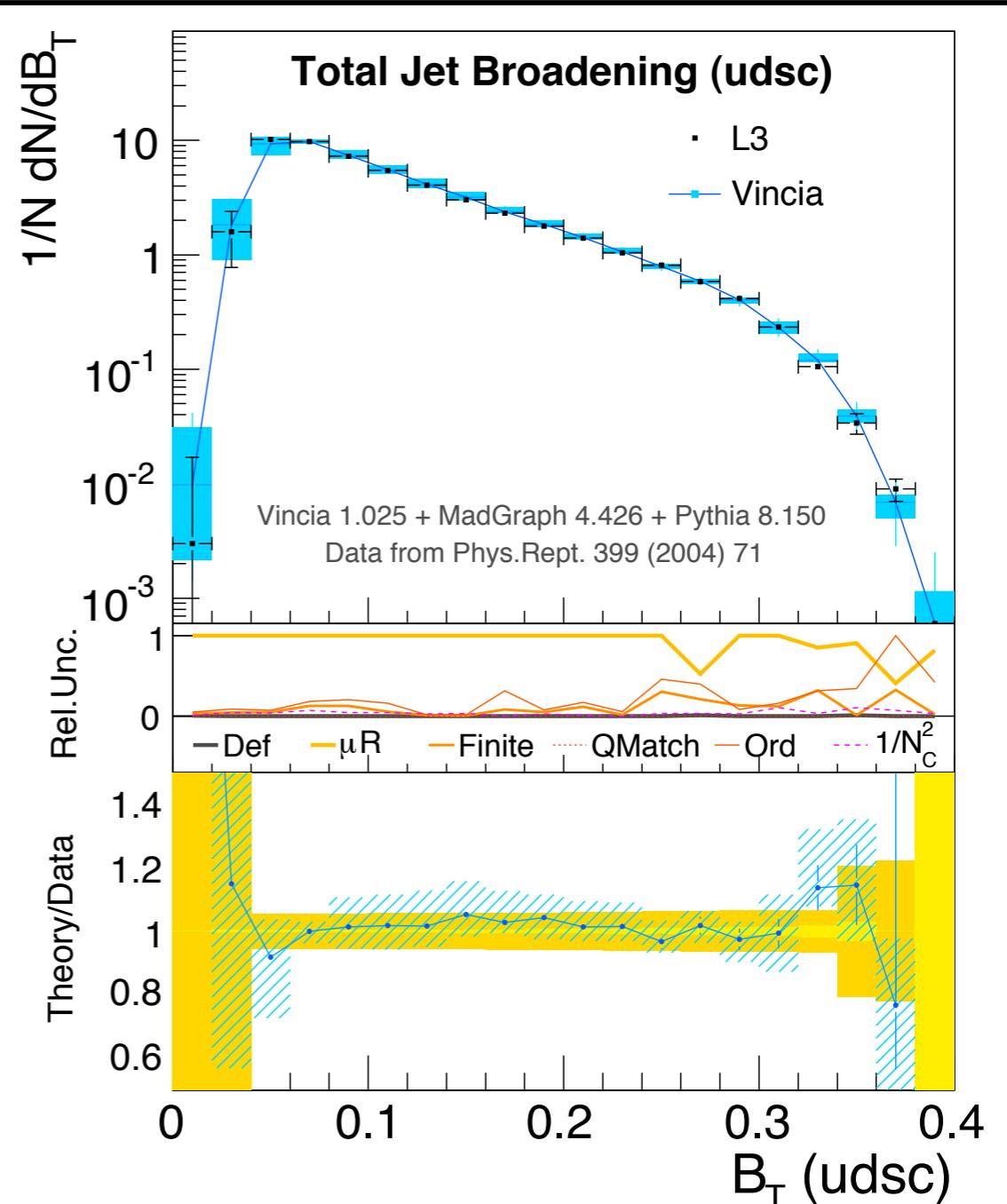
Variation of “finite terms” (no matching)

# Putting it Together

VinciaMatching:order = 0



VinciaMatching:order = 3



## VINCIA STATUS

PLUG-IN TO PYTHIA 8

STABLE AND RELIABLE FOR FINAL-  
STATE JETS (E.G., LEP)

AUTOMATIC MATCHING AND  
UNCERTAINTY BANDS

IMPROVEMENTS IN SHOWER  
(SMOOTH ORDERING, NLC, MATCHING, ...)

PAPER ON MASS EFFECTS ~ READY  
(WITH A. GEHRMANN-DE-RIDDER & M. RITZMANN)

## NEXT STEPS

MULTI-LEG ONE-LOOP MATCHING  
(WITH L. HARTGRING & E. LAENEN, NIKHEF)

“SECTOR SHOWERS”  
(WITH J. LOPEZ-VILLAREJO, CERN)

POLARIZED SHOWERS  
(WITH A. LARKOSKI, SLAC, & J. LOPEZ-VILLAREJO, CERN)

→ INITIAL-STATE SHOWERS  
(WITH W. GIELE, D. KOSOWER, G. DIANA, ...)

THE  
**VINCIA**  
CODE

[HTTP://PROJECTS.HEPFORGE.ORG/VINCIA](http://PROJECTS.HEPFORGE.ORG/VINCIA)

## VINCIA STATUS



## NEXT STEPS

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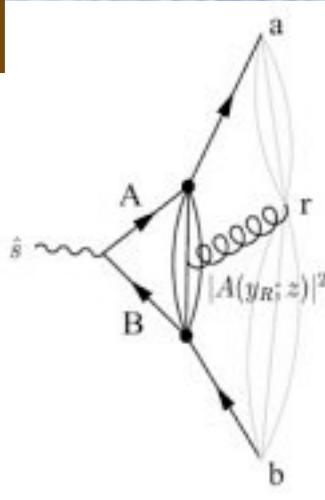
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# SECTOR SHOWERS

Kosower, D. A. Phys.Rev. D57 (1998) 5410-5416 ; Gehrmann-De Ridder, A. et al. JHEP 0509 (2005) 056 ; G. Gustafson Phys.Lett. B175 (1986) 453

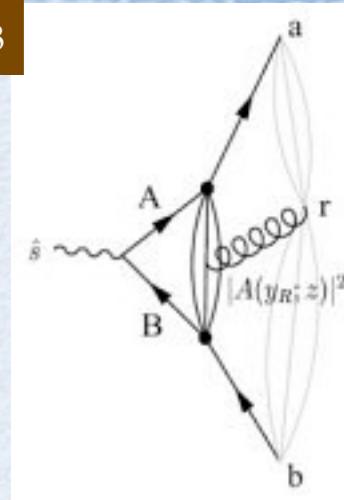


----- \* ) shows Global without any ordering condition imposed → overcounting

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- Dipole-antenna formalism ( $2 \rightarrow 3$ )



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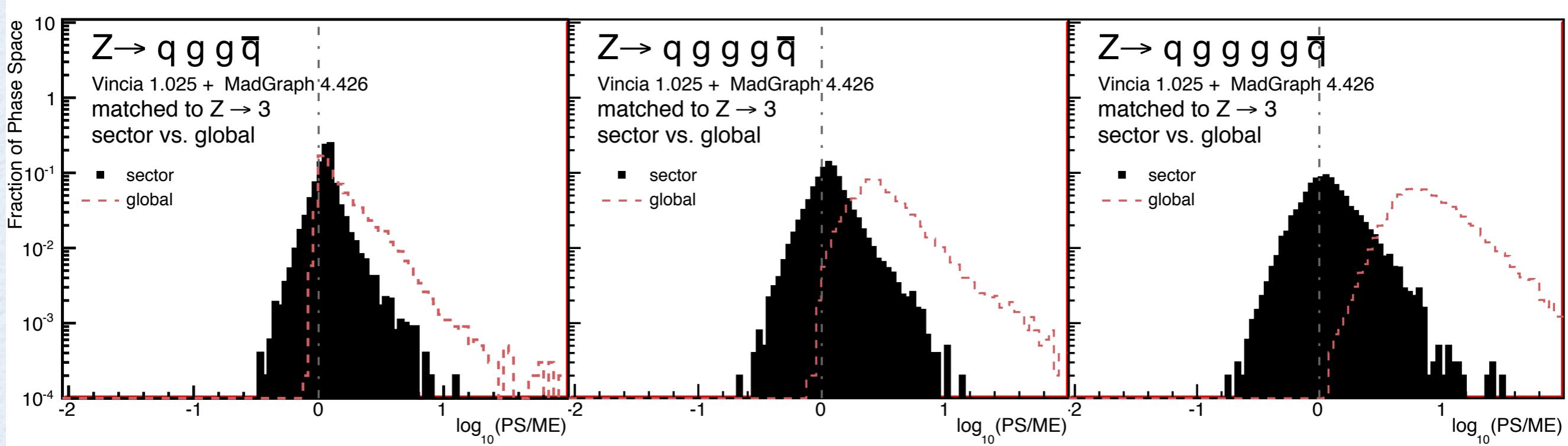
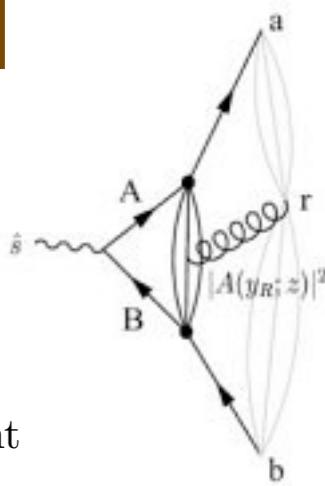
- Dipole-antenna formalism ( $2 \rightarrow 3$ )

- Two types: {
  - Global
  - Sector

$$|M^{(n)}|^2 \sim \sum_{i \in \text{clust.}} a_i |M_i^{(n-1)}|^2 \quad \text{for any P.S. point}$$

$$|M^{(n)}|^2 \sim \sum_{i \in \text{clust.}} \tilde{a}_i |M_i^{(n-1)}|^2 \quad \Theta_i(\text{P.S.}) \sim \tilde{a}_j |M_j^{(n-1)}|^2$$

for some clust. j



\*)shows Global *without* any ordering condition imposed  $\rightarrow$  overcounting

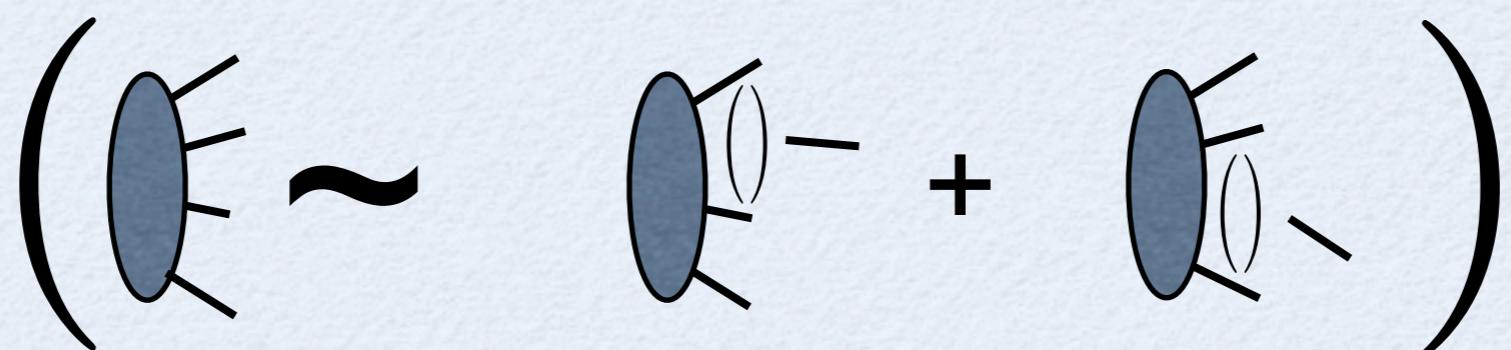
# NUMBER OF TERMS



## Global FSR shower (default VINCIA)

	"Traditional" parton shower	Vincia Markov global antenna shower	Vincia Markov sector antenna shower
# of terms produced in the shower	$2^N N!$	N	1

N = number of  
emitted partons



3 → 4  
2 terms per phase-space point

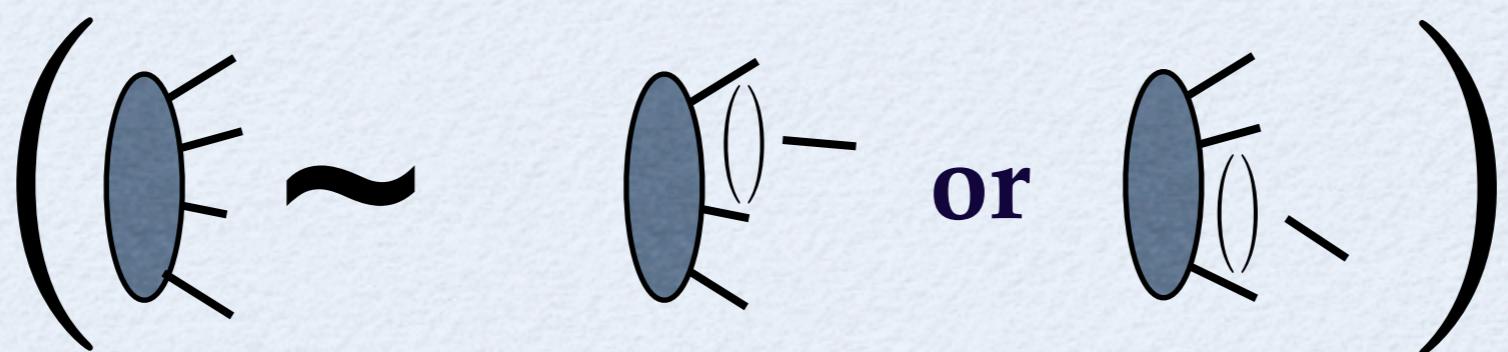
# NUMBER OF TERMS



→ Sector shower

	“Traditional” parton shower	Vincia Markov global antenna shower	Vincia Markov sector antenna shower
# of terms produced in the shower	$2^N N!$	$N$	1

$N$  = number of emitted partons



$3 \rightarrow 4$   
1 term per phase-space point

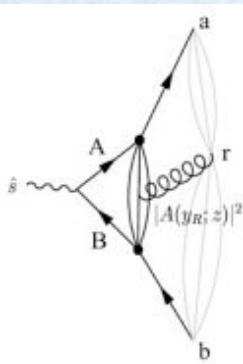
# SECTOR IMPLEMENTATION

# SECTOR IMPLEMENTATION

- Implementation based on the global shower setup.

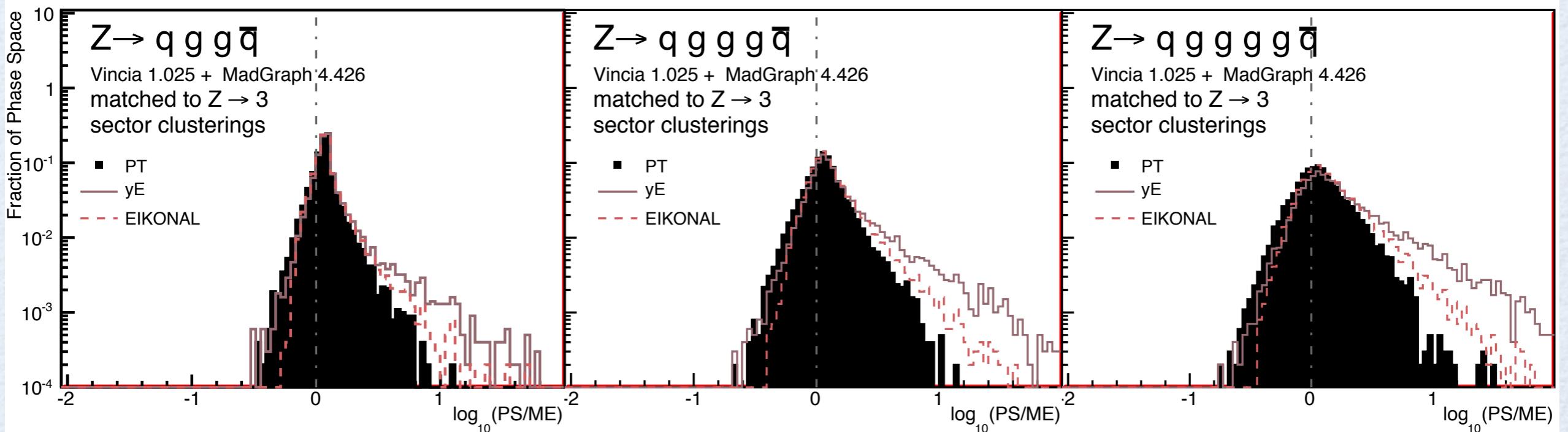
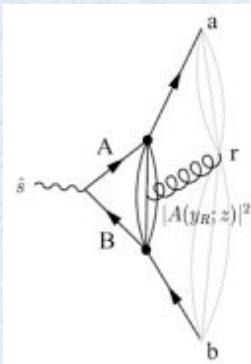
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- Implementation based on the global shower setup.
- Antenna functions are different than in the global case.  
→ Challenges (partitioning of collinear radiation singularities)



# SECTOR IMPLEMENTATION

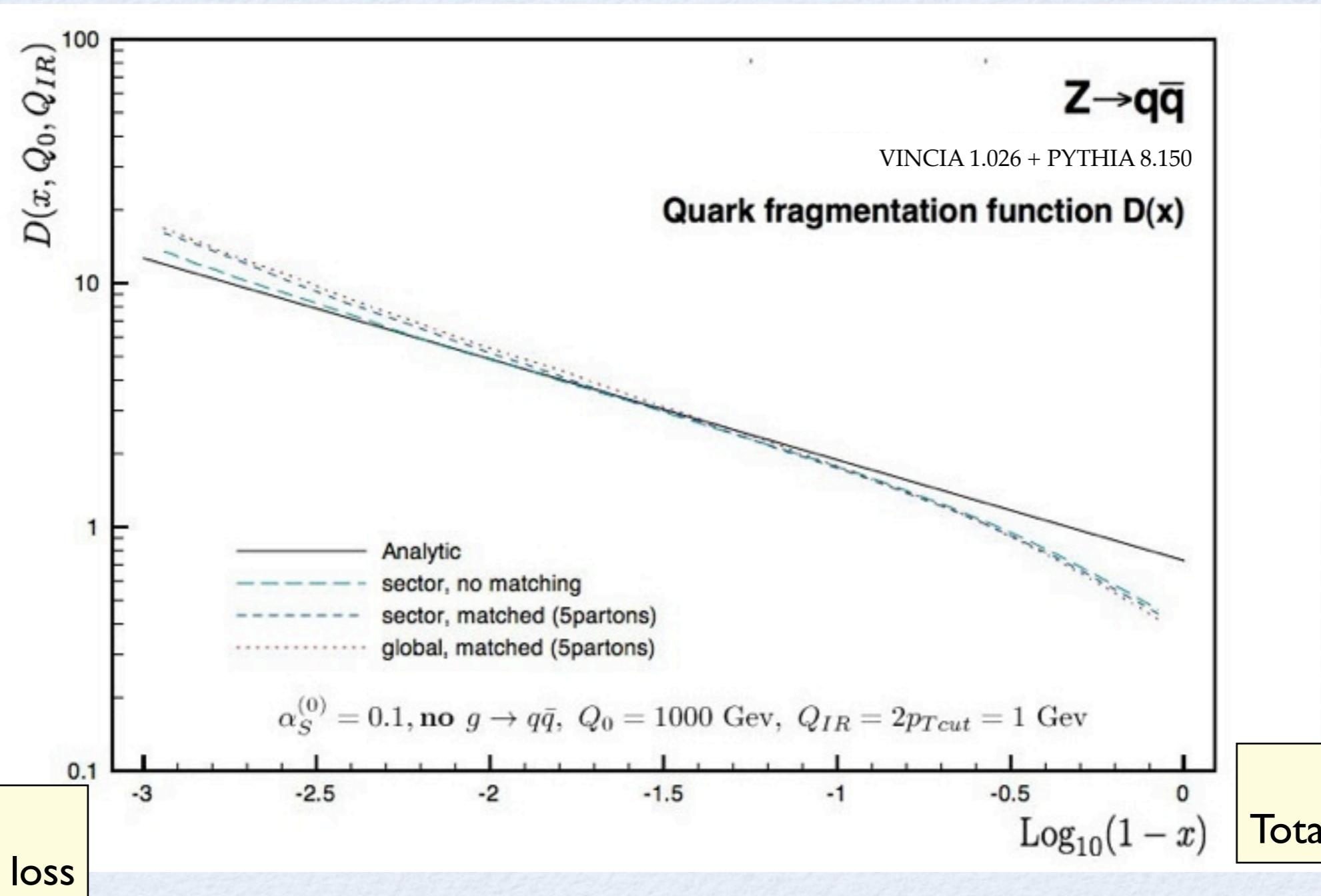
- Implementation based on the global shower setup.
- Antenna functions are different than in the global case.  
→ Challenges (partitioning of collinear radiation singularities)
- Different criteria for separating sectors in phase space  
Looking for “best” sub-LL behavior.



# RESULTS->FF

Skands, Weinzierl: Phys.Rev.D79 (2009) ; Nagy, Zoltan et al. JHEP 0905 (2009) 088

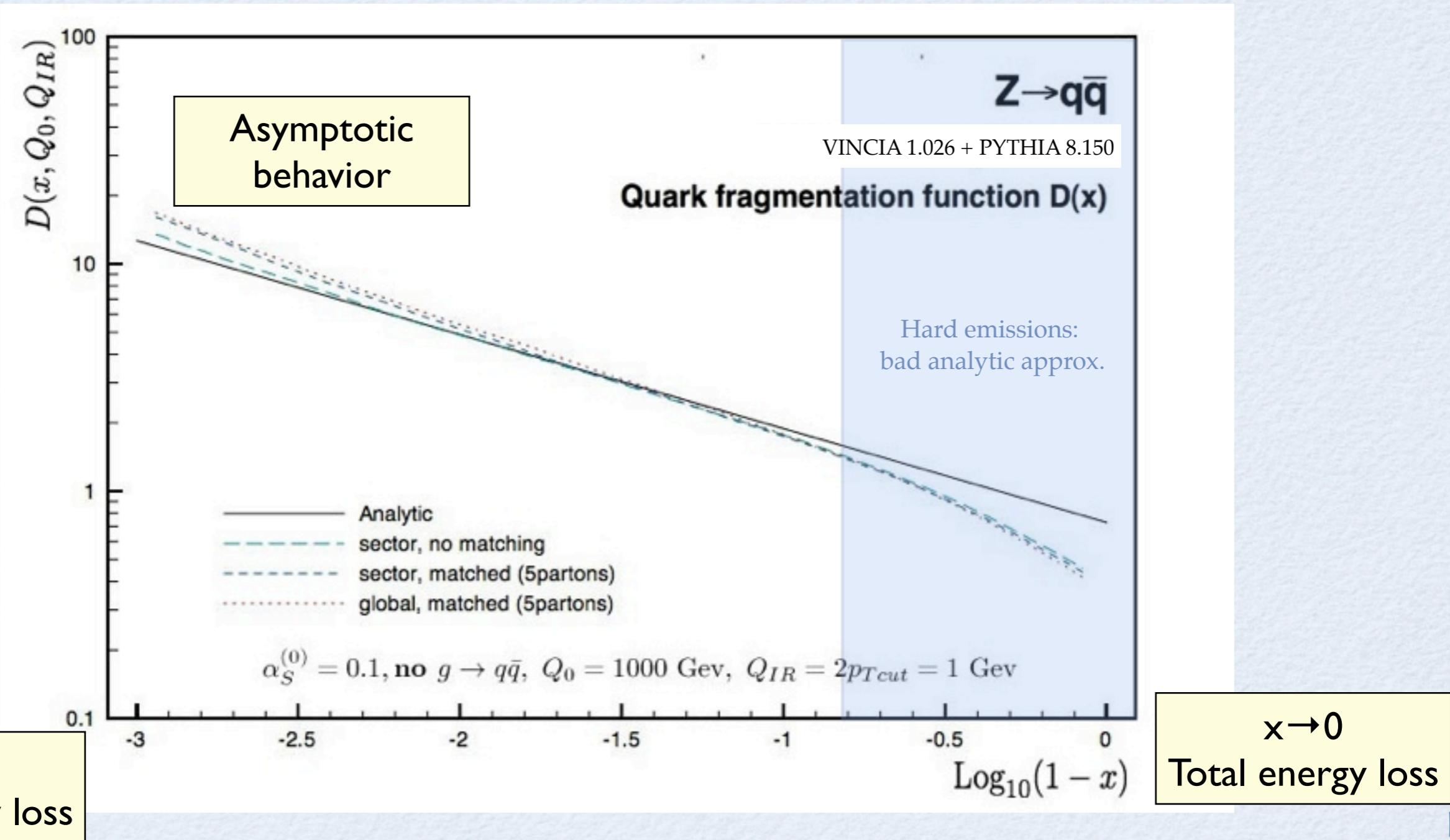
## Test: fragmentation function for a quark



# RESULTS->FF

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## Test: fragmentation function for a quark



# RESULTS -> SPEED



<b>Matched through:</b>	Z→3	Z→4	Z→5	Z→6
<b>Pythia 6</b>	0.20	<b>ms/event</b>		
<b>Pythia 8</b>	0.22	Z→qq ( $q=udscb$ ) + shower. Matched and unweighted. Hadronization off gfortran/g++ with gcc v.4.4 -O2 on single 3.06 GHz processor with 4GB memory		
<b>Vincia Global</b>	0.30	0.77	6.40	130.00
<b>Vincia Sector</b>	0.27	0.63	6.90	52.00
<b>Vincia Global (<math>Q_{match} = 5 \text{ GeV}</math>)</b>	0.29	0.60	2.40	20.00
<b>Vincia Sector (<math>Q_{match} = 5 \text{ GeV}</math>)</b>	0.26	0.50	1.40	6.70
<b>Sherpa (<math>Q_{match} = 5 \text{ GeV}</math>)</b> * + initialization time	5.15*	53.00*	220.00*	400.00*
	1.5 minutes	7 minutes	22 minutes	2.2 hours
Generator Versions: Pythia 6.425 (Perugia 2011 tune), Pythia 8.150, Sherpa 1.3.0, Vincia 1.026 (without uncertainty bands, NLL/NLC=OFF)				

J.J. Lopez-Villarejo & Peter Z. Skands. "Efficient matrix-element matching with sector showers":

Arxiv soon ...

# RESULTS -> SPEED



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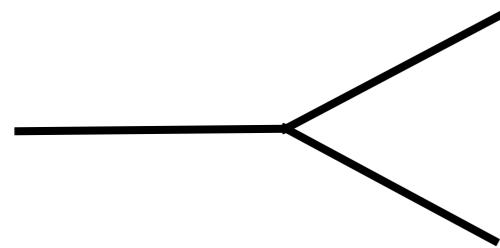
Next steps: ISR, polarization, NLO, faster MEs? (now using MadGraph), etc. . . .



# Backup Slides

# pQCD as Markov Chain

**Start from Born Level:**



$$\frac{d\sigma_H}{d\mathcal{O}} \Big|_{\text{Born}} = \int d\Phi_H \quad \begin{matrix} \xleftarrow{\text{Born-Level Phase Space}} \\ |M_H^{(0)}|^2 \end{matrix} \delta(\mathcal{O} - \mathcal{O}(\{p\}_H))$$

H = Arbitrary hard process

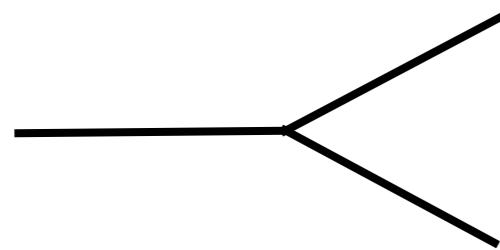
On-Shell Momentum Configuration

Born-Level Matrix Element

Born-Level Phase Space

# pQCD as Markov Chain

## Start from Born Level:

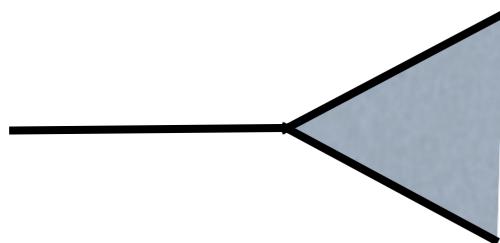


$$\frac{d\sigma_H}{d\mathcal{O}} \Big|_{\text{Born}} = \int d\Phi_H \quad |M_H^{(0)}|^2 \delta(\mathcal{O} - \mathcal{O}(\{p\}_H))$$

Born-Level Phase Space  
Born-Level Matrix Element  
On-Shell Momentum Configuration

H = Arbitrary hard process

## Insert Evolution Operator, S:



$$\frac{d\sigma_H}{d\mathcal{O}} \Big|_S = \int d\Phi_H \quad |M_H^{(0)}|^2 \mathcal{S}(\{p\}_H, \mathcal{O})$$

Evolution operator

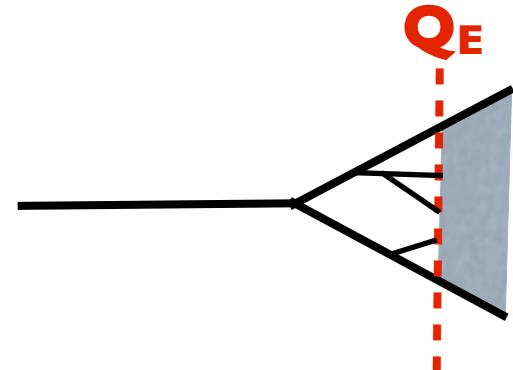
Think: starting a shower off an incoming on-shell momentum configuration  
Postpone evaluating observable until shower “finished”

# The Evolution Operator

**Depends on Evolution Scale :  $Q_E$**

$$\mathcal{S}(\{p\}_H, s, Q_E^2, \mathcal{O}) = \underbrace{\Delta(\{p\}_H, s, Q_E^2) \delta(\mathcal{O} - \mathcal{O}(\{p\}_H))}_{H+0 \text{ exclusive above } Q_E}$$

$$+ \sum_r \underbrace{\int_{Q_E^2}^s \frac{d\Phi_{H+1}^{[r]}}{d\Phi_H} S_r \Delta(\{p\}_H, s, Q_{H+1}^2)}_{\substack{\text{Sum over} \\ \text{radiators}}} \underbrace{\Delta(\{p\}_H, s, Q_{H+1}^2) \mathcal{S}(\{p\}_{H+1}, Q_{H+1}^2, Q_E^2, \mathcal{O})}_{\substack{\text{“Corrected” Radiation Functions} \\ \text{Exact} \\ \text{Phase Space} \\ \text{Factorization}}} \underbrace{\text{Continue Markov Chain off } H+1}_{H+1 \text{ inclusive above } Q_E}$$



**Legend:**

$\Delta$  represents *no-evolution probability (Sudakov)*: conserves probability = preserves event weights

$S_r$  = Emission probability (partitioned among radiators r)

According to best known approximation to  $|H+I|^2$  (e.g., ME or LL shower)

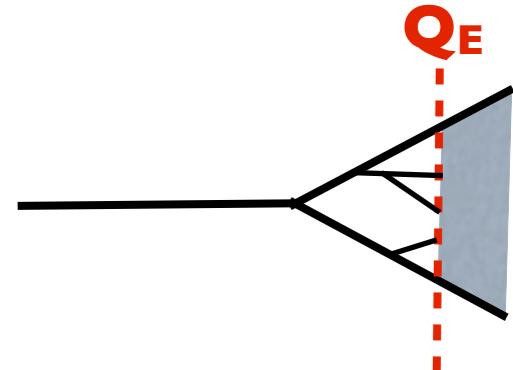
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Sum over radiators No-evolution Probability  
 $\Delta(\{p\}_H, s, Q_E^2)$   
 $\delta(\mathcal{O} - \mathcal{O}(\{p\}_H))$   
 $\int_{Q_E^2}^s \frac{d\Phi_{H+1}^{[r]}}{d\Phi_H}$   
 $S_r$   
 $\Delta(\{p\}_H, s, Q_{H+1}^2)$   
 $\mathcal{S}(\{p\}_{H+1}, Q_{H+1}^2, Q_E^2, \mathcal{O})$



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# (Expand S to First Order)

## Equivalent to Sjöstrand/POWHEG

$$\mathcal{S}^{(1)}(\{p\}_H, s, Q_E^2, \mathcal{O}) = \left( 1 + K_H^{(1)} - \int_{Q_E^2}^s \frac{d\Phi_{H+1}}{d\Phi_H} \frac{|M_{H+1}^{(0)}|^2}{|M_H^{(0)}|^2} \right) \delta(\mathcal{O} - \mathcal{O}(\{p\}_H))$$

↑ Unitarity

$$+ \int_{Q_E^2}^s \frac{d\Phi_{H+1}}{d\Phi_H} \frac{|M_{H+1}^{(0)}|^2}{|M_H^{(0)}|^2} \delta(\mathcal{O} - \mathcal{O}(\{p\}_{H+1})) .$$

↑ Sudakov Expansion  
↑ “NLO” virtual correction  
↑ Torbjörn’s trick

## Virtual Correction (NLO normalization)

$$\frac{2\text{Re}[M_H^{(0)} M_H^{(1)*}]}{|M_H^{(0)}|^2} = K_H^{(1)} - \int_0^s \frac{d\Phi_{H+1}}{d\Phi_H} \frac{|M_{H+1}^{(0)}|^2}{|M_H^{(0)}|^2}$$

$\overbrace{\frac{a}{\epsilon^2} + \frac{b}{\epsilon} + c + \mathcal{O}(\epsilon)}$        $c' \uparrow \downarrow$        $\overbrace{\frac{a}{\epsilon^2} + \frac{b}{\epsilon} + c' + \mathcal{O}(\epsilon)}$

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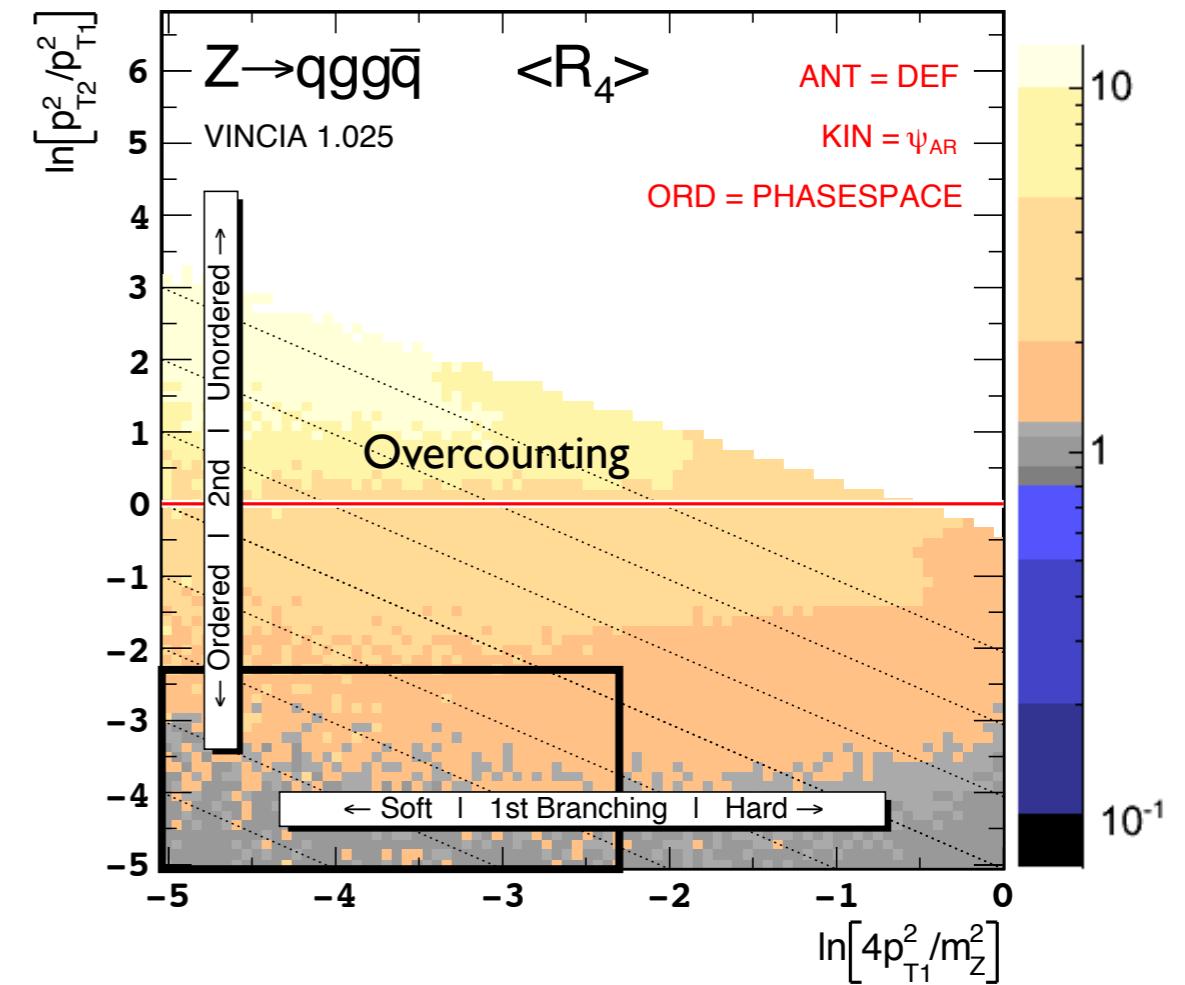
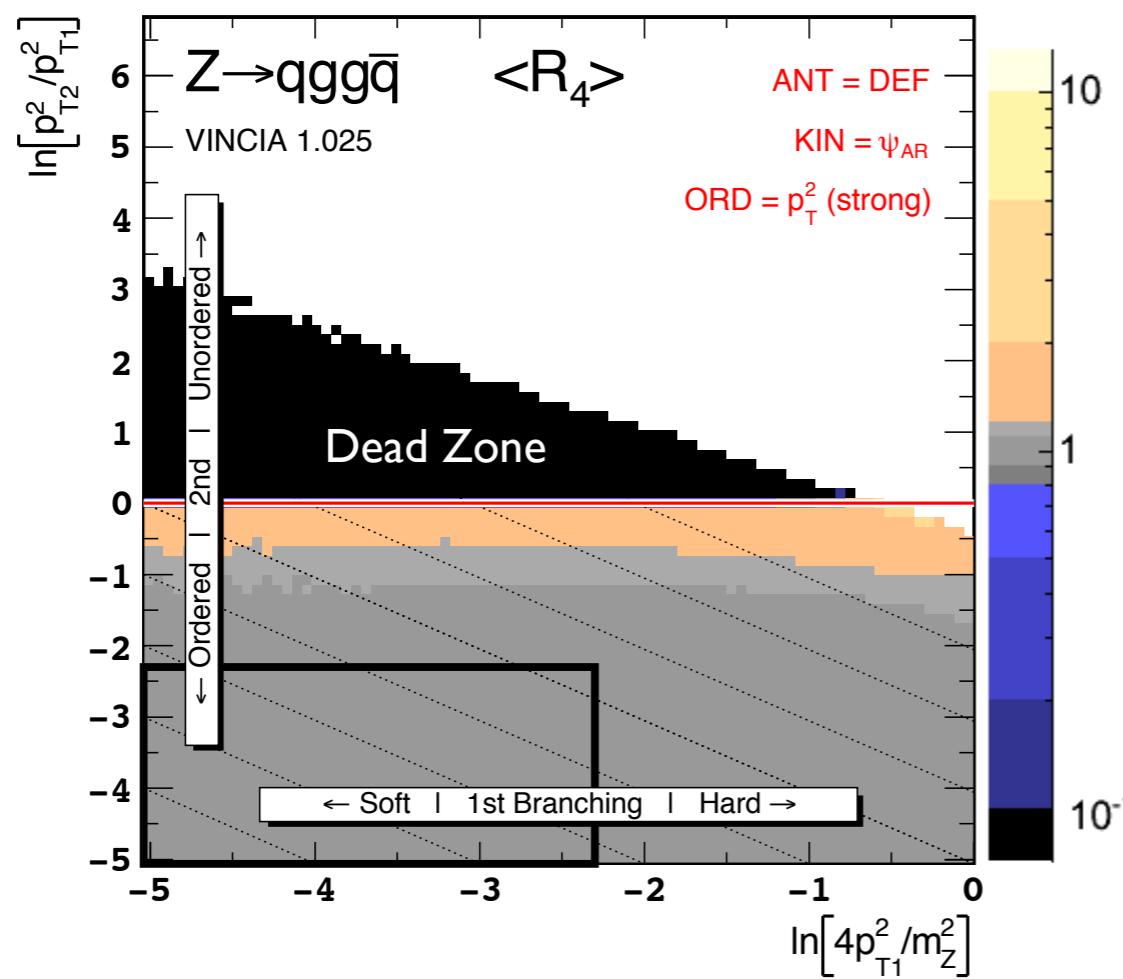
# Simple Solution

## Generate Trials **without** imposing strong ordering

At each step, each dipole allowed to fill its entire phase space

*Overcounting removed by matching*

*(revert to strong ordering beyond matched multiplicities)*

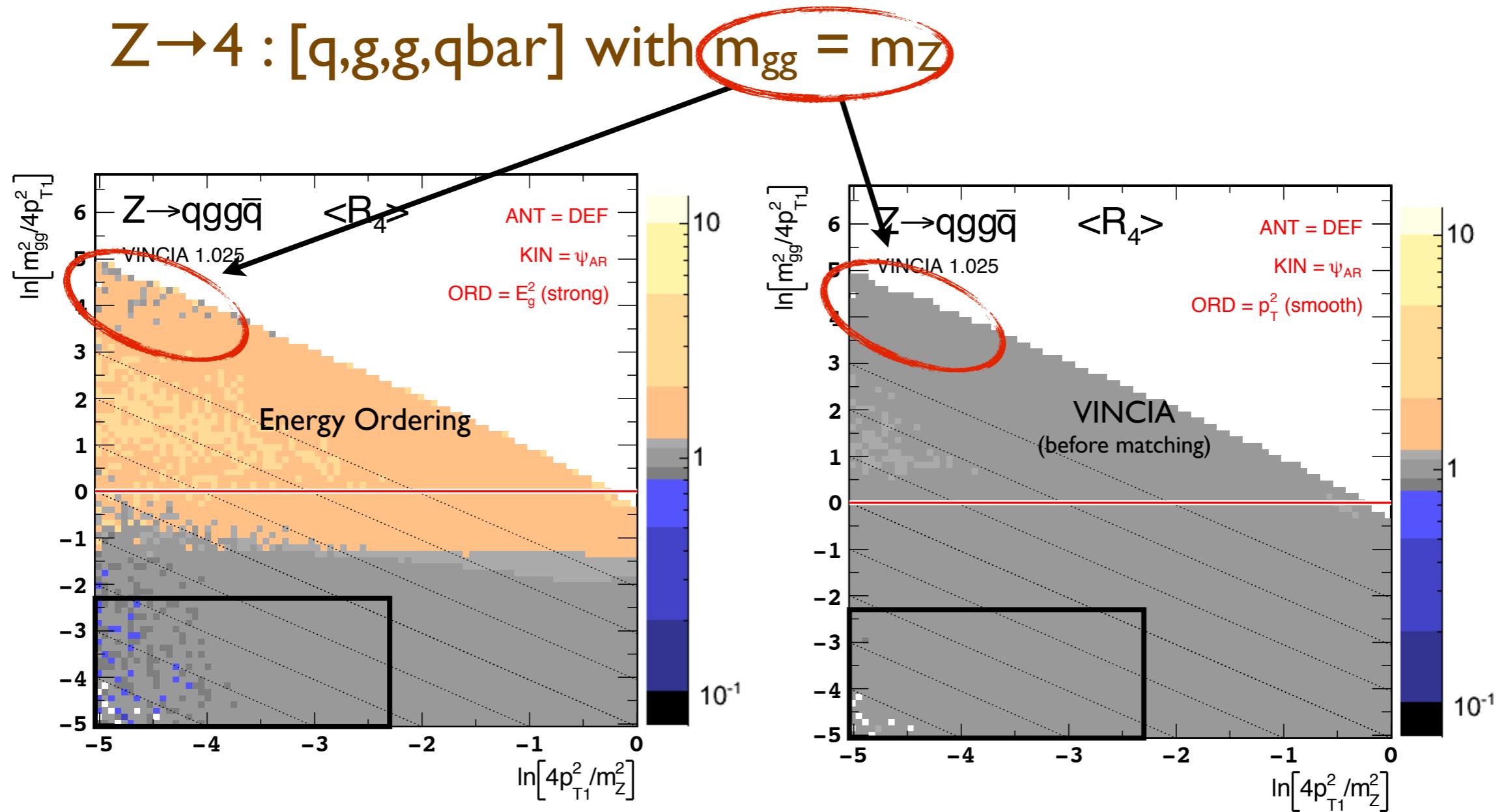


# (Subleading Singularities)

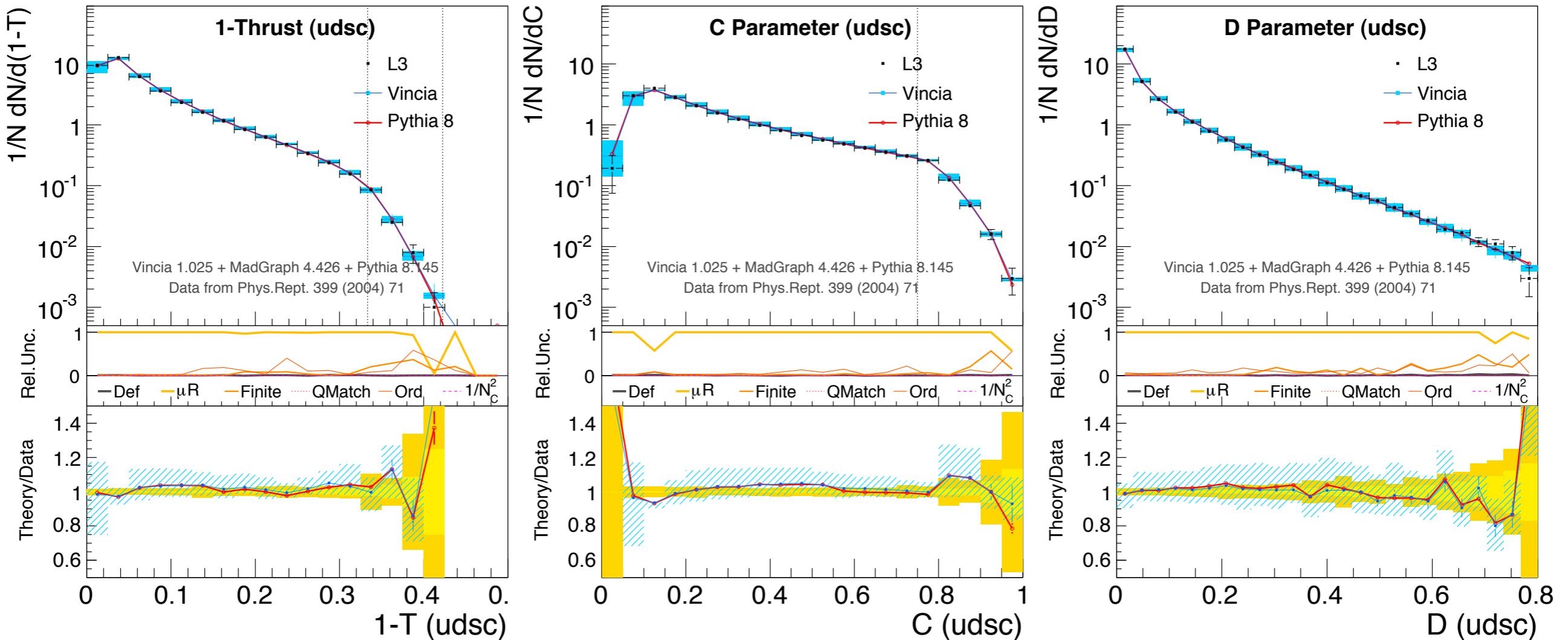
**Isolate double-collinear region:**

$$\alpha_s^2 \ln^2$$

$Z \rightarrow 4 : [q, g, g, q\bar{q}]$  with  $m_{gg} = m_Z$

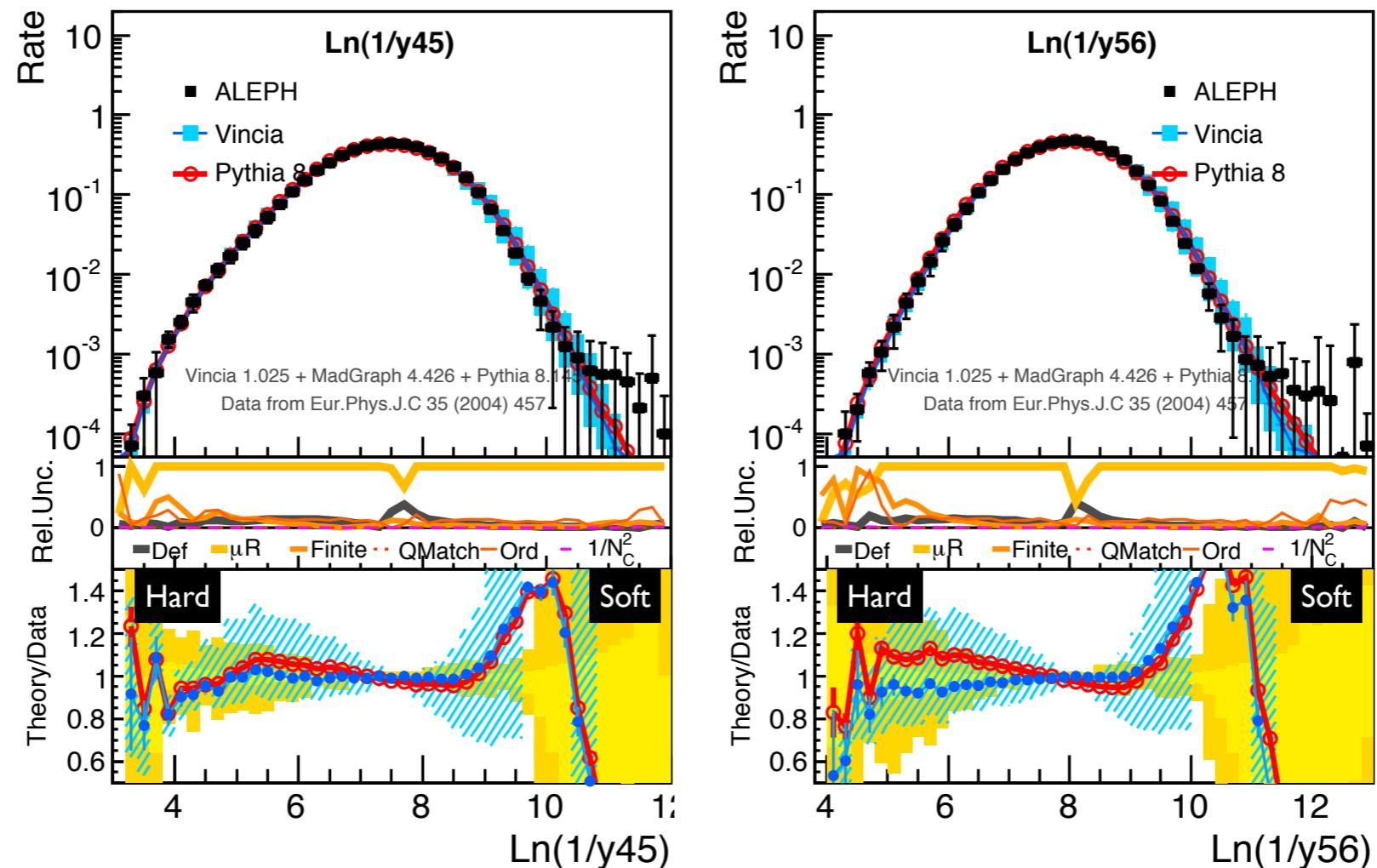


# LEP event shapes



PYTHIA 8 already doing a very good job  
 VINCIA adds uncertainty bands + can look at more exclusive observables?

# Multijet resolution scales



$y_{45}$  = scale at which 5<sup>th</sup> jet becomes resolved ~ “scale of 5<sup>th</sup> jet”

# 4-Jet Angles

## 4-jet angles

Sensitive to  
polarization effects

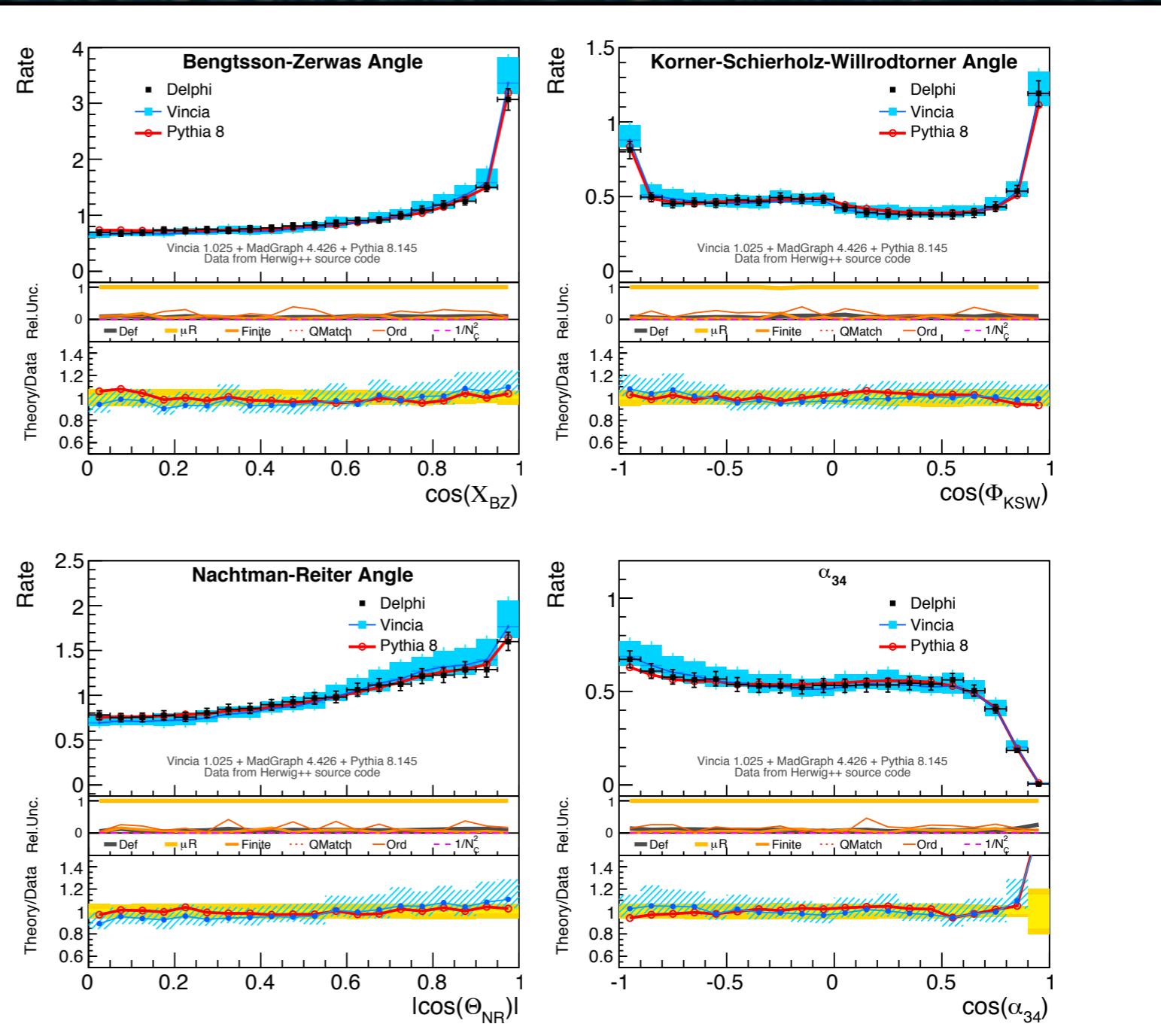
## Good News

VINCIA is doing  
reliably well

Non-trivial verification  
that shower+matching  
is working, etc.

## Higher-order matching needed?

PYTHIA 8 already  
doing a very good job  
on these observables



Interesting to look at more exclusive observables, but which ones?